Decaying of a Scalar Particle

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Consider the Following Lagrangian involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \mu\Phi\phi\phi$$

The last term allows a Φ particle to decay into two ϕ's provided that M¿2m. Assuming this is true, calculate the lifetime of Φ in the lowest order of μ.

1 Attempt

We know that the particle *tau*'s lifetime is the reciprocal sum of its decay rates into all possible final states.

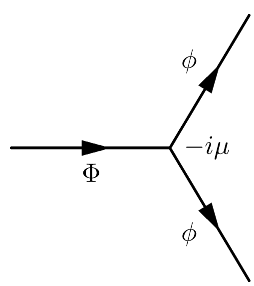

Figure 1: Feynman Diagram of the Decay

Using the standard cross-section formula:

$$d\sigma = \frac{1}{4E_{\mathcal{A}}E_{\mathcal{B}}|v_{\mathcal{A}} - v_{\mathcal{B}}|}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \times |\mathcal{M}|^2(2\pi)^4\delta^{(4)}(P - \sum p_f) \quad (1)$$

If we just consider a single initial particle, and work in the frame of decaying particle Φ, we get the expression:

$$d\Gamma = \frac{1}{2m_\Phi}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \times |\mathcal{M}(m_\Phi \to \{p_f\}^2)|^2(2\pi)^4\delta^{(4)}(P_\Phi - \sum p_f) \quad (2)$$

Evaluating this in term of $\phi(p_i)$ for i =1,2, and plugging everything in, we get that:

$$\int d\Gamma = \frac{1}{2m_\Phi}\int \frac{d^3p_1 d^3p_2}{(2\pi)^6}\frac{1}{4E_1E_2}\times|\mathcal{M}(\Phi(0)\to\phi(p_1)\phi(p2))|^2(2\pi)^4\delta^{(4)}(p_\Phi - p_1 - p_2) \tag{3}$$

Now looking at our Lagrangian, we see that at the Vertex, the lowest order will give us a term of $-i\mu$, and when we consider the combinatorial factor due to the different ways we can contract the fields, we get that the contribution to the amplitude will be

$$-2i\mu = i\mathcal{M} = -i\mu\langle p_1p_2|\int d^4xT\{\Phi\phi(p1)\phi(p2)\}|p_\Phi\rangle$$

Seperating the 3 momentum from energy indices of the delta function, we get that:

$$\Gamma = \frac{1}{2m_\Phi}\int \frac{d^3p_1 d^3p_2}{(2\pi)^6}\frac{1}{4E_1E_2}\times(4\mu^2)(2\pi)^4\delta(m_\Phi - E_1 - E_2)\delta^{(3)}(\mathbf{p_1}+\mathbf{p_2}) \tag{4}$$

Hence, by the constraint imposed by the delta function: $\mathbf{p_1} = -\mathbf{p_2}$ and $p_1^2 = p_2^2 = (\frac{m_\Phi}{2})^2 - m_\phi^2$, where the M/2 combinatorial factor once again arises from the possible field contractions.

Plugging these in equation 4, we get that:

$$\Gamma = \frac{\mu^2}{m_\Phi}\int \frac{d^3p_1}{(2\pi)^2}\frac{1}{4(m^2+p_1^2)}\delta(m_\phi - 2E_1) \tag{5}$$

Evaluating this gives us our final Decay Rate:

$$\Gamma = \frac{\mu^2}{8\pi m_\Phi}(1-\frac{4m_\phi^2}{m_\Phi^2})^{1/2} \tag{6}$$

Now using the fact that $\tau = \Gamma^{-1}$, we get that:

$$\tau = \frac{8\pi m_\Phi}{\mu^2}(1-\frac{4m_\phi^2}{m_\Phi^2})^{-1/2}$$