PARIS: A PARALLEL RSA-PRIME INSPECTION TOOL

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Abstract

PARIS: A PArallel RSA-prime InSpection tool

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Modern-day computer security relies heavily on cryptography as a means to protect the data that we have become increasingly reliant on. As the Internet becomes more ubiquitous, methods of security must be better than ever. Validation tools can be leveraged to help increase our confidence and accountability for methods we employ to secure our systems.

Security validation, however, can be difficult and time-consuming. As our computational ability increases, calculations that were once considered “hard” due to length of computation, can now be done in minutes. We are constantly increasing the size of our keys and attempting to make computations harder to protect our information. This increase in “cracking” difficulty often has the unfortunate side-effect of making validation equally as difficult.

We can leverage massive-parallelism and the computational power that is granted by today’s commodity hardware such as GPUs to make checks that would otherwise be impossible to perform, attainable. Our work presents a practical tool for validating RSA keys for poor prime numbers: a fundamental problem that has led to significant security holes.

Our implementation using NVIDIA’s CUDA framework offers a 27.5 times speedup over a reference sequential implementation. This level of speedup brings this validation into the realm of runtime reachability.

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Background

RSA is an asymmetric key encryption scheme. Keys come in matched pairs: public keys and private keys. Using the same algorithm, information encrypted with the one key, can be decrypted with the other and vice versa. A party must keep the private keep secret, but the public portion of the key can be seen and used by anyone in the world. To generate a pair, an algorithm is performed on two randomly-generated prime numbers whose product is of a certain bit-length (e.g. 1024 bits). These are the primes that our work looks to discover inadequacies with.

The work presented by Lenstra et. al. [10] shows that around 0.2% of RSA keys collected from sources including the SSL Observatory suffer from inadequate random prime generation. They show that when the primes used to build an RSA public-private key pair are not actually random, factoring does not need to be done to break the keys. Instead, only a greater common divisor needs to be calculated between two RSA moduli. When anything greater than 1 is found as a result of this calculation, the keys can be considered broken and offer no protection. This is because once a GCD greater than 1 is found, the private keys

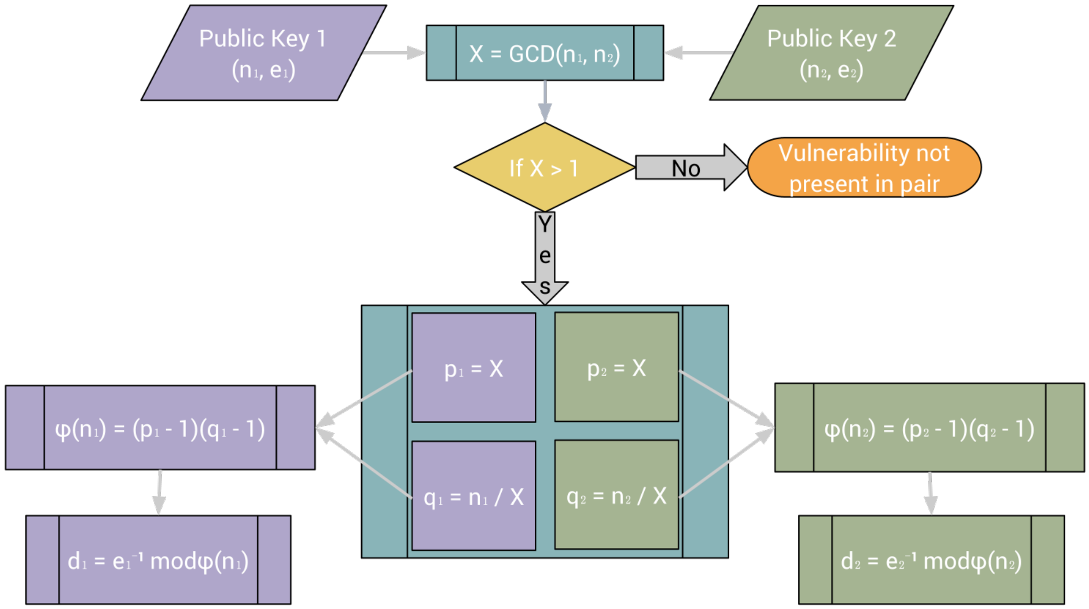

Public Key 1
(n_1, e_1)
$X = GCD(n_1, n_2)$
Public Key 2
(n_2, e_2)
If $X > 1$
No
Vulnerability not present in pair
Yes
$p_1 = X$
$p_2 = X$
$q_1 = n_1 / X$
$q_2 = n_2 / X$
$\varphi(n_1) = (p_1 - 1)(q_1 - 1)$
$\varphi(n_2) = (p_2 - 1)(q_2 - 1)$
$d_1 = e_1^{-1} \mod \varphi(n_1)$
$d_2 = e_2^{-1} \mod \varphi(n_2)$

Figure 2.1. Explanation of poor-prime vulnerability

can be generated from the publicly available information. The vulnerability is explained in detail in Figure 2.1.

A tool by Scharfglass et. al. [14] performs a parallelized version of this GCD calculation (using the binary GCD algorithm) between every pair combination in a given set of keys. It structures these calculations so that as many as possible can be done at once. To increase parallelism, and because the keys are so large, they are broken up into 32 chunks, which are also operated on in parallel using the GPU. Through these 2 levels of parallelism, a speedup of 27.5 is achieved. This tool provides a practical tool for checking a given set of keys for the vulnerability we are focusing on.

2.1 Motivation

Scharfglass et. al. use NVIDIA's CUDA framework to significantly speedup a check for the vulnerability presented in [10]. The work in [14] lacks any context or explanation of usage for RSA keys in general, and although their data set is security- and encryption-focused, no discussion is given to the security aspects and the potential impacts of the work. Some research was done in an attempt to add context to the work, and the implications it might have.

Most initial research yielded general statements that did not explain or give appropriate context to RSA (statements including "...all over the Internet..." were accurate, but not informative). More in-depth research yielded two primary use-cases of RSA: Digital Rights Management (DRM) and Secure Socket Layer (SSL) and Transport Layer Security (TLS) Internet security protocols. Other uses included password alternatives (e.g. ssh connections or command line interface tools like `git`) but these are more difficult to collect data for and analyze since they tend to be done on an individual basis, and are managed by individual users.

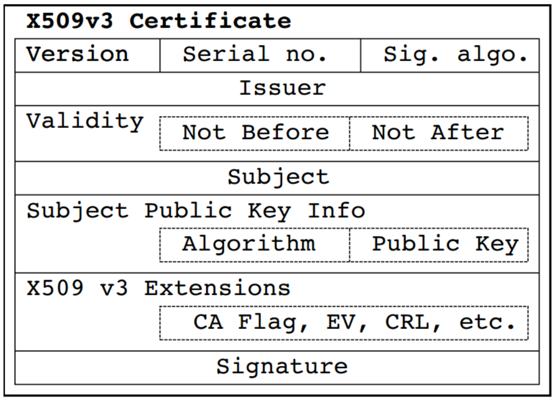
2.1.1 Digital Rights Management

Digital rights management is a protocol used to secure the usage and distribution of various types of media content. It is adopted by content providers and device manufacturers to ensure users don't misuse or wrongfully share protected, copyrighted content. The RSA Association proposed a protocol that could be applied to various types of media content that secured its usage [1, 2]. These announcements argued that using RSA for DRM offered benefits to the content providers, the device manufacturers, and, arguably, the consumers of the content. It apparently allowed device manufactures and content providers to ensure proper usage of protected content while allowing users to consume and playback their rightfully-owned media on an array of their own devices.

2.1.2 Internet Security and Certificates

Internet security relies on a particular protocol called SSL and its successor TLS. These protocols define how to securely transfer information over the Internet by using encryption and signing mechanisms. One aspect of both the SSL and TLS protocols involve certificates to ensure the expected party is actually being communicated with. These certificates provide information about a server that a user is connected to (e.g. Amazon or Google) and is signed by a certificate authority (e.g. VeriSign). These certificates are also encrypted with a subject's (e.g. Amazon) public key to ensure that the entity is who they say they are. Furthermore, the public key is then used to transfer information back to the subject in a secure way. Figure 2.3 describes the TLS process between a client and server.

The most widely used type of certificate is the X.509 certificate, whose current revision is 3. A breakdown of the major sections of the certificate is shown in Figure 2.2. The "Subject Public Key Information" section of the certificate holds the relevant data (the subject's RSA public key) for our work. The public key portion of the certificates can actually use one of several algorithms defined in the specification. RSA is by far the most popular, accounting for over half of the found certificates (as shown in Figure 2.4). Other options include DSA and Diffie-Hellman. For more detailed information about X.509 certificates and analysis on their infrastructure, see [8].



X509v3 Certificate

Version | Serial no. | Sig. algo.

Issuer

Validity: Not Before | Not After

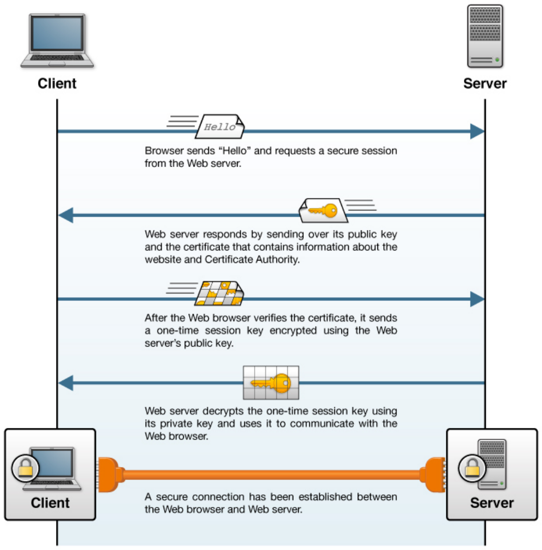
Subject

Subject Public Key Info: Algorithm | Public Key

X509 v3 Extensions: CA Flag, EV, CRL, etc.

Signature

Figure 2.2. X.509 Fields[8]



Browser sends "Hello" and requests a secure session from the Web server.

Web server responds by sending over its public key and the certificate that contains information about the website and Certificate Authority.

After the Web browser verifies the certificate, it sends a one-time session key encrypted using the Web server's public key.

Web server decrypts the one-time session key using its private key and uses it to communicate with the Web browser.

A secure connection has been established between the Web browser and Web server.

Figure 2.3. TLS Overview[3]

2.1.3 Data Collection

The proposed protocol given by the RSA actually modeled the certificate model used by SSL/TLS. For this reason, the SSL certificate model was the primary focus of the data collected. Moreover, the data set of RSA keys in SSL certificates is much larger than DRM implementations that use RSA. Thus, the data set that was collected and then analyzed consisted entirely of X.509 certificates.

Since a primary motivation of this work is the potential impact that vulnerable RSA keys may have, a data set representative of a large portion of traffic was desired. A survey was done over 2007-2010 [9] that gathered some useful data on the Internet as a whole. Relevant to web traffic, this study showed that together, the major content data networks (CDN) (i.e. Google, Facebook, Amazon, etc.) account for nearly 17% of web traffic. Specific breakdowns and conversions can be found in Table 2.1.

Alexa was used to find websites with large amounts of traffic[1], as it serves as one of the Internet's most prevalent sources for traffic information. In addition to individualized site traffic data, Alexa provides a daily-updated list of the top one million websites ordered by traffic[2]. In regards to the previous point concerning CDNs and traffic breakdowns, it was noted that all web sites hosted by the major CDNs from [9] are present in the top one million list. Additionally, the vast majority of sites from the top one million list were not provided by the CDNs mentioned in [9]. This means that a much larger percentage than 17% is actually represented by the sites in the list, although quantifying precisely how much becomes difficult, and lies beyond the scope of this project.

[1] http://www.alexa.com/

[2] http://s3.amazonaws.com/alexa-static/top-1m.csv.zip

Table 2.1. Major CDN Traffic (2010)

CDN	**Percent of all Internet traffic**	**Approx. percentage of web traffic**
Google	7	12.72
Facebook	0.45	0.818
Amazon CDN (NASA/JPL, PBS,. . .)	0.55	1
EdgeCast (Yahoo!, Break, Imgur,. . .)	0.5	0.909
LeaseWeb (Heineken, Starbucks,. . .)	0.8	1.454

A Python script was used to parse the CSV file provided by Alexa. Each extracted URL was visited on port 443 (corresponding to "https://") using `openssl`. If a site responded on that port, it provided an X.509 certificate to be parsed and verified. Most of the time, this is done by a user's web browser which has certificate authorities' private keys hidden within. However, since the work here was not concerned with the actual web content, the certificate was simply saved. When certificates were found, the "Subject Public Key Algorithm" section was inspected. If this contained some form of RSA, `openssl` was used again to extract the RSA key and save it into a PEM file (a standard format for saving public and private key information in). Additionally, since part of this work's motivation lies in expanding the work done in [14], the keys were also stored into a SQLite3 database so that later retrieval by the tool was trivial.

After writing an initial implementation of the Python script, it was realized that the collection process was too slow. Therefore, some of Python's multiprocess capabilities were utilized to speed up the collection of data. Eight processes were spawned to perform the work described above. Seven of the processes read URLs from a deque (double-ended queue) and attempted to obtain an X.509 certificate and RSA key from each. When found, these keys were added to a queue. These data structures not only offered convenient structures for organizing the data, but were also thread-safe and could be used with our updated implementation. An eighth thread read keys from the queue and entered them into the SQLite3 database. This reimplementation offered a 15× speedup over the previous, naïve approach.

2.1.4 Data Analysis

After collecting data from each of the top one million Alexa sites, some surface-level analysis was performed in order to gain an overview of the security of these one million sites. This analysis included overall security breakdowns (i.e. whether a site used any kind of security) and bit-length breakdowns of the RSA keys that were found. This data analysis can be found in Figures 2.4 and 2.5. Table 2.2 offer specific breakdown for bit-lengths, and offers details that cannot be seen in the figure.

Interestingly (and slightly worrisome), is that the majority of web sites in this top one million list did not even respond on port 443, implying that they do not allow any option for secure traffic to their site. Nearly 60% of websites in the list fell into this category.

Also of note are the specifics of bit-lengths found. It is a surprising data set for various reasons. First, bit-lengths as small as 384, 512, and 768 were not

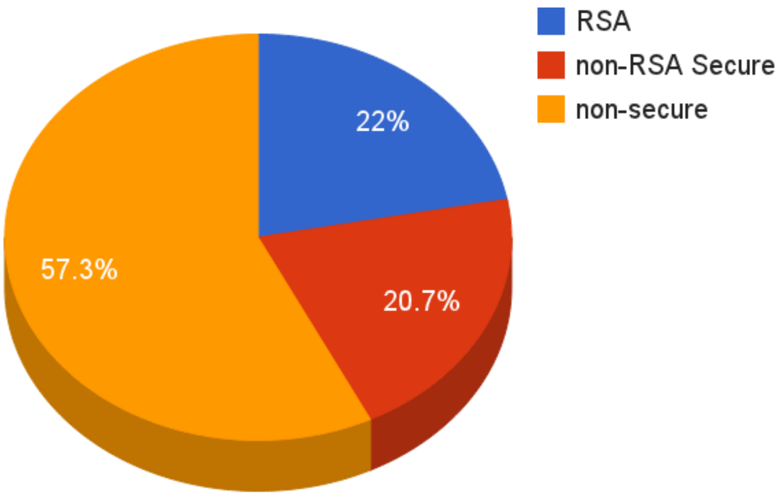


Figure 2.4. Presence of RSA key or other security in Alexa top 1M sites

expected as keys of this size were factored years ago [4], and cannot be expected to offer much security[3].

On the other hand, the high number of longer bit-lengths was also surprising, but comforting. The fact that the vast majority of keys were 2048-bit was not expected, but means that a large number of high-traffic websites have adequate (for now) encryption. Moreover, there were quite a number of sites with 4096, and, even more surprisingly, 8192-bit.

Further analysis was done using the tool described in [14]. This tool performs a complete check of all RSA keys in a given set for the previously mentioned vulnerability described in [10]. This check is done using NVIDIA's CUDA framework to parallelize the work as much as possible which allows for a practical security validation tool. At the time of writing, the tool's current implementation is limited to 1024-bit RSA keys. Due to this limitation, the data set was reduced to 46,736 RSA keys. Due to the $O(n^2)$ nature of this problem, this check resulted

[3] Interesting note: The author attempted to visit the sites 384-bit keys. Google Chrome did not find a web page on port 443 for any of the sites.

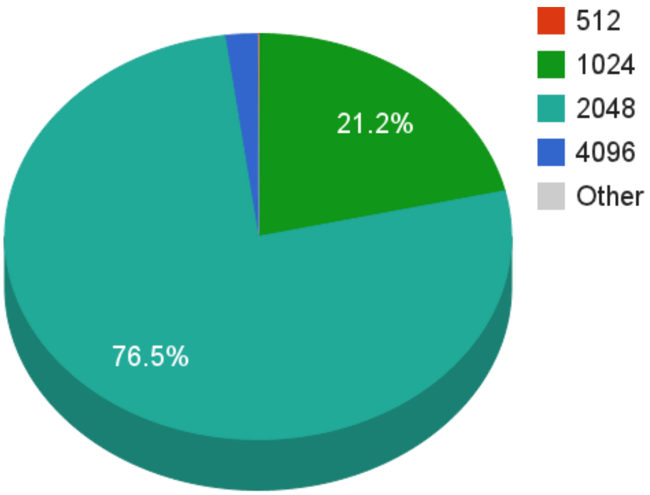


Figure 2.5. Breakdown of Alexa top 1M sites' RSA keys by bit-length

Table 2.2. Alexa top 1M X.509 RSA bit-length

Bits	**Count**
384	4
512	309
768	4
1024	46736
2048	168391
2432	37
3072	17
4096	4511
8192	14
other	44

in $1,092,150,216$ comparisons. The run of the tool using this data set took 138 minutes, 37 seconds or $131,305$ comparisons per second. After applying the tool to this set of keys, it was found that no keys were susceptible to the vulnerability (insofar as none of the keys in the set shared primes with a given key). Although no vulnerabilities were detected, this does not mean the set is completely secure from the previously mentioned vulnerability. Since such a limited data set was used, and the security tool relies on large sets to accumulate many primes, the analysis presented can only be considered an initial pass.

2.1.5 Implications

For the same reasons the data set was chosen to be X.509 certificates, presence of the vulnerability mentioned here would have the largest effect on Internet security: specifically in the certificate infrastructure. If servers were unknowingly providing compromised public keys in their certificates, their traffic could offer no security if this became known. This could offer ill-effects since the vulnerability is agnostic of the type of data being being encrypted.

Additionally, the password-alternatives that were mentioned briefly before could be particularly impacted. If a system was generating many new RSA keys (e.g. a user is building new accounts for various websites, and tying RSA keys to them) with inadequate primes, potentially all these accounts for this user could end up compromised, despite the sense of stronger security.

Along these lines, a system administrator could potentially suffer from similar issues. If many machines and key-pairs are being setup and generated as part of an initial setup process for a new network of users, poor random number generation could have severe consequences when done at scale. This would likely create many new, vulnerable keys at once. By being aware that this problem could exist, and that a tool such as [14] exists, an insecure set of users could be avoided or detected much earlier than would have otherwise occurred.

2.1.6 Conclusions

This survey determined that the primary use of RSA keys in practice is the signing of SSL certificates. A data set was gathered that accounts for a significant amount of certificates in the context of web traffic. Some surface-level analysis was done to gain a general understanding of the collected data. A high-performance tool was then applied to a subset of the data to check for a vulnerability within the data subset. No vulnerabilities were found; however, due to limitations in the tool used, as well as limitations in the data set, this cannot be considered a complete analysis and requires further investigation.

Though limited, this survey does successfully add real-world application context to the work done in [14]. Specific implications are able to be draw from the practicality of the tool, and more tangible examples of what vulnerabilities in RSA keys looks like have been realized.

Chapter 3

Related Works

3.1 The Vulnerability

An RSA vulnerability was discovered and detailed in [10] in early 2012 that forms the basis of the exploit the work presented here builds upon. Namely, that poor random number generation in RSA keys results in insecure encryption using these keys. Specifically, a key's private components can be generated using publicly available information by taking advantage of the work presented in [14].

The exploit explored in [10] makes use of the two prime values that play a fundamental role in an RSA key. These values must be sufficiently random such that repeated values are infeasible and mathematically extremely improbable to encounter due to the scope of the set (this work focuses on 1024-bit keys). However, on some systems and due to some less-than-adequate code, this may not always be the case. When these values are repeated between keys, the greatest-common-divisor algorithm can be applied to find the shared value between keys. This is significant because the GCD calculation is significantly less computationally expensive than the alternative: factoring the large values. With this new information about each key, they private components are straight-forward to generate since the RSA key-generation process is public and not difficult to implement.

A large sample set was obtained via the OpenSSL Observatory and by crawling the Internet for SSL certificates (which use RSA as an encryption scheme). Since it was thought this would provide an adequate sample set, all these keys were analyzed. The set contained 1024-bit keys, as well as 2048-bit keys. The findings were that 0.2% of these keys were vulnerable to this exploit, and thus offered no security. This included 2048-bit keys as well, despite the conception 2048-bit keys are more secure (this would be true if this vulnerability were avoided).

3.2 The Process

As mentioned, the vulnerability is exploited by performing greatest common divisor (GCD) calculations with pairs of keys. The research presented in this paper looks at how to most quickly and efficiently analyze a large database of RSA keys and detect if this vulnerability is present within the set. Traditionally, this would be a tedious process that would likely be infeasible due to time constraints. Since the calculations are independent of one another (between pairs of keys) they can be done in parallel, given the proper system. One such system, and the focus of this work, is NVIDIA's CUDA. This will be detailed later, however, it relates to work done in [7].

This work optimized a specific version of the GCD (binary GCD). This method reduces the algorithm to three repeated operations. This algorithm was then implemented in CUDA for 1024-bit numbers (something CUDA does not offer native support for). This allows much of each operation to be performed in a parallel fashion, further optimizing the process.

3.3 Completed Areas

As a groundwork for the research presented here, the work done in [14] will be used extensively. Here, the ideas present in both [10] and [7] are combined with further optimization and parallelization of the data to achieve a significant speedup of the overall process (when compared to the sequential runtimes). This work makes use of CUDA to parallelize the binary GCD algorithm as presented in [7], but additionally parallelizes each GCD calculation as each is independent of the other (that is, many complete GCD comparisons are performed in parallel). This same approach will be used in the work that follows.

This work, however, does not offer a complete solution and suffers from several limitations. The work in [14] does not actually produce the private components of the RSA keys found to be vulnerable by the algorithm. Furthermore, the set of keys that can be used in the algorithm as presented, is limited by the memory present on the GPU. To offer a complete solution, this must be overcome to allow RSA key-sets of arbitrary size to be used for the validation. The usefulness of such work is severely impacted due to these two limitations.

3.4 Multiple GPUs

As an extension to the work in [14], multiple GPU support will be explored. This concept is discussed at length in [13]. Here, several GPU configurations are investigated to analyze how a distributed system may benefit over a single-GPU design. Work is divided intelligently among the systems in the distributed system to gain more performance increase than would be possible with a single system.

At the time [13] was written, single-system, multiple-GPU configurations were not possible using CUDA over SLI (Scalable Link Interface, NVIDIA's brand for the interface between multiple GPUs). However, this is possible now, and will be explored at length in our work. This configuration offers benefits including transfer speeds over alternative configurations.

The work done in [15] helps to make a case for the additional speedup that making use of multiple GPUs can offer. This work was done after CUDA had added adequate support for using multiple GPUs simultaneously, and configurations with two-GPU and four-GPU systems were used.

Significant additional speedups were gained, ranging up to three times speedup using four GPUs over the single-GPU system (which translates to 100 times speedup compared to the original, non-GPU benchmark being used). This kind of speedup offers great optimism in how multiple GPUs may offer significant increases to performance of the GCD key algorithm.

Another interesting aspect of the work done in [15] is the varied domain sizes that were used. The data sets were broken down into several different configurations which was each tested on the single, dual, or quad-GPU configuration. This was done to achieve a more accurate answer to how much faster the multi-GPU system could be. The largest domain set ($1024 \times 32 \times 1024$), interestingly, offered the greatest additional speedup between configurations—the quad-GPU configuration was 3 times faster than the single-GPU, compared to 2 times for the $256 \times 32 \times 256$ domain. This is significant, and implies that exploring different ways to organize our own data may offer increased advantages when moving to a multi-GPU system.

3.5 Large Data

Since one of our primary goals is to remove the limitation present in [14] that prevents larger sets of keys from being analyzed, work investigating how to manage large data sets is relevant. Specifically, large data sets that must be managed by the GPU.

One such work is [16]. In this work, like our own, the data is too large to fit within the extremely-limited memory of the GPU. Thus, the data must be partitioned. In this work, the data was first prepared by organizing it so that it could be effectively partitioned. There was a straightforward way of doing so for this application (K-means).

Additionally, some features of the GPU were considered and used, namely asynchronous memory transfer, a type of multi-tasked streaming. This feature allows memory transfers to be done between the GPU and CPU while the GPU is processing data it already has. This can increase performance significantly when compared to having the GPU move from processing to data transfer (i.e. having a single tasked thread). Optimization of threads was looked into in this work as well, as the authors attempted to find how many of these threads would add the positively impact speedup the most. They found that only 2 threads (i.e. one processing, one transferring) were needed to gain the greatest performance benefit. Adding additional streams after this point did not offer additional benefit.

Decomposition of data is the primary problem when attempting to perform work with a GPU due to the limitations of current hardware. Therefore, much work and thought has been given to optimizing the structure of large data sets. Although a different type of parallelism is looked at in [6], the strategies to data decomposition are relevant to most parallel problem in general, including CUDA. The focus in this survey is matrix multiplication, a standard parallel problem. The most effective way to chunk the data in this work appears to be entire rows/columns of the element matrices. This is logical, and the approach is mimicked (as much as it can be since matrix multiplication is not perfectly synonymous) in [14]. However, a slight modification was found to make this strategy most effective: hold the results in memory until an entire set of iterations completes (all that will fit in memory) and only then transfer the results back to the CPU. Although this does not decrease the amount of memory sent from the GPU to the CPU, it reduces the number of total memory transfers which is often more important. Such fine details must be found and applied to our work here to further increase our performance.

3.6 Optimization

As mentioned, data decomposition and organization around the architecture are some of the most important (if not *the* most important) aspects of creating a parallel solution for use with a GPGPU. The work done in [12] gives a detailed overview of how one might optimize work done with CUDA. It discusses the memory model, and how certain techniques such as tiling are used to optimize the performance gains. Although some of these techniques are used in the original implementation found in [14], it does not appear that they were used extensively.

It seems that the research and surveying done in [12] can be applied to our work here and our speedup will benefit from their results.

Unfortunately, this work was done before multi-GPU systems were fully available and accessible using CUDA. The optimization strategies presented in this work will no doubt be useful; however, other strategies also must be employed that considered systems with more than a single GPU (such as those mentioned here from [15]). Leveraging techniques in both realms will offer the greatest amount of performance gain.

3.7 Similar Algorithms

A similar algorithm that suffers from the same problems that pairwise comparisons does (each element much be traversed and compared), is search. The work done in [11] explores how search can be optimized for the CUDA framework. Two approaches are given to the bitonic search algorithm that is used: the initial, most straight-forward implementation, and another implementation optimized around the CUDA memory model. The latter offered significant performance increases over the original.

A similar approach will be applied to the work that is used from [14]. Namely, shared memory will be used as efficiently as possible to allow CUDA to perform at its greatest capacity. This is a vital concept in general when optimizing for CUDA; however, the work and challenges from [11] are similar to our own, so the implementation overlaps more than arbitrary examples.

Another important contribution to the field and concepts of distributed systems is the work presented in [5]. MapReduce has become an extremely useful tool for companies like Google with massive data centers that must distribute the work to perform as reasonable (real-time) levels. MapReduce explores how work might be delegated to multiple workers that exist in a particular distributed system. These concepts apply to the problem of performing the necessary GCD comparisons using multiple GPUs in a single system.

Chapter 4

Validation

Accuracy and speedup will both be validated to gain a measure of success.

To validate accuracy, a sequential implementation will be compared against. Much as the work found in [14] performed accuracy validation, a separate, sequential implementation will be created and ran alongside the parallel implementation. Since this work will function as an extension to [14], the validation process will likely be identical or even reused. The results each implementation produces must match for a given set of keys (i.e. both implementations must find the same set of vulnerable keys given identical sets to start with).

An additional limitation realized in [14] was the runtime of the sequential version becoming too long for large sets of keys. Since this is known to be an extremely time-consuming process, it will not always be performed with larger sets of keys. If correct operation of the parallel implementation for small (i.e. sets that can be run sequential in a reasonable amount of time) can be validated using many varying sets of keys of increasing number, it can be assumed the same level of accuracy will continue for larger sets that will not be run sequentially.

Thus, accuracy will be validated for sets no larger than 200,000 keys (as in [14]), and assumed to be accurate (given success of the aforementioned validation) for sets larger than this.

Speedup validation also involves comparing with a sequential implementation; however, this time the runtimes will be compared so an overall speedup may be calculated. Again, much as in [14], speedup will be calculated by taking the ratio of the sequential and parallel runtimes.

As seen in [14], the speedup reached an upper threshold, namely, 27.5. Since this work will expand upon on that, it seems natural to use 27.5 as a baseline. However, there are some differences to consider. First, in [14], sets of keys that could fit entirely in the GPU's memory were tested and presented. This ensures that the number of memory transfers to the memory card is 1: this significantly reduces memory transfer time, and would help increase the speedup. Since one goal of this work is to allow an arbitrary number of keys to be used in a set, this will likely affect the overall speedup since memory transfers will become smaller and more frequent.

On the other hand, another goal is to add multi-card support which would further increase the total speedup of the final, parallel implementation. Since we will be looking at how fast additional cards make the process (compared to the single-card implementation), the sequential runtime will become less important. It will continue to serve as a baseline to the entire algorithm though.

Considering the modifications done, speedup near 25 seems like a reasonable expectation. Functionality that is expected to decrease the original 27.5, as well as additions that are expected to increase it will both be researched and added. Therefore, a speedup in the same area appears to be a fair expectation of success.

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