* Bitwise Manupulation :-

→ operators :-

1. AND

a	b	a & b
0	0	0
0	1	0
1	0	0
1	1	1

→ when you '&' 1 with any no.
digits remains same

eg:→ 1 1 0 0 1 0 1 0 0
& 1 1 1 1 1 1 1 1 1
1 1 0 0 1 0 1 0 0 → Same

2. OR

a	b	a or b
0	0	0
0	1	1
1	0	1
1	1	1

3. Ex-OR (^) (if & only if → iff)
→ Exclusive

a	b	a ^ b
0	0	0
0	1	1
1	0	1
1	1	0

• Observations :-

$a \wedge 1 = \bar{a}$

$a \wedge 0 = a$

$a \wedge a = 0$

4. Complement (~)

a = 10110

$\bar{a}$ = 01001

5. Left Shift Operator (<<)

1010 << 1 $(1010)_2 = (10)_{10}$

10100 → $(10100)_2 = (20)_{10}$

→ $\boxed{a << 1 = 2a}$ $\boxed{a << b = a \times 2^b}$

6. Right Shift Operator (<<)

eg:- 0011001 >> 1 → 001100 ⇒ (1100)

$(000\,11234)_{10} = (11234)_{10}$

ignored → Same for all Number System

$\boxed{a >> b = \frac{a}{2^b}}$ $\boxed{a >> 1 = \frac{a}{2}}$

Made with Goodnotes

→ Number System :

1. Decimal :→ 0, 1, 2, 3, ..., 9 [Base : 10]

$(357)_{10}$, $(10)_{10}$

2. Binary :→ 0, 1 [Base : 2]

eg:- $(10)_{10} \rightarrow (1010)_2$

$(7)_{10} \rightarrow (111)_2$

3. Octal :→ 0, 1, 2, 3, ... 7 [Base : 8]

eg:- Decimal :– 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ...

Octal :→ 0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, ...

$(9)_{10} \rightarrow (11)_8$

4. Hexadecimal :→ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F [Base : 16]

eg:- $(10)_{10} \rightarrow (A)_{16}$, $(12)_{10} = (C)_{16}$

→ Conversion :

1. Decimal to base 'b'

eg:- Convert $(17)_{10}$ to base 2.

→ Keep dividing by base, take remainder, write in opp.

$$\begin{array}{r|l}
2 & 17 \\
2 & 8 \rightarrow 1 \\
2 & 4 \rightarrow 0 \\
2 & 2 \rightarrow 0 \\
 & 1 \rightarrow 0
\end{array}$$

$\Rightarrow (10001)_2 = (17)_{10}$

$(17)_{10} = (?)_8$

$$\begin{array}{r|l}
8 & 17 \\
 & 2 \rightarrow 1
\end{array}$$

$\Rightarrow (21)_8$

2. Convert any base 'b' to decimal.

eg:- $(1001)_2 = (?)_{10}$

$1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \rightarrow (9)_{10}$

$(21)_8 = (?)_{10}$

$2 \times 8^1 + 1 \times 8^0 = 16 + 1 = 17 \rightarrow (17)_{10}$

Q. Find if the no. is even or odd.

A:→ Every no. is calculated in binary form internally.

$12+7 \rightarrow$ 1100 + 0111 = 10011 → 19 $(19)_{10} = (10011)_2$

(1001**1**) → leaving this, every other no. is power of 2.
2^0
This will always be even

So, if 2^0 place == 1 ⇒ Odd

otherwise → Even

eg:- 0110110**1** → 1 → Odd No. (LSB)

0111**0** → 0 → Even

100101
& 000001
000001 ⇒ (1) → odd

→ if n & 1 == 1 → Odd
else Even

Q. Every no. in an array is repeating twice except one. Find that number.

arr = [2, 3, 4, 1, 2, 1, 3, 6, 4]

→ $a \wedge 0 = a$
$a \wedge a = 0$ → Ex-OR all Nos.

Q. Find the i^{th} bit of a no.

10110110 → 5^{th} Bit = ?

8 7 6 5 4 3 2 1
10110110
& 00010000
00010000 → Ans → This is called Mask

n → mask with (n-1) zero → 1<<(n-1)

(1<<4) → 10000

Ans: → (n & (1<<(i-1))) >> (i-1)

Q. Set the i^{th} bit

4^{th}
1010110

Make bit '1' → 1→1, 0→1

7 6 5 4 3 2 1
1010110
OR 0001000 → Mask
1011110

Q. Reset the i^{th} bit

Make 0 → 0→0, 1→0

5 4 3 2 1
1010110
& 1101111
1000110

Mask:- ~(1<<(n-1))

→ Negative of a number in Binary form:

1 byte = 8 bits

10 → | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | → A number Stored like this (LSB)

-10 = ?

MSB → tell us if no. is +ve or -ve
→ 1 → -ve
→ 0 → +ve

→ represents value of a No.

eg:- $(10)_{10} = (00001010)_2$

1. 11110101
2. 11110101 + 1
$(11110110)_2$ ⇒ $(-10)_{10}$ → -ve 10 in Binary

→ Steps:

1. Take Complement of a no.
2. Add 1 to it

} → 2's Complement Method

→ Range of Numbers :-

Eg. 1 byte = 8 bits

→ Sign of No.

0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Total = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

Nos. can be store in 1 byte $= 2^8 = 256$

Actual no. is stored in bits → n-1 → Total bits ⇒ In 1 byte = 7 bits

Total Nos. that can be made from 7 bits $= 2^7 = 128$

→ 128 Nos. in +ve
→ 128 Nos. in -ve
→ Total 256 Nos. ⇒ X Wrong

-128 to 128 → Total No. = 257 (0 is also there)

⇒ -128 to 127 → 127 cuz → -ve of 0 is not a -ve no. (-0 = 0)

(0 is also there)

∴ 0 0 0 0 0 0 0 0

1y. 1 1 1 1 1 1 1 1

2y. + 1

$\underbrace{1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0}_{9 \text{ bits}}$

→ will be discarded

→ $\boxed{\text{Range formula for n-bits :- } -2^{n-1} \text{ to } 2^{n-1}-1}$

Q7 Find the position of the right most Set bit

$\underbrace{10110}_{a}\ 1\ \underbrace{00}_{b}$

→ N = a 1 b a = 10110 1

-N = $\bar{a}$ 1 b b = 00

Ans: → N & (-N)

* Prime Numbers :→

eg:- 2, 3, 5, 7, 11, 13, ...

```
for (i=2 ; i<N ; i++){
    if (N%i==0){
        Not Prime
    }
}
Prime
```

eg:→

1 ×	36
2 ×	18
3 ×	12
4 ×	9
6 ×	6
9 ×	4
12 ×	3
18 ×	2
36 ×	1

→ repeating → ignore

So, only make check for numbers $\leq \sqrt{N}$

↓

More Optimised

Sieve of Eratosthenes :→

N=40 → 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~

O → False

× → True

→ Time Complexity Analysis:→

$$\rightarrow \frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \dots\dots$$

Numbers divisible by 2 till 40

similarly by 3 till 40

$$= n\left\{\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right\}$$ → Harmonic Progression

$$= \log(\log(N))$$ → By Taylor Series

Time Complexity :→ $O(N * \log(\log(N)))$

* Newton Raphson Method (for Sqrt) { Binary Search Method for Sqrt is in Code }

→ $\underset{\text{actual sq. root}}{root} = \dfrac{\left(X + \frac{N}{X}\right)}{2}$ { X → sq. root that you have assumed

1. Assign X to N
2. error = |root - X|

   You will find your Ans when error < 1
3. Update the value of, X = root

Q) why the formula works?

→ $\sqrt{N} = \dfrac{\left(X + \frac{N}{X}\right)}{2}$ { let $X = \sqrt{N}$ → guess

$= \dfrac{\sqrt{N} + \frac{N}{\sqrt{N}}}{2} = \dfrac{\sqrt{N} + \sqrt{N}}{2} = \sqrt{N}$

Time Complexity :→ O(log(N) F(N))

F(N) → cost of calculating f(x)/f'(x) with n-digit precision

* Factors of a Number :-

eg:- n = 20 → 1, 2, 4, 5, 10, 20 → Factors

20 % 1 → 20 × 1 = 20

20 % 2 → 10 × 2 = 20

20 % 4 → 5 × 4 = 20

20 % 5 → 4 × 5 = 20

20 % 10 → 2 × 10 = 20

20 % 20 → 1 × 20 = 20

repeated

O → repeating

So, only check till $\sqrt{n}$

* Modulo Properties :

→ $(a+b)\%m = ((a\%m) + (b\%m))\%m$

→ $(a-b)\%m = ((a\%m) - (b\%m) + m)\%m$

→ $(a \times b)\%m = ((a\%m) \times (b\%m))\%m$

→ $\left(\frac{a}{b}\right)\%m = ((a\%b) \times (b^{-1}\%m))\%m$ { $b^{-1}\%m$ → Multiplicative Modulo Inverse (MMI)

↳ means that b & m → are Co-primes → only one factor i.e 1 eg:→ 6 & 7

eg:– $(6 \times y)\%7 = 1$ → y = MMI for 6 & $y = 6$

$(6 \times 6)\%7 = 36\%7 = 1$

→ $(a\%m)\%m = a\%m$

→ $m^x\%m = 0 \quad \forall x \in Z^+$

→ If 'p' is a prime no. which is not a divisor of 'b'

then $ab^{p-1}\%p = a\%p$

↳ Due to Fermat's Little Theorem

* Die-Hard Example :

$\boxed{3}$ $\boxed{5}$ = $\boxed{4}$
a b

{ Two buckets give, $a = 3L$ & $b = 5L$ → limits
You had to fill a new bucket with 4L

1st :→ (a, b) $(0,0) \xrightarrow{\text{fill with 3L}} (3,0) \xrightarrow{\text{pass}} (0,3)$

2nd :→ $(0,3) \rightarrow (3,3) \rightarrow (1,5) \rightarrow (1,0) \rightarrow (0,1)$

↳ fill bucket a with 3L ↳ now empty bucket b

3rd :→ $(0,1) \rightarrow (3,1) \xrightarrow{\text{pass}} (0,4)$ → Ans

↳ fill bucket a with 3L

bucket a → S_1 times filled

bucket b → S_2 times emptied

remainder in the bucket b $= r = aS_1 - bS_2$, $L = S_1 a + t_1 b$

$= aS_1 + (-bS_2)$

$r = S_1 a + t_1 b - t_1 b - bS_2$

$= L - (t_1 + u)b$

→ If $t' + u \neq 0 \Rightarrow r < 0$ or $r > b$ → which is not true

if $t' + u = 0 \Rightarrow u = -t'$ → $r = S_1 a + t_1 b = L$

$\equiv r = ax + by$

eg:- $3x + 5y = 4$

Put x \$ y as integers,

Q. What is the min^m +ve value you can have for eq^n?

$x = -3$, $y = 2$

$3x + 5y = 1$ → Min^m +ve value that can form

→ This is called HCF

→ HCF of a \$ b or GCD → Min +ve value of eq^n $ax + yb$ when x \$ y are integers.

eg:- HCF(4, 18) = 2 → Highest Factor Common in both

1, 2, 4   1, 2, 3, 6, 9, 18

eg:→ Min^m $(3x + 9y) = 3$

$\therefore$ $3x + 9y = 3(x + 3y)$

Put $x = -2$, $y = 1$

$3(-2 + 3) = 3 \times 1 = 3$

eg:- a, b

$ax + by = L$

$2x + 4y = 5$

→ $2(x + 2y) = 5$

⇒ $x + 2y = 2.5$ → X Should not be Decimal

eg:- $3x + 6y = 9$

⇒ $3(x + 2y) = 9$

⇒ $x + 2y = 3$ → ✓ You can fill 9L bucket with 3L & 6L bucket

HCFs

eg:- $3x + 5y = 17$

⇒ $1(3x + 5y) = 17$ → 1 divides 17 → ✓ You can fill 17 bucket with 3L & 5L bucket

N:→ Whatever HCF you will get that will come out as Common

* Euclidean Algorithm :→

→ $gcd(a, b) = gcd(rem(b, a), a)$

eg:- $gcd(105, 224) = gcd(rem(224, 105), 105)$

$= gcd(14, 105)$ → Apply Same Formula Again → to Reduce

Q. Why do this?

→ $gcd(105, 224)$ → $105x + 224y$ → $14x + 105y$

Why Subtract? Why not changing this?

→ because the $gcd(105, 224)$ also divided a Linear Combination of 105 \$ 224   eg:→ → $224 - 2 \times 105 = 14$ (rem)

* LCM (Least Common Multiple) :-

→ lcm(a,b) → Min^m no. i.e divisible by both a & b

eg:- lcm(2,4) → 4

lcm(3,7) → 21

N :→ Say we have a, b

$d = gcd(a,b)$

$f = \frac{a}{d}$, $g = \frac{b}{d}$

$\Rightarrow a = fd$, $b = gd$

lcm = c , lcm(a,b) = lcm(fd, gd)

→ We know that f & g have no other common factors

eg:- 9 & 18

a = 9, b = 18, d = HCF(9,18) = 9

$f = \frac{a}{d} = 1$, $g = \frac{b}{d} = 2$

→ No other common factors of (2,1)

$a = fd$, $b = gd$

$a \times b = f\,d \times g\,d$ → repeating twice → remove it

→ $lcm = f \times g \times d$ → This is how above conditions are satisfied → lcm → $f \times g \times d$

→ $a \times b = fd \times gd = d \times dfg$

$= hcf \times lcm$

→ $hcf \times lcm = a \times b$ → { multiplication of two nos (a & b) = hcf(a,b) × lcm(a,b) }

→ $LCM = \frac{a \times b}{HCF(a,b)}$

LCM(a,b) → contains both a & b

$2\ 7\ 8\ 3 \longrightarrow 3\ 8\ 7\ 2$

$r = n \% 10$

$S = (S \times 10) + r$

$\rightarrow (n/10)$

$\rightarrow 3$

$S = 0 + 3 \rightarrow 3$

(278)

$\hookrightarrow 8$

$S = (3 \times 10) + 8 \rightarrow 38$

(27)

$\hookrightarrow 7$

$S = (38 \times 10) + 7$

3872

$3 \times 10 + 8$

$38 \times 10 + 7$

$387 \times 10 + 2$

3872

10

(4)

$\hookrightarrow$ (3)

$\hookrightarrow$ (2)

$\hookrightarrow$ (1)

$\hookrightarrow$ 0

10

10

$1+2+3 - - - +10$

4672

2464

$4\times10^3 + 6\times10^2 + 7\times10^1 + 2\times10^0$

$2\times10^3 + (3, 467) \rightarrow 200 + 764 \rightarrow 2764$

$\hookrightarrow 7\times10^2 + (2, 46) \rightarrow 700 + 64 \rightarrow 764$

$\hookrightarrow 6\times10^1 + (1, 4) \rightarrow 60 + 4 \rightarrow 64$

$\hookrightarrow 4\times10^0 + (0, 0) \rightarrow 4$

40

arr = [1, 2, 3, 4, 3, 5]

0: → (arr, 3, 1)

↳ (arr, 3, 2)

↳ list(2) → (arr, 3, 3)

↳ (arr, 3, 4) → list(4)

↳ (arr, 3, 5)

↳ (arr, 3, 6) [ ]

[4]

[4]

[ ]

5 → (4)

↳ 4 → (3)

↳ 2 → (1)

↳ 0

5 4 3 2 1 1 2 3 4 5

(5) → (4) → (3) → (2) → (1) → (0)

1

1 2 3 4 5

4 3 2 8 1

↓

4 3 2 1 8

↓

1 3 2 4 8

↓

1 2 3 4 8

↓

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	5	4	3	2	1

$n = 5$

$0 + 5$

$1 +$

$a[0+5] = n[5]$

1950   1961   , $m = 1961$, $C = 1$

1960   1971   , $m = 1971$, $C = 1+1$ ✓

1970   1981   , $m = 1981$, $C = 1+1$

Made with Goodnotes

$n=5$

$\underbrace{-3\ -2}_{-5}\ \ 0\ \ \underbrace{2\ \ 3}_{5} \rightarrow 0$

$n=6 :\rightarrow\ -3\ -2\ -1\ 0\ 1\ 2\ 3 \rightarrow 0$

$n=7 :\rightarrow\ -4\ -3\ 0\ 3\ 4 \rightarrow 0$

$n=8 :\rightarrow\ -4\ -3\ -1\ 0\ 1\ 3\ 4 \rightarrow 0$

0 1 2 3 4 5

$-2\ -1\ 1\ 2$

$6 = \frac{n}{2}(a+l)$

$\boxed{6 = \frac{nl}{2}}$

3

0 1 2 3 4 5

0 1 2 3 4 5

$n=9 :-$

$-4\ -3\ -2\ 0\ 2\ 3\ 4$

0 1 2 3 4 5

$n = 11 : \to \; -5 \; -4 \; -2 \; 0 \; 2 \; 4 \; 5$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$-2 \; -1 \; 0 \; 1 \; 2$

$n = 5 : \to \; -3 \; -2 \; 0 \; 2 \; 3 \qquad \frac{5}{2} = 2.5$

$n = 6 : - \; -3 \; -2 \; -1 \; 1 \; 2 \; 3 \qquad \frac{6}{2} = 3$

$n = 7 : - \; -4 \; -3 \; 0 \; 3 \; 4 \qquad \frac{7}{2} = 3.5$

$-4 \; -3 \; -2 \; -1 \; 0 \; 1 \; 2 \; 3 \; 4$

$-3 \; -2 \; -1 \; 0 \; 1 \; 2 \; 3$

$n = 5$:

$s = \frac{5}{2} = 2.5 = 2$

$$\left\{ \begin{array}{l} a(0) = -2 \\ a(1) = -1 \\ a(2) = 0 \\ a(3) = 1 \\ a(4) = 2 \end{array} \right\}$$

$n = 4 = -2 \ -1 \ 0 \ 1 \ 2$

$\frac{4}{2} = 2$

$n = 6$

$\frac{6}{2} = 3$

$a(0) = -3$

$a(1) = -2$

$a(2) = -1$

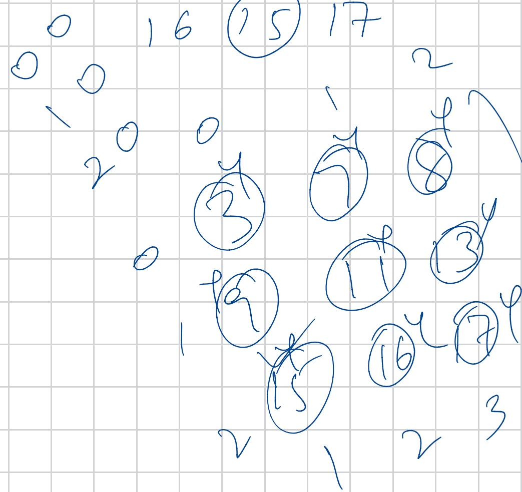
$a(3) = 1$

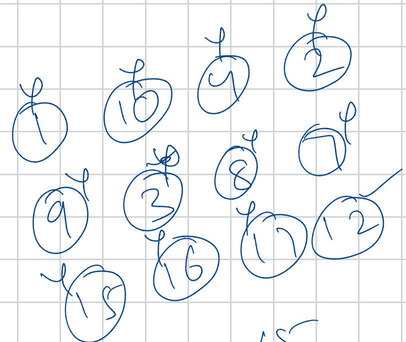
$a(4) =$

3	7	8
9	11	13
15	16	17

7	3	8
11	9	13
16	15	17

11	9	13
7	3	8
16	15	17





3	9	15
7	11	16
8	13	17

$m_{ij} = m_{ji}$

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 10 & 4 & 13 \\ 1 & 9 & 3 & 8 & 7 \\ 2 & 15 & 16 & 17 & 12 \end{array}$$

3	7	8
9	11	13
15	16	17

(2, 2)

0 0
1 0
2 0

a[j][i]
1 0

3

i = 0
min = 1
j = 0,
max = m[0][0]
x = 0
j = 1 → max[1][0] = max | x = 1
[2][0], x = 2
j = 2 → max = 15
min = 12
i = 2

3 7 8 1

9 11 13 3

15 16 17 12

1 10 4 2

12 3 8 7

15 16 17 12

15

15 16 16 9 17 12

12

1 8 12 11

0 3 15 16 17 12 = 11 Ans

0 3

2 3

0 0

1 0

2 0

	0	1	2	3
0	1	10	4	2
1	9	3	8	7
2	15	16	17	12

0 0
1 0
2 0

0

0	1
1	1

@ min = [1, 3, 9, 12]

max = [15, 16, 17, 12]

7 8
1 2

min: [7, 1]

max = [7, 8]

3	7	8
9	11	13
15	16	17

min = 3, 9, 15

max = 15, 16, 17

Made with Goodnotes

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1 & 2 \\ 1 & 3 & 4 \end{array}$$

2x2 → 1x4

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 2 & 3 & 4 \end{array}$$

→ 1x4

0,0 → 0,0

0,1 → 0,1

0,2 → 1,0

0,3 → 1,1

2x2 → 1x4

0,0 → 0,0

0,1 → 0,1

1,0 → 0,2

1,1 → 0,3

$3 \times 4 = 12$

$2 \times 6 = 12$

0 1 2 3

0 1 2 3 4

1 5 6 7 8

9 9 10 11 12

3×4 | 2×6

0, 0 → 0, 0

0, 1 → 0, 1

0, 2 → 0, 2

0, 3 → 0, 3

1, 0 → 0, 4

1, 1 → 0, 5

1, 2 → 1, 0

1, 3 → 1, 1

2, 0 → 1, 2

2, 1 → 1, 3

2, 2 → 1, 4

2, 3 → 1, 5

0 1 2 3 4 5

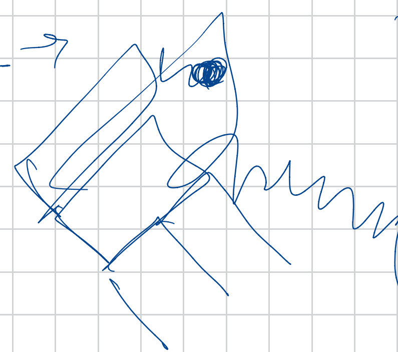
0 1 2 3 4 5 6

1 7 8 9 10 11 12

all

1 if $c_2 > c_2$

m[



→ [2, 2, 2, 3, 3] [0, 3, 2]

[1, 1, 1]

[1, 3, 2, 4, 2, 0]

5

4 :- 2 + 2 + 0 + 1 = 5

1 :- 3 + 0 + 4 + 0 = 7

5

8

3 :- 1 + 2 + 0 + 2 = 5

2 :- 4 + 0 + 3 = 7 → 8

0 :- 2 + 0 + 2 + 0 + 1 = 5

1 3 2 2 3 4 4 3

0

1 → $3 + 0 + \cancel{8} + 0 + \cancel{4} + 0 = \cancel{12}$ — 9

2 → $1 + 2 + 0 + 3 + 0 + 4 = 10$

3 → $3 + \cancel{8} + 0 + 4 + 0 = \cancel{12}$ — 9

4 → $1 + 0 + 2 + 3 + 0 + 4 = 10$

5 → $0 + 3 + 0 + \cancel{8} + 4 + 0 = \cancel{12}$ — 9

6 → $1 + 0 + 2 + 0 + 3 + 4 = 10$ — 9

7 — $0 + 3 + 0 + 8 + 0 + 4 = 12$ — 9

1, 2, 3, 4, 5, 6, 7

19

1 :- $3 + 2 + 4 = 9$

2 :- $1 + 2 + 3$ = 6

6 :- $1 + 2 + 3 = 6$

5 :- 9

9 :- 6

3 :- 9

[1, 2, 3]

| 2 2 2 3 3 |

$\frac{0}{0}$ $\frac{0}{1}$ $\frac{0}{2}$ $\frac{0}{3}$ | $\frac{0}{4}$

0 $\frac{0}{1}$ $\frac{00}{2}$ $\frac{00}{3}$

1 : − 1

2 : − 2

3 : − 1

ev = 1 + 1

od = 1

$\frac{00}{0}$ $\frac{000}{1}$ $\frac{00}{2}$ $\frac{0000}{3}$

Shubham Kumar

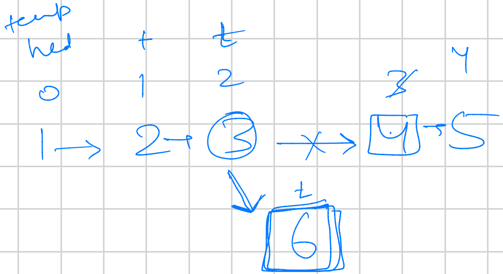
0 :- $3 + 4 = 7$
1 :- $2+2 = 4$
2 :- $3+4 = 7$
3 :- $2+2 = 4$

---

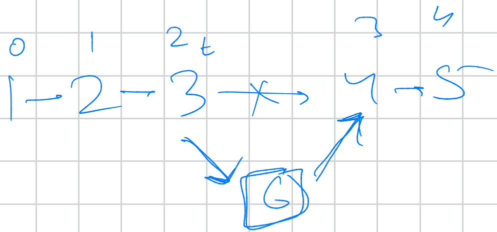
0 :- 2
1 : $\rightarrow$ 3
2 : $\rightarrow$ 2
3 : $\rightarrow$ 2
4 : $\rightarrow$ 1

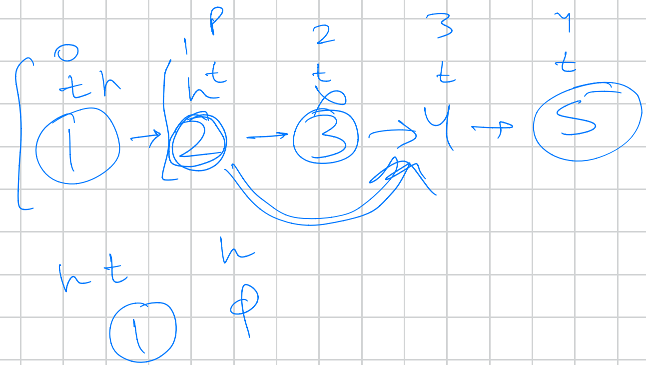
$l_1 = 10$

$l_2 = 5$



3





		r		d
		0	1	2
0		1	2	3
1		4	5	6
2		7	8	9
	t			l

| 1 | 2 | 3 | 4 | 5 |

t

	0	1	2	3
	l			r
t	1	2	3	4
	5	6	7	8
b	9	10	11	12

left = 0
right = 3
top = 0
bottom = 2

m[0][0]
m[0][1]
m[0][2]
m[0][3]

m[1][r]
m[2][r]

$n = 3$, $n^2 = 9$

	0	1	2
	l		r
t	1	2	3
	8	9	4
b	7	6	5

$n = 4$, $n^2 = 16$

	l			r
t	1	2	3	4
t	12	13	14	5
	11	16	15	6
b	10	9	8	7

n^2
n^2

		0	1	2
		l	r	~~r~~
0	~~t~~	1	2	3
1	t b	8	9	4
2	~~b~~	7	6	5

a[0][0] = 1
a[0][1] = 2
a[0][2] = 3
a[2][1] = 6
a[2][0] = 7

a[1][2] = 4
a[2][3] = 5

0 1 2 3 4 5

1 2 3 4 5 6

2 240 120 4

0 1 2 3 4 5

arr = 1 1 2 6 24 120

arr = 120

0 1 2 3

n = 1 2 3 4

0 1 2 3

arr = 1 1 2 6

0 1 2 3

arr = 6

$$\boxed{\frac{9}{3} = 3}$$

a[0] = 1

p = 1x1 = 1

a[1] = 1

p = 1x2 = 2

a[2] = 2

p = 2x3 = 6

a[3] = 6

p = 6x4 = 24

a[3] = 1

p = 1x6 = 6

a[2] = 6

p =

Made with Goodnotes

	0	1	2	3
$n =$	1	2	3	4
$a =$	1	1	2	6
$a =$	24	12	8	6

$p = 1$

$p = 1 \times 1$

$p = 1 \times 2 = 2$

$p = 2 \times 3$

0 1 1 1 1

0 2 7 9 3 1 10 6 8

1. $t = 0 = p1$
   $p1 = M(0+2, 0) = 2$
   $p2 = 0 = t$

2. $t = 2$
   $p1 = M(0+7, 2) = 7$
   $p2 = 2$

3. $t = 7$
   $p1 = M(2+9, 7) = 11$
   $p2 = 7$

4. $t = 11$
   $p1 = M(7+3, 11) = 11$
   $p2 = 11$

5. $t = 11$
   $p1 = M(11+1, 11) = 12$
   $p2 = 11$

6. $t = 12$
   $p1 = M(11+10, 12) = 21$
   $p2 = 12$

7. $t = 21$
   $p1 = M(12+6, 21) = 21$
   $p2 = 21$

8. $t = 21$
   $p1 = M(21+8, 21) = 29$
   $p2 = 21$

0 1 2 3 4 5 6 7 8 9

0 2 4 6 7 5 4 2 1 0

b a n k

b n a k

b a c n d k

b m c a d k

0 1 2 3 4 5 6

4 5 6 7 0 1 2

① $a[m] > a[m+1]$ && $a[m] > a[m-1] \rightarrow m$

② $a[m] < a[m+1]$ && $a[m] < a[m-1] \rightarrow m-1$

③ $a[m] < a[m+1]$ && $a[m] > am[m-1] \rightarrow m+1$

④ $a[s] > a[e]$ && $a[m] < a[e] \longrightarrow e = m-1$

⑤ $s = m+1$

Made with Goodnotes

0	1	2	3	4	5	6	7	8
s				m				e
2	7	11	13	14	16	19	20	22

t = 20

t(20)

7 + 13 = 20

26

27

11 + 16 = 27

log(N)

$\boxed{19 + 7 = 26}$

12 + 14 = 26

14 + 12 = 26 | 16 + 12 = 28

12 = 26 − 14

12 = 28 − 16

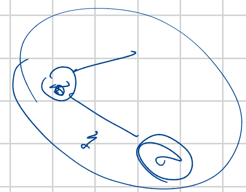
s		m		e
1	2	3	4	5

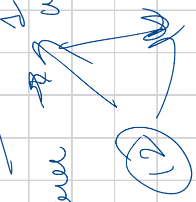
dir

2D array

1 func

Tree





0	1	2	3	4
~~s~~		~~m~~ e	s m	e
2	3	4	7	11

$\boxed{2 + 2 + 1 = 5}$

7 − (3 + 1) = 3

mN ≥ 2

1 < 2

mN = 4 − (2 + 1) = 4 − 3 = 1

(mN < 5)

s = 3

4 + 2 | 4 + 2 + 1 = 7

mN = 11 − (4 + 1) = 6

(mN > 5)

e = 4 − 1 = 3

5 + 3 + 1 = 9

mN = 7 − (3 + 1) = 7 − 4 = 3

(mN < 5) → s = 4

0 1 2 3 4 5 6 7 8 9 10

s s m m → s m e

0 0 2 3 3 4 4 5 5 6 7

s e

6 7 8 9

$x \le m - e$

$x \le a[s]$

1 2 3 4

s m e

5 5 6 7

$s - e = 4$

$a[s] \le s - e$

0 1 2 3 4

s → s m e

4 5 5 6 7