PHYS815 Project 4: Estimating Average Muon Lifetime

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1 Introduction

In this project, the goal is to estimate the average lifetime of a muon, which is a commonly found particle of charge -1. The lifetime can be estimated using log-likelihood minimization methods, which were developed in class.

For this type of experiment, typically a scintillator is used, which is a device or material that emits illumination or a flash when it is excited by ionization radiation from a charged particle, like the muons.

When muons enter the scintillator, they will trigger a flash, likewise, when they decay, a second flash is emitted, by taking many measurements of the time between these flashes, the average life time of a muon in our lab can be estimated.

This report is organized as follows: Sec. 2 discusses the experimental set up and how the data is collected, and a description of the computer simulation and minimization techniques used is shown in Sec. 3, with an analysis of the estimated parameter as well as it's associated error in Sec. 4. Finally, conclusions are presented in Sec. 5.

2 Discussion of the experiment and data collection

As mentioned above, this experiment typically uses a scintillator, which makes use of a type of material, which absorbs the electromagnetic radiation of an incoming charged particle, and re-emits it as light (fluorescence).

Negatively charged muons, found on the Earth, are typically the result of cosmic rays interacting with particles in the Earth's atmosphere. Muons raining down on the Earth will, after traveling down to our lab, decay into an electron, as well as a electron-type antineutrino, and a muon-type neutrino (shown in FIG. 1).

When a muon collides with our scintillator, since it is charged, it will trigger a flash in the scintillator apparatus. The apparatus has a built in timer, which will activate when a flash is measured from the incoming muon. Once the initial timer is started, it will run until the muon decays. When the muon decays, producing the electron, the produced electron will trigger a second flash, stopping the timer. The differences in the start and stop time are recorded. These times are expected to follow an exponential distribution, which will give us information about the average muon life time.

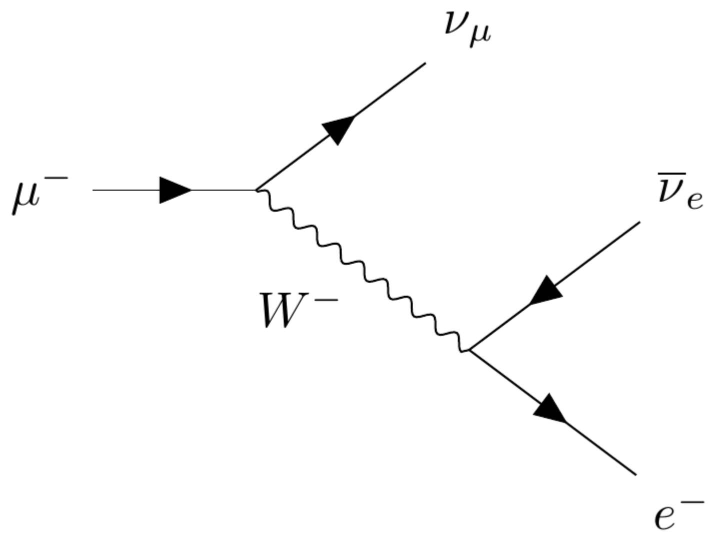

Figure 1: Feynman diagram of a typical negative muon decay

3 Code and Experimental Simulation

The data generation, and analysis is done using three separate codes: the first being a random class, which contains all of the code used to generate the distribution; second, there is the data generation code, which uses the random class to generate a list of data according to our chosen distribution; and, lastly, there is the analysis code, which collects and reads our data, uses the data to estimate the value of the fitting parameter and its error interval, then plots the data in a histogram.

In this experiment, the time-difference data will be randomly generated according an exponential distribution, this was chosen since exponential distributions are typically used to analyze decay rates. Our code takes a random float number R, between 0 and 1, and returns a random variable X from 1 to ∞, thrown according to an exponential distribution:

$$X = -\frac{1}{\lambda} ln\left(R\right) \tag{1}$$

Here, λ is the rate parameter, which controls how fast or slow the exponential decays. The Average lifetime of our muon is related to the rate parameter by $\tau = \frac{1}{\lambda}$

The data is generated using a distribution with a true rate parameter, $\lambda = 0.4545455 \frac{1}{\mu s}$ or $\tau = 2.2 \mu s$. To estimate the value of our parameter given the data, which was collected, the negative log of the likelihood, shown in equation 3, was minimized with respect to our parameter of interest, τ:

$$\frac{d(-ln(L(\tau)))}{d\tau} = 0 \tag{2}$$

such that the likelihood, $L(\tau)$, is given by:

$$L(\tau) = \prod_{i=1}^{N} P(x_i|\tau) \tag{3}$$

Here, N is the total number of measurements, and P is the probability density function (PDF) for an exponential distribution given by:

$$P(x_i|\tau) = \frac{1}{\tau} e^{-\frac{1}{\tau} X} \tag{4}$$

Finding the value of τ, which minimizes $-ln(L(\tau))$, will be a good estimate for the true value, τ_{true}.

The rest of the code is devoted to determining the upper and lower error bounds of our estimation. To find these bounds, the log likelihood ratio is examined. The upper and lower bounds of the estimated parameter are the roots of:

$$-\frac{1}{2} \leq ln\left(\frac{L(\tau)}{L(\tau_{best})}\right) \tag{5}$$

such that, $\tau_{low} \leq \tau_{best} \leq \tau_{high}$. These values are the bounds of the FWHM, which is width or difference between the log likelihood at 1/2 of the maximum. From equation 5, it is also clear that, at $\tau = \tau_{best}$, the function is at a max, since $ln(1) = 0 \geq -0.5$.

The next section will discuss and analyze the outputs of the experiment as well as the corresponding figures.

4 Output Analysis

In this experiment, 10,000 events were generated using the distribution, and plotted into a histogram (FIG. 2). After being fitted to an exponential curve, eq.4, minimization techniques were used to estimate the average muon life time: $\tau = 1/\lambda$. The average lifetime is estimated to be 2.194 μs. The error bounds are estimated to be $2.173 \leq \tau_{best} \leq 2.217$. This is close to the predicted error of $1/\sqrt{N} = 0.01$. The true value of the lifetime is around 2.2 μs, so the estimation is within 0.27 %.

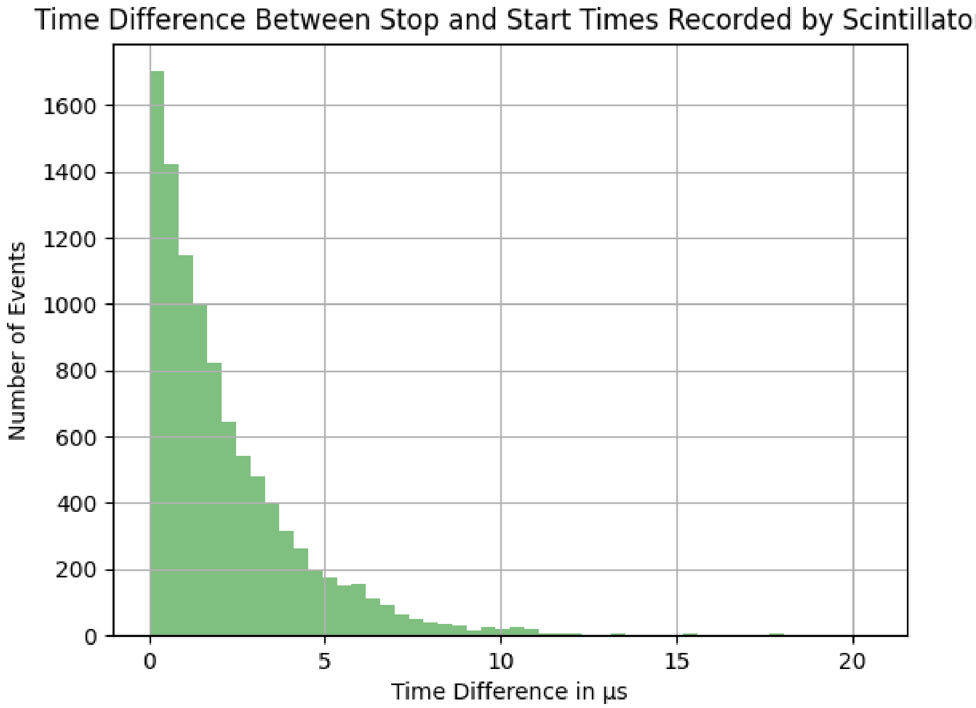

Figure 2: Histogram of time difference data taken for 10,000 events

5 Conclusion

To Conclude, in this experiment, the difference in time between a muon entering a scintillator, and the subsequent time taken when the muon decay is triggered, is recorded for 10,000 such events. The data is expected to follow an exponetial distribution, where the fitted rate parameter gives us information about the average life time of the negative muon. By preforming log-likelihood minimization techniques, this life time is estimated to be 2.194 μs, with error margains of 2.217 μs on the upper bound, and 2.173 μs on the lower bound. This gives us around 0.27 % error when compared to the true value of the average life time: 2.2 μs.

References

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