Stat 5814 Homework 2

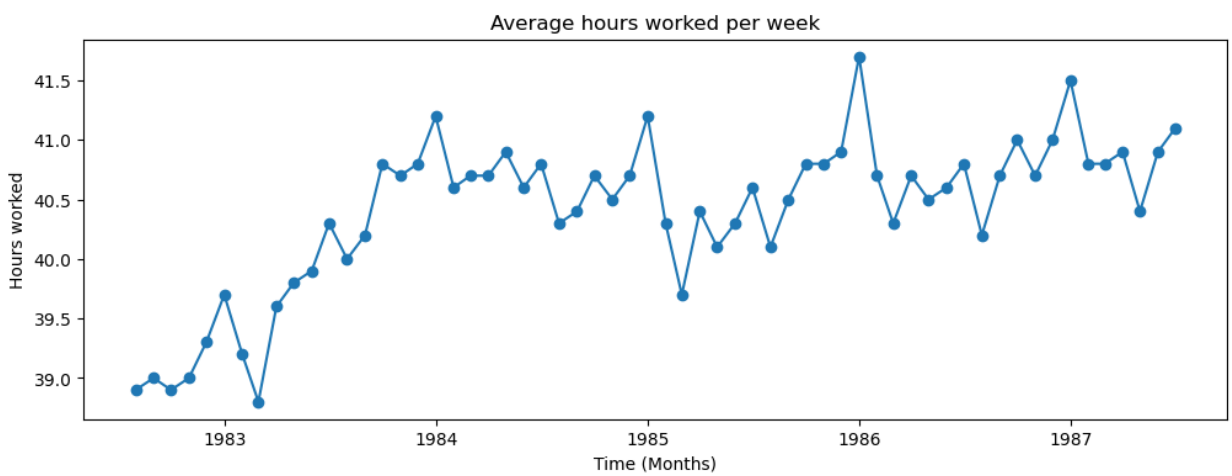
3.4

setup

```
# read the hours dataset into memory
hours = pd.read_csv(os.path.join(cs.DATASETS, "hours.dat"))
# create timestamps for the data
hours.set_index(pd.date_range("1982-07", periods=len(hours.index), freq=
```

a

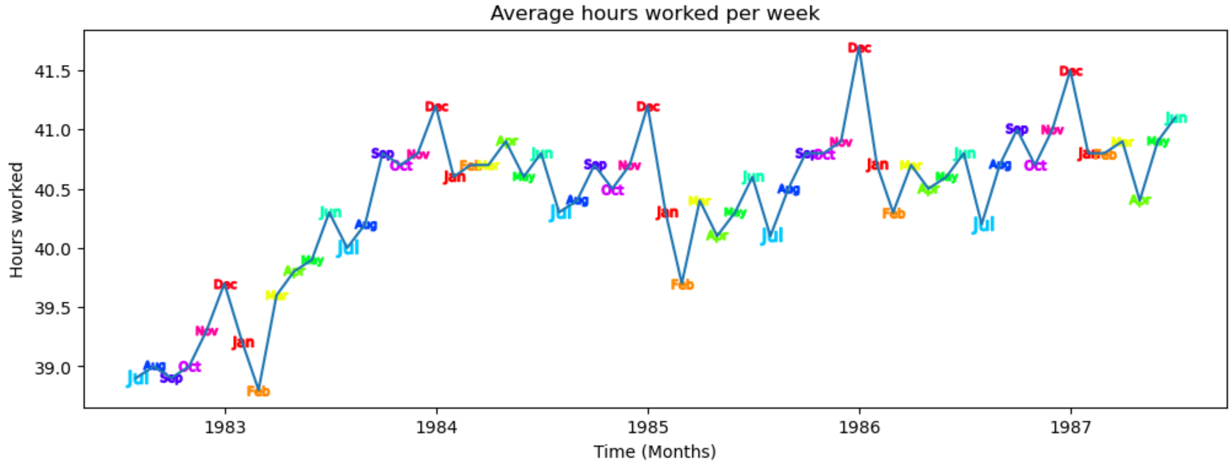
```
# plot the data
plot_ts(hours.index, hours["hours"], "Time (Months)", "Hours", "Plot of
```



Trends are difficult to clearly see using only this plot.

b

```
# plot the data with unique monthly markers
plot_monthly_markers(
    hours.index, hours["hours"], "Time (Months)", "Hours", "Plot of Hour
)
```



Using monthly markers, this data exhibits strong seasonality. For example, we can see that the number of hours worked always peaks in December

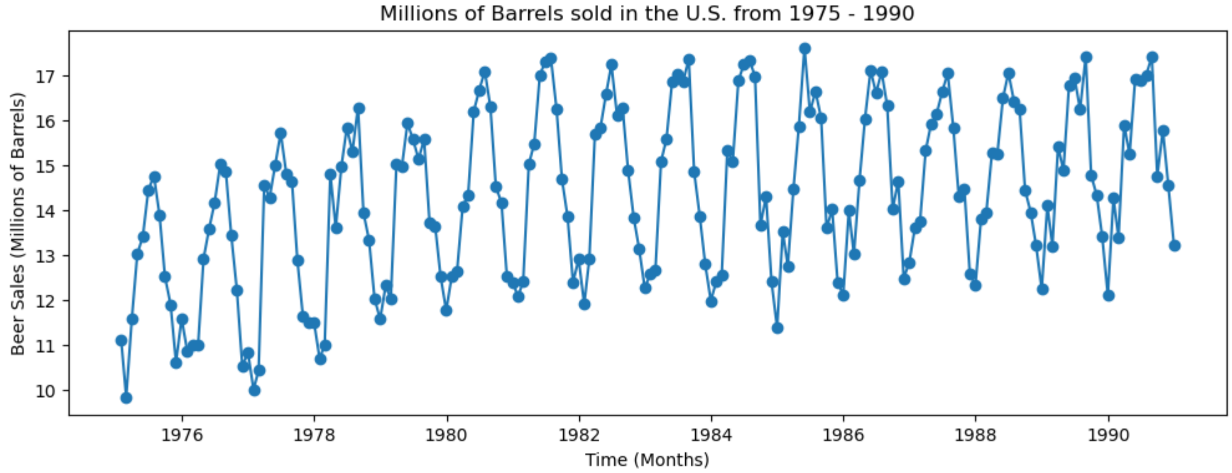
3.6

setup

```
# read dataset and set timestamp
beersales = pd.read_csv(os.path.join(cs.DATASETS, "beersales.dat"))
beersales.set_index(
    pd.date_range("1975-01", periods=len(beersales.index), freq="M"), in
)
beersales["month"] = beersales.index.month
beersales["t"] = (beersales.index - beersales.index[0]).days
```

a

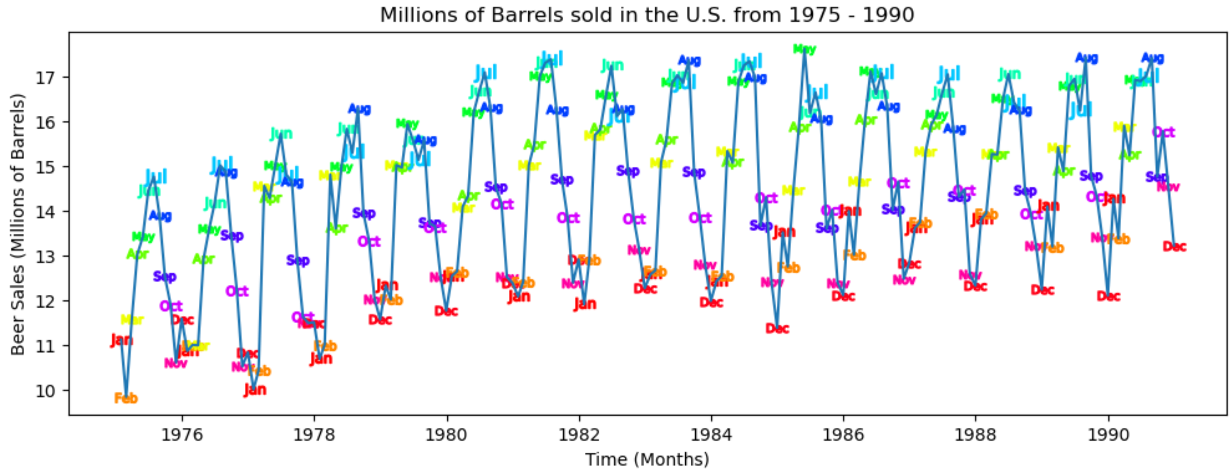
```
plot_ts(
    beersales.index,
    beersales["beersales"],
    "Time (Months)",
    "Beer Sales (Millions of Barrels)",
    "Millions of Barrels sold in the U.S. from 1975 - 1990",
)
```



Even without monthly markers, this data clearly exhibits a strong level of seasonality. However, we cannot easily see which months repeat.

b

```
plot_monthly_markers(
    beersales.index,
    beersales["beersales"],
    "Time (Months)",
    "Beer Sales (Millions of Barrels)",
    "Millions of Barrels sold in the U.S. from 1975 - 1990",
)
```



Using monthly markers, we can see that the number of beer sales peaks in the summer months and decreases in the winter months

c

```
model = smf.ols("beersales ~ C(month)", data=beersales).fit()
model.summary()
```

OLS Regression Results

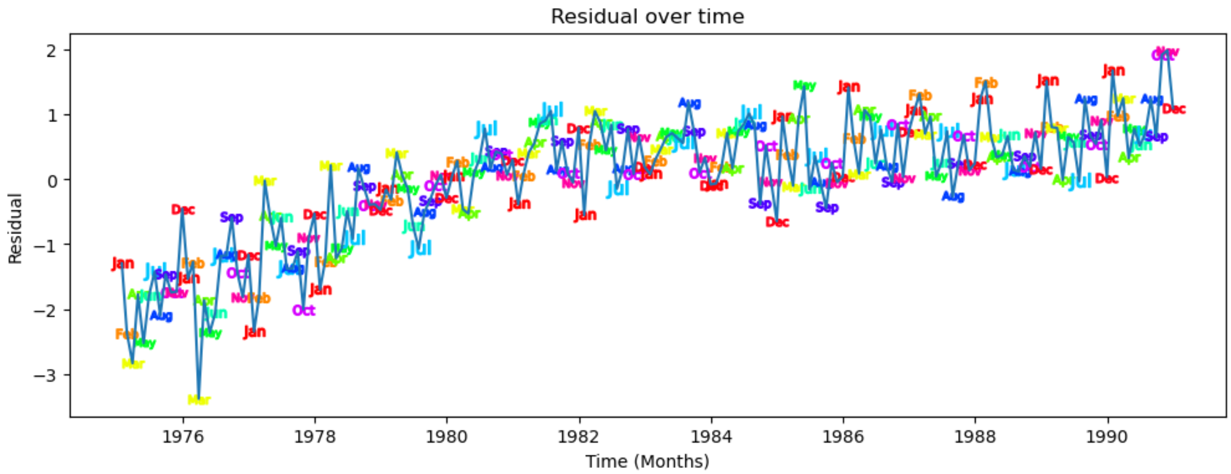
Dep. Variable:	beersales	R-squared:	0.710
Model:	OLS	Adj. R-squared:	0.693
Method:	Least Squares	F-statistic:	40.12
Date:	Sat, 10 Feb 2024	Prob (F-statistic):	1.12e-42
Time:	15:35:29	Log-Likelihood:	-276.64
No. Observations:	192	AIC:	577.3
Df Residuals:	180	BIC:	616.4
Df Model:	11		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	12.4857	0.264	47.309	0.000	11.965	13.006
C(month)[T.2]	-0.1426	0.373	-0.382	0.703	-0.879	0.594
C(month)[T.3]	2.0822	0.373	5.579	0.000	1.346	2.819
C(month)[T.4]	2.3976	0.373	6.424	0.000	1.661	3.134
C(month)[T.5]	3.5990	0.373	9.643	0.000	2.862	4.335
C(month)[T.6]	3.8498	0.373	10.314	0.000	3.113	4.586
C(month)[T.7]	3.7687	0.373	10.097	0.000	3.032	4.505
C(month)[T.8]	3.6088	0.373	9.669	0.000	2.872	4.345
C(month)[T.9]	1.5728	0.373	4.214	0.000	0.836	2.309
C(month)[T.10]	1.2544	0.373	3.361	0.001	0.518	1.991
C(month)[T.11]	-0.0480	0.373	-0.129	0.898	-0.784	0.689
C(month)[T.12]	-0.4231	0.373	-1.134	0.258	-1.160	0.313

Omnibus:	23.704	Durbin-Watson:	0.485
Prob(Omnibus):	0.000	Jarque-Bera (JB):	28.359
Skew:	-0.890	Prob(JB):	6.95e-07
Kurtosis:	3.614	Cond. No.	12.9

d

```
plot_monthly_markers(beersales.index, model.resid_pearson, "Time (Months
```



Seasonality across years seems to be gone. Sometimes the Winter months are now on top and the Summer months are on the bottom - or vice versa. However, each year still has many rises and falls int he beer sales. Additionally, the amount of beer sold seems to be increasing with each year.

e

```
model = smf.ols("beersales ~ t + np.power(t, 2) + C(month)", data=beersa
model.summary()
```

OLS Regression Results

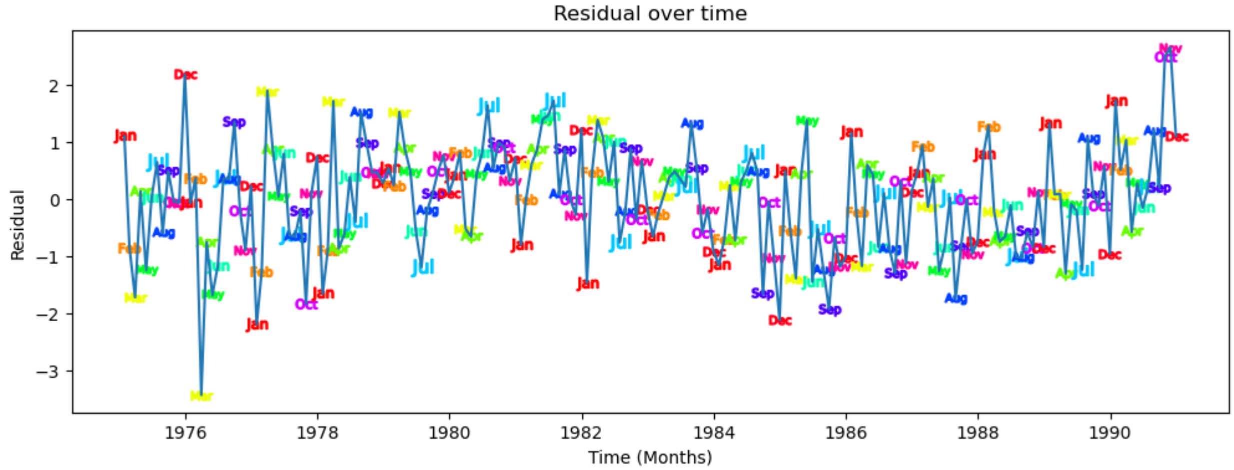
Dep. Variable:	beersales	R-squared:	0.910
Model:	OLS	Adj. R-squared:	0.904
Method:	Least Squares	F-statistic:	138.7
Date:	Sat, 10 Feb 2024	Prob (F-statistic):	9.47e-86
Time:	15:40:37	Log-Likelihood:	-164.23
No. Observations:	192	AIC:	356.5
Df Residuals:	178	BIC:	402.1
Df Model:	13		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	10.4667	0.188	55.629	0.000	10.095	10.838
C(month)[T.2]	-0.1568	0.209	-0.750	0.454	-0.569	0.256
C(month)[T.3]	2.0527	0.209	9.822	0.000	1.640	2.465
C(month)[T.4]	2.3535	0.209	11.261	0.000	1.941	2.766
C(month)[T.5]	3.5401	0.209	16.937	0.000	3.128	3.953
C(month)[T.6]	3.7768	0.209	18.068	0.000	3.364	4.189
C(month)[T.7]	3.6814	0.209	17.611	0.000	3.269	4.094
C(month)[T.8]	3.5075	0.209	16.777	0.000	3.095	3.920
C(month)[T.9]	1.4583	0.209	6.974	0.000	1.046	1.871
C(month)[T.10]	1.1264	0.209	5.387	0.000	0.714	1.539
C(month)[T.11]	-0.1889	0.209	-0.903	0.368	-0.602	0.224
C(month)[T.12]	-0.5770	0.209	-2.759	0.006	-0.990	-0.164
t	0.0012	0.000	12.412	0.000	0.001	0.001
np.power(t, 2)	-1.357e-07	1.68e-08	-8.095	0.000	-1.69e-07	-1.03e-07

Omnibus:	1.442	Durbin-Watson:	1.557
Prob(Omnibus):	0.486	Jarque-Bera (JB):	1.077
Skew:	-0.141	Prob(JB):	0.584
Kurtosis:	3.235	Cond. No.	1.89e+08

f

```
plot_monthly_markers(beersales.index, model.resid_pearson, "Time (Months
```



Similar to part d, the seasonal component across years is gone. However, this graph seems to have also removed the increasing trend of beer sales, leaving only the rises and falls in each year.

3.10

a

```
# create time index for estimation
hours["t"] = (hours.index - hours.index[0]).days
model = smf.ols("hours ~ t + np.power(t, 2)", hours).fit()
model.summary()
```

OLS Regression Results

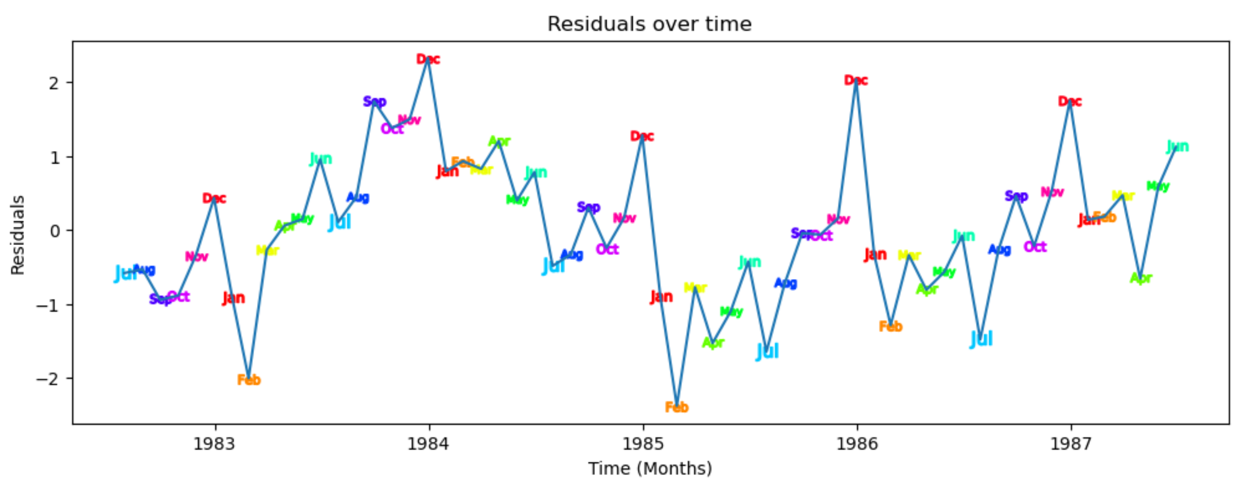
Dep. Variable:	hours	R-squared:	0.592
Model:	OLS	Adj. R-squared:	0.578
Method:	Least Squares	F-statistic:	41.38
Date:	Sat, 10 Feb 2024	Prob (F-statistic):	7.91e-12
Time:	16:20:15	Log-Likelihood:	-31.964
No. Observations:	60	AIC:	69.93
Df Residuals:	57	BIC:	76.21
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	39.1462	0.159	246.886	0.000	38.829	39.464
t	0.0026	0.000	6.297	0.000	0.002	0.003
np.power(t, 2)	-9.733e-07	2.2e-07	-4.425	0.000	-1.41e-06	-5.33e-07

Omnibus:	0.143	Durbin-Watson:	0.974
Prob(Omnibus):	0.931	Jarque-Bera (JB):	0.103
Skew:	0.090	Prob(JB):	0.950
Kurtosis:	2.904	Cond. No.	4.24e+06

b

```
plot_monthly_markers(hours.index, model.resid_pearson, "Time (Months)",
```



These residuals still show the same seasonality from the original hours worked dataset

c

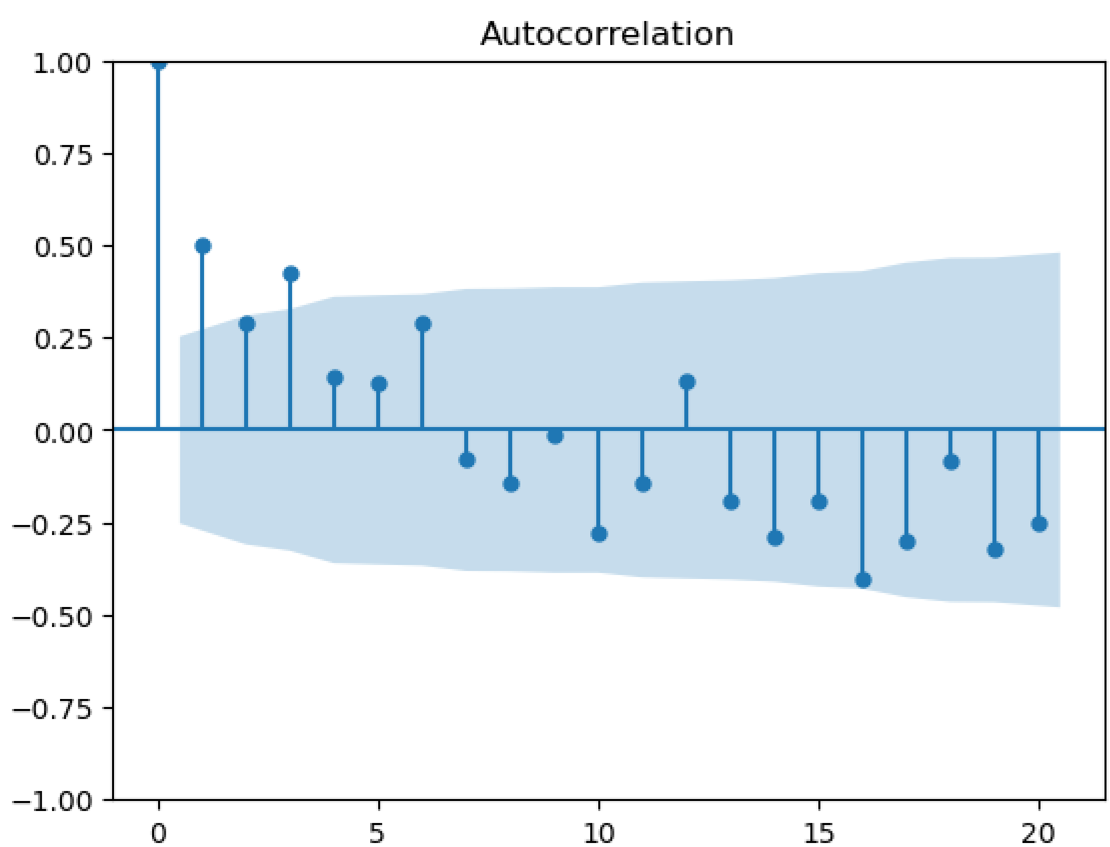
```python
# 3.10 c
from statsmodels.sandbox.stats.runs import runstest_1samp

z_score, p_value = runstest_1samp(model.resid_pearson, cutoff=0, correct
print(f'Z-score:\t {z_score}')
print(f'p-value:\t {p_value}')
```

```
Z-score:        -3.9019582005393216
p-value:        9.54176373202256e-05
```

The low p-value corresponds to the graph of residuals from part b, indicating that the quadratic least squares model is likely not a good fit for this dataset because the seasonality is preserved.

d

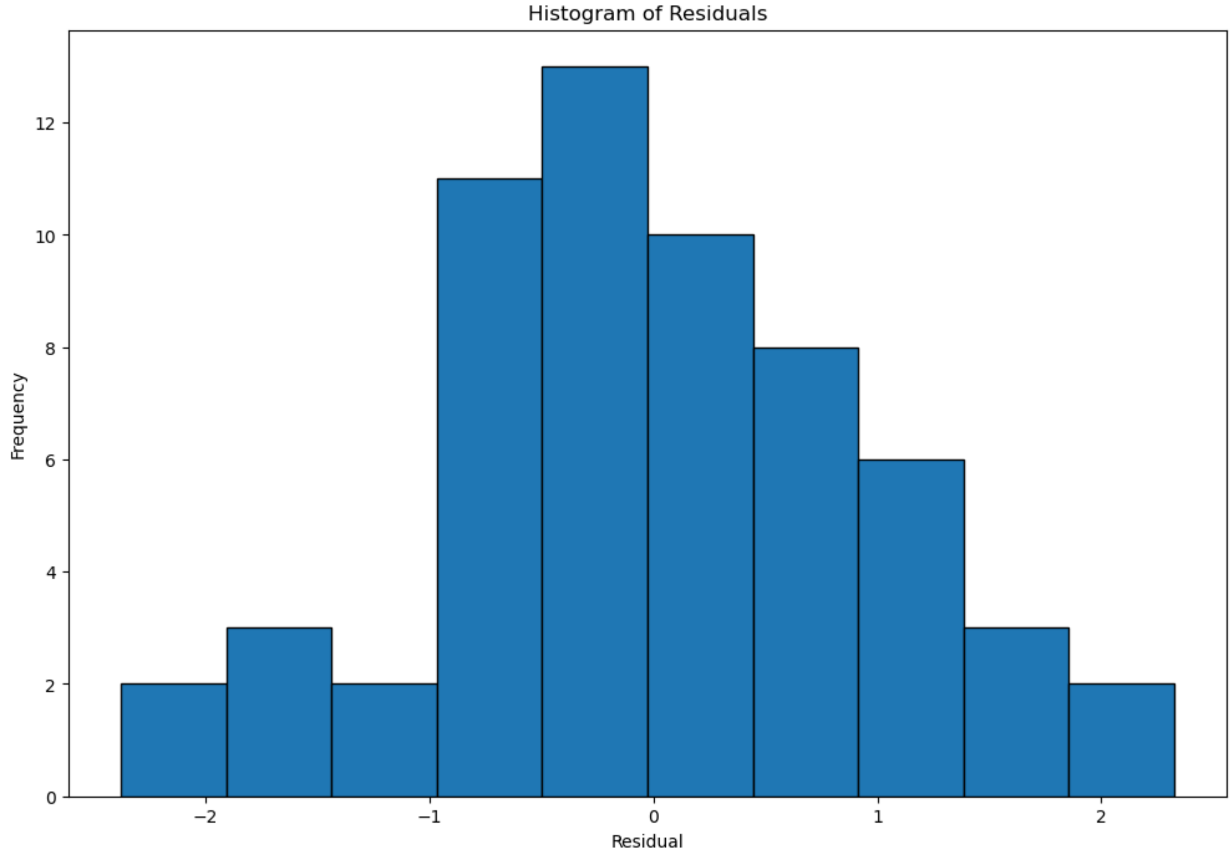
```python
sm.graphics.tsa.plot_acf(model.resid_pearson, lags=20)
```

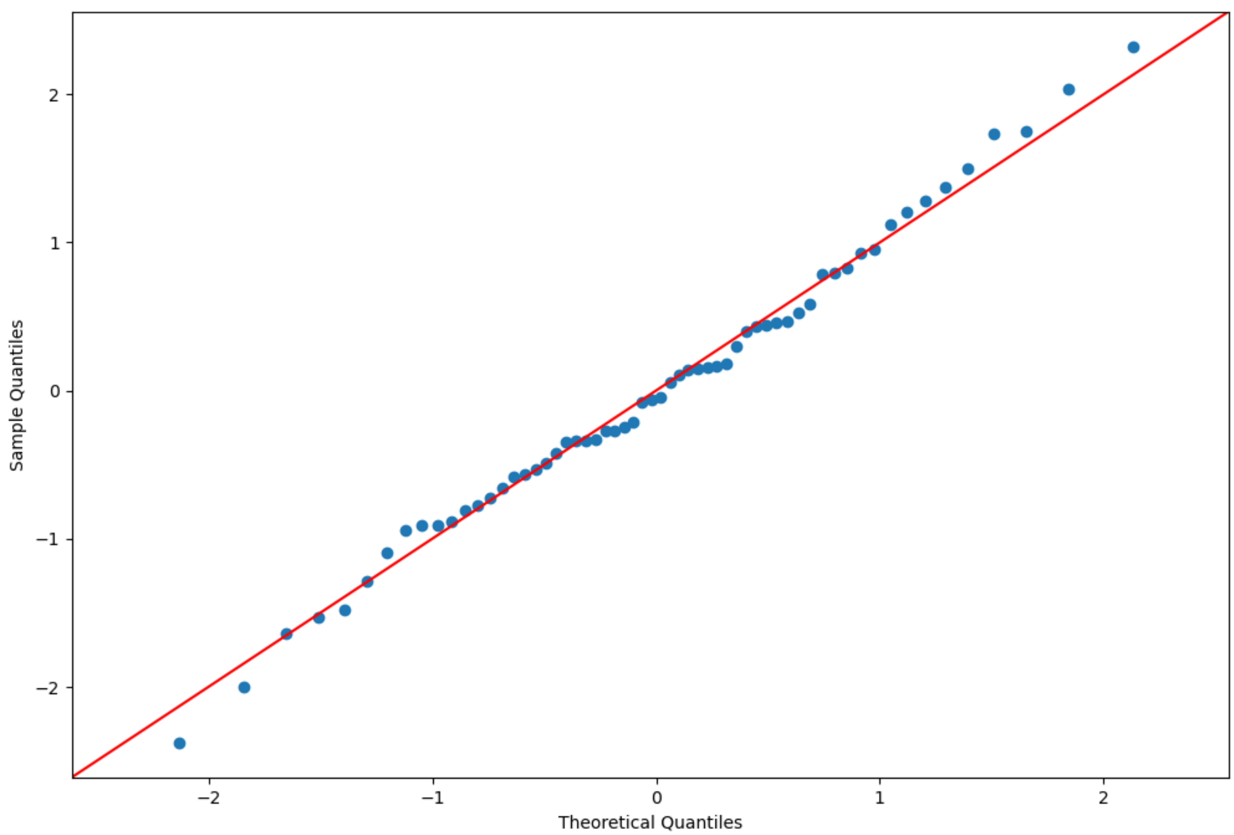


The autocorrelation functions shows that the first two lags are significant. The fourth lag also exits the boundary but may not be significant.

e

```
plt.figure(figsize=(12, 8))
plt.hist(model.resid_pearson, edgecolor="black")
plt.xlabel("Residual")
plt.ylabel("Frequency")
plt.title("Histogram of Residuals")
plt.show()

with plt.rc_context(plt.rc("figure", figsize=(12, 8))):
    pp = sm.ProbPlot(model.resid_pearson)
pp.qqplot(line="45")

plt.show()
```





The histogram appears to be slightly negatively skewed. The qq-plot seems to have more points on the top right side of the graph than it does on the bottom left.

3.12

a

```
model = smf.ols("beersales ~ t + np.power(t, 2) + C(month)", data=beersa
model.summary()
```

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C(month)[T.10]	1.1264	0.209	5.387	0.000	0.714	1.539
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t	0.0012	0.000	12.412	0.000	0.001	0.001
np.power(t, 2)	-1.357e-07	1.68e-08	-8.095	0.000	-1.69e-07	-1.03e-07

Omnibus:	1.442	Durbin-Watson:	1.557
Prob(Omnibus):	0.486	Jarque-Bera (JB):	1.077
Skew:	-0.141	Prob(JB):	0.584
Kurtosis:	3.235	Cond. No.	1.89e+08

b

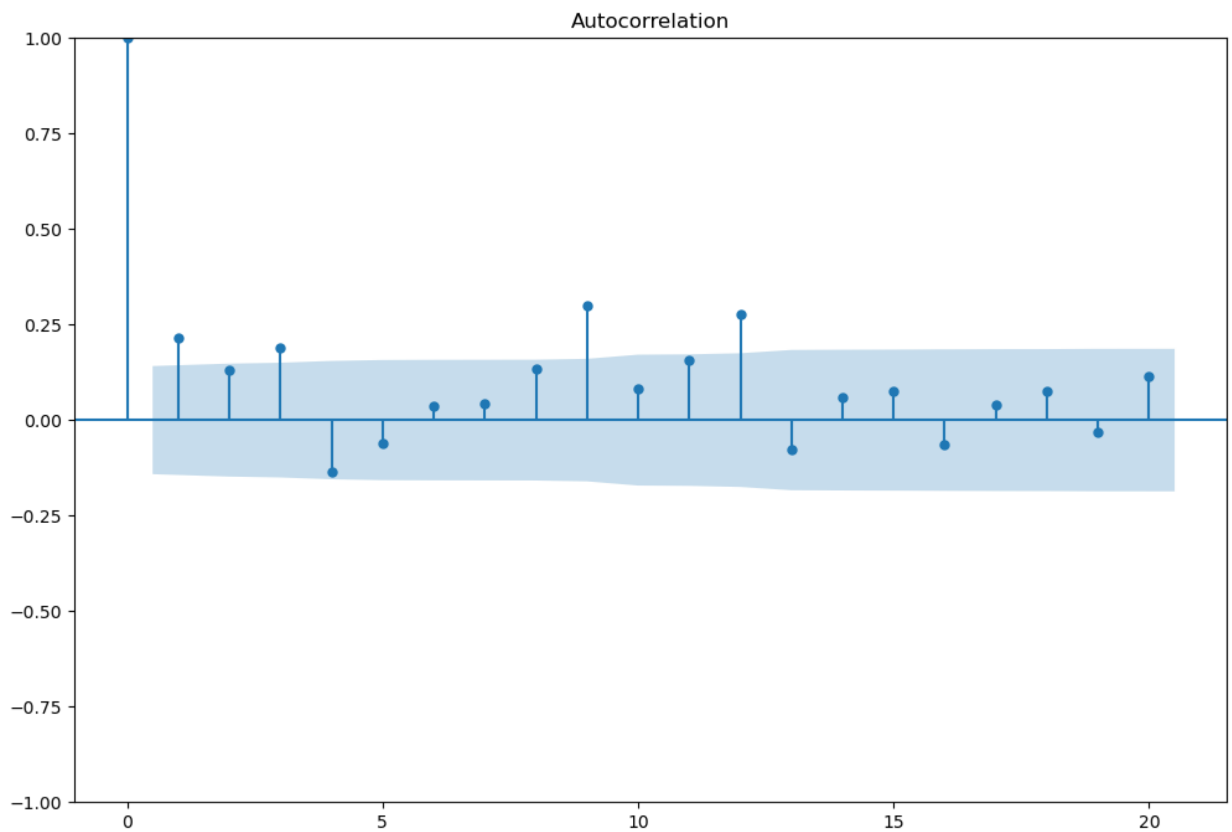
```
from statsmodels.sandbox.stats.runs import runstest_1samp

z_score, p_value = runstest_1samp(model.resid_pearson, cutoff=0, correct
print(f'Z-score:\t {z_score}')
print(f'p-value:\t {p_value}')
```

```
Z-score:        -2.5606907870154956
p-value:        0.010446428739886405
```

The small p-value indicates that the residuals till have exhibit a trend

c

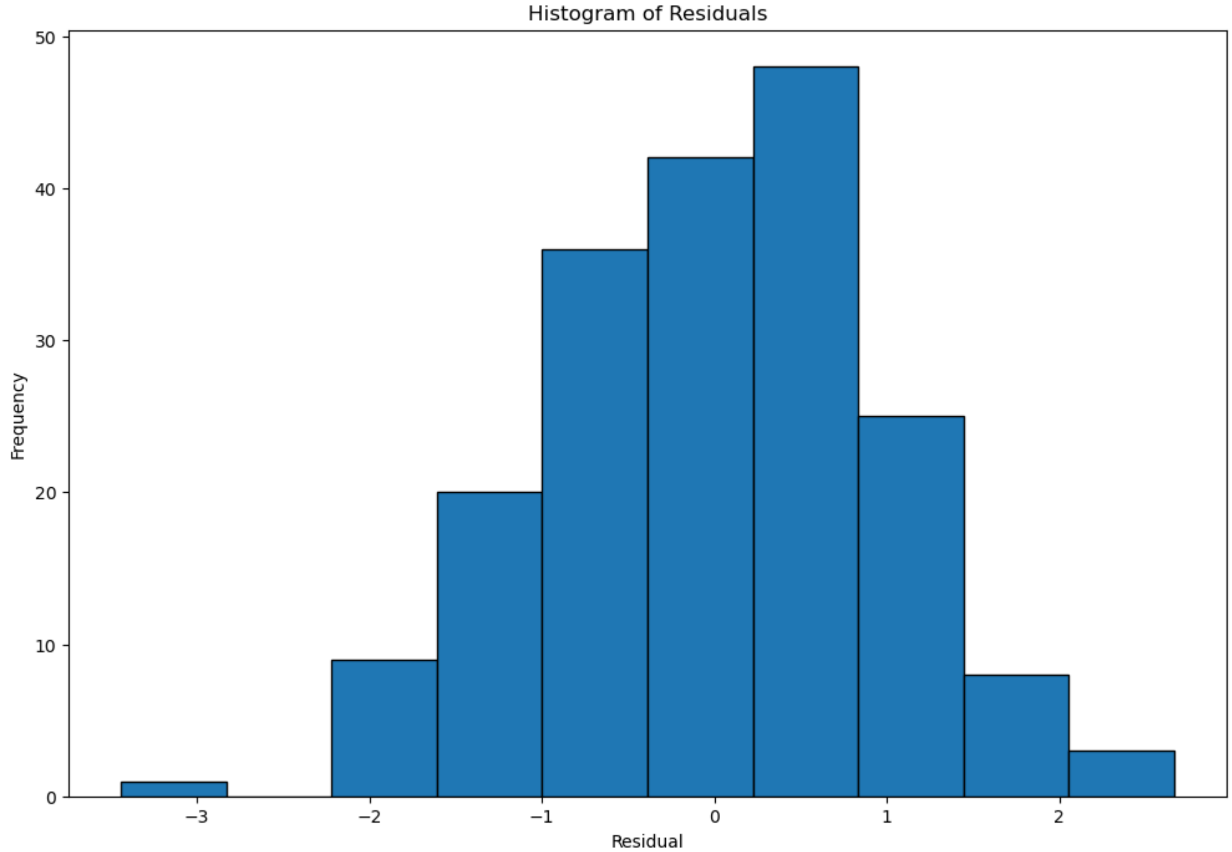
```
sm.graphics.tsa.plot_acf(model.resid_pearson, lags=20)
```

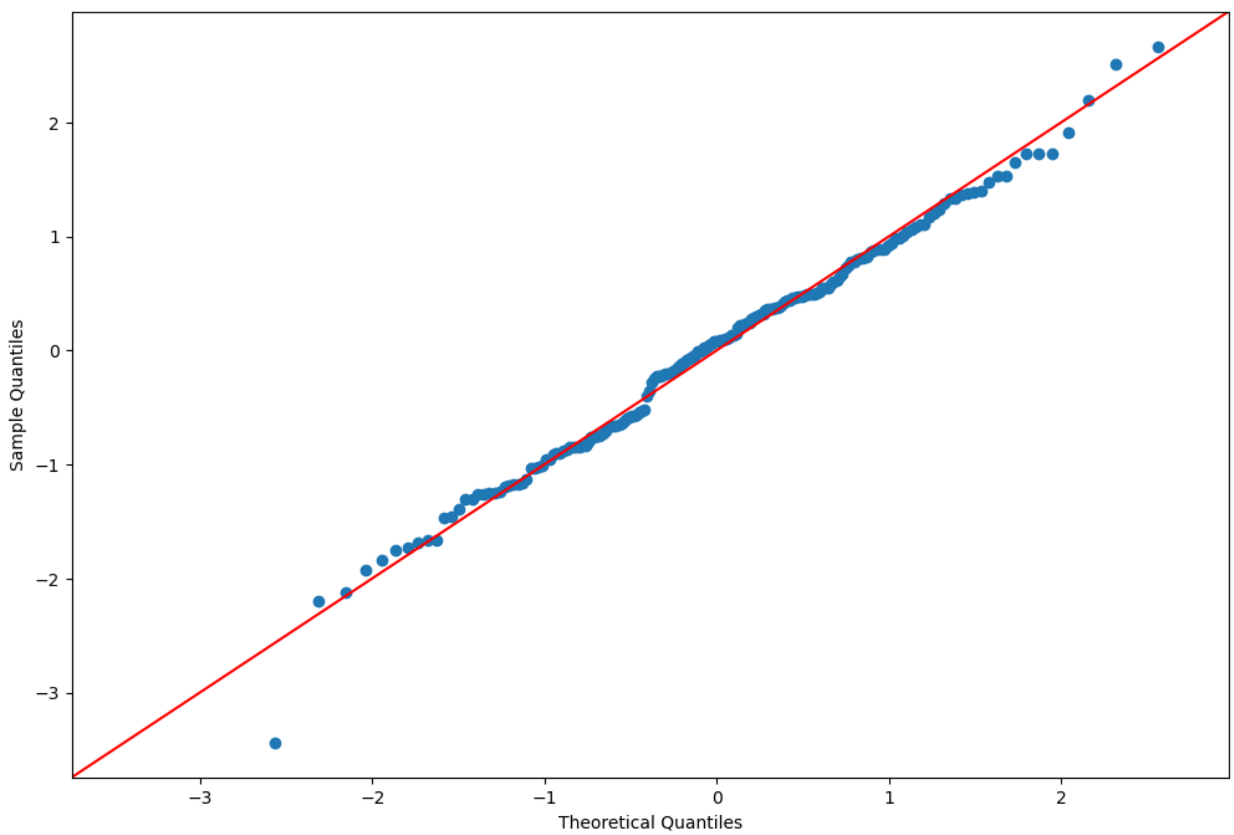


Some of the lags around 10 exit the boundary by a pretty significant margin. This could also indicate that some trends are apparent in the residuals

d

```
plt.figure(figsize=(12, 8))
plt.hist(model.resid_pearson, edgecolor="black")
plt.xlabel("Residual")
plt.ylabel("Frequency")
plt.title("Histogram of Residuals")
plt.show()

with plt.rc_context(plt.rc("figure", figsize=(12, 8))):
    pp = sm.ProbPlot(model.resid_pearson)
pp.qqplot(line="45")

plt.show()
```



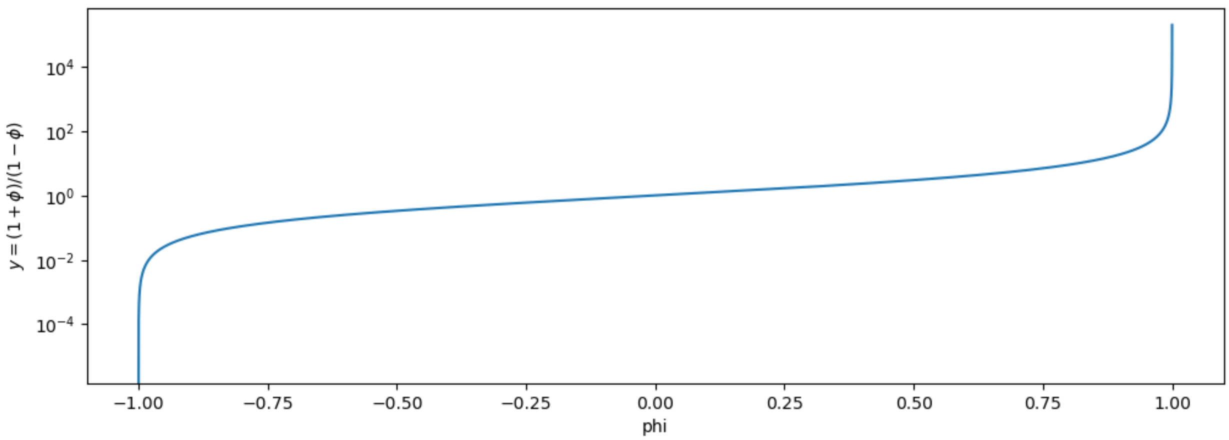


Much of the data from the qq-plot appears to follow the red line closely, indicating the residuals closely follow a normal distribution. The histogram also shows this, but additionally indicates that the data is slightly positively skewed

3.16

c

```
num_points = int((1 - (-1)) / 0.00001) + 1
phi = np.linspace(-1, 1, num_points)
y = (1 + phi) / (1 - phi)
plt.figure(figsize=(12, 4))
plt.plot(phi, y)
plt.xlabel("phi")
plt.ylabel(r"$y = (1 + \phi) / (1-\phi)$")
plt.yscale("log")
```



The process mean appears to get much more difficult to estimate as phi tends closer to +1 or -1. This is shown by the plot trending towards infinity as phi gets close to +1, and the plot trending towards 0 as phi gets close to -1.