MITALI Bela - PEER MEMABE 1 : λ_1

Given Matrix:

$$A = \begin{bmatrix} 4 & 8 & -1 & -3 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

Step 1: CHARACTERISTIC POLYNOMIAL

find eigenvalues, solve: $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{matrix} 4-\lambda & 8 & -1 & -3 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{matrix}$$

Step 2: DETERMINANT CALCULATION

$$\det(A - \lambda I) = (4-\lambda) \times M_{11} - 8 \times M_{12} + (-1) \times M_{13} + (-3) \times M_{14}$$

Where M_{11} M_{12} M_{13} M_{14} are the 3x3 minor

$\lceil\lambda_{11} = \det \; -9-\lambda \;\; -2 \;\; -4 = (-9-\lambda)[(5-\lambda)(-13-\lambda)$
$+ 140] - (-2)[(-2)(-13-\lambda) + 40]$
$+ (-4)[(-2)(10) - 10(5-\lambda)]$

$$\begin{bmatrix} 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

CHARACTERISTIC POLYNOMIAL:

$$\lambda^4 + 24\lambda^3 + 167\lambda^2 + 462\lambda + 420 = 0$$

$\lambda_1 = -21.186$
$\lambda_2 = -5.61$
$\lambda_3 = 2.8$
$\lambda_4 = 10.9$

Step 4: $\lambda_1 = -21.18634915637137$

Find the eigenvector:

$v_1: (A - \lambda_1 I) v_1 = 0$

Step 5: Verification

$$A \cdot \vec{v}_1 = \lambda_1 \cdot \vec{v}_1$$

Given Matrix A:

$$A = \begin{bmatrix} 4 & 8 & -1 & -3 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -3 \end{bmatrix}$$

eigen value $\lambda_1 = -21.186$

eigen vector $\vec{v}_1$:

$$\vec{v}_1 = \begin{bmatrix} 0.0113 \\ 0.3696 \\ 0.2407 \\ 1.000 \end{bmatrix}$$

Left side: $A \cdot \vec{v}_1$

$$A \cdot \vec{v}_1 = \begin{bmatrix} -0.2387 \\ -7.8301 \\ -5.0976 \\ -21.186 \end{bmatrix}$$

Right side: $\lambda_1 \cdot \vec{v}_1$

$$\lambda_1 \cdot \vec{v}_1 = -21.186 \times$$

$$\begin{bmatrix} 0.0113 \\ 0.3696 \\ 0.2407 \\ 1.0000 \end{bmatrix} = \begin{bmatrix} -0.2387 \\ -7.8301 \\ -5.0976 \\ -21.186 \end{bmatrix}$$

Verified.

$$(A - \lambda_1 I) = \begin{bmatrix} 25.186 & 8 & -1 & -3 \\ -2 & 12.186 & -2 & -4 \\ 0 & 10 & 26.186 & -10 \\ -1 & -13 & -14 & 8.186 \end{bmatrix}$$

~~25.186 8 -1 -3 -2 0 -2 12.186 -2 -4~~

$$\begin{bmatrix} 25.186 & 8 & -1 & -3 \\ -2 & 12.186 & -2 & -4 \\ 0 & 10 & 26.186 & -10 \\ -1 & -13 & -14 & 8.186 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 0.011 \\ 0.369 \\ 0.240 \\ 1 \end{bmatrix}$$

IMPORTANCE CALCULATION

$$\text{Total magnitude} = |\lambda_1| + |\lambda_2| + |\lambda_3| + |\lambda_4| = 21.186 + 5.613 + 2.812 + 10.987$$

$$= 40.598$$

$$\lambda_1 \text{ importance} = (21.186 / 40.598) \times 100\%$$

$$= 52.19\%$$

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$A - \lambda I$ for $\lambda = 2.675$

$$A - 2.675I = \begin{bmatrix} 4-2.675 & 8 & -1 & -2 \\ -2 & -9-2.675 & -2 & -4 \\ 0 & 10 & 5-2.675 & -10 \\ -1 & -13 & -14 & -13-2.675 \end{bmatrix}$$

$$= \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

$(A - 2.675I)V = 0$

$$\begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & 13 & -14 & -15.675 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = 0$$

Reduction results; $V_1 = 0.9583$, $V_2 = -0.1610$, $V_3 = 0.2074$

$V_4 = -0.1128$

$$\therefore \boxed{\text{Eigenvector } V_3 = \begin{bmatrix} 0.9583 \\ -0.1610 \\ 0.2074 \\ -0.1128 \end{bmatrix}}$$

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -4 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

① Eigen Values :

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -4-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

First term :

$$4-\lambda \begin{vmatrix} -4-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$$

Second term :

$$+8 \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$$

Third term

$$-1(-1)\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$$

Fourth term

$$-2\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

Subtiting and Simplifying the Minors.

$$\Rightarrow (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(-2\lambda^2 - 22\lambda + 370) - 1(\lambda^3 + 390) + 2(\lambda^2 + 22\lambda + 275)$$

$$\Rightarrow \lambda^4 + 73\lambda^3 - 214\lambda^2 - 835\lambda + 3080 = 0$$

Find the Eigen Values yields:

$\lambda_1 \simeq -21.125$, $\lambda_2 \simeq -5.604$, $\lambda_3 \simeq 2.675$

$\lambda \simeq 11.054$

Finding eigen Vectors
Subtituting $\lambda \approx -5.604$

$A + \cancel{2.6}\,5.604\, I =$

$$\Rightarrow \begin{vmatrix} 9.604 & 8 & -1 & -2 \\ -2 & 1.604 & -2 & -4 \\ 0 & 10 & 10.604 & 10 \\ -1 & -13 & -14 & \cancel{-7.604}\ -7.396 \end{vmatrix}$$

$$\text{let } \vec{V} = \begin{vmatrix} x \\ y \\ z \\ w \end{vmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 9.604 & 8 & -1 & -2 & 0 \\ -2 & 1.604 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & 10 & 0 \\ -1 & -13 & -14 & \cancel{-3.604}\ -7.396 & 0 \end{array}\right]$$

Gaussian Elimination

Row 1: $9.604V_1 + 8V_2 - V_3 - 2V_4 = 0$

Express V_1 in terms of V_2, V_3, V_4

$$V_1 = \frac{-8V_2 + V_3 + 2V_4}{9.604}$$

Row 2: $-2V_1 + 1.604V_2 - 2V_3 - 4V_4 = 0$

Substitute V_1 from Row 1

$$-2\left(\frac{-8V_2 + V_3 + 2V_4}{9.604}\right) + 1.604V_2 - 2V_3 - 4V_3 = 0$$

Simplify to relate V_2, V_3, V_4.

③ Row 3: $10V_2 + 10.604V_3 - 10V_4 = 0$

$$V_2 + 1.0604V_3 - V_4 = 0$$

$$V_2 = -1.0604V_3 + V_4$$

Row 4: $-V_1 - 13V_2 - 14V_3 - 7.396V_4 = 0$

Substitute V_1 and V_2 to express everything in terms of V_3 and V_4

$V_4 = t$ (a free parameter)

Row$_3$: $V_2 = -1.0604 V_3 + t$

Row$_1$ to find V_1 in terms of V_3 and t

Row 4: to solve for V_3 in terms of t

$$V \approx \begin{vmatrix} -0.48 \\ 1 \\ -0.52 \\ 1 \end{vmatrix}$$

my final Eigen Vector for $\lambda = 5.604$

$$\begin{vmatrix} -0.48 \\ 1 \\ -0.52 \\ 1 \end{vmatrix}$$

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$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

Subtracting λ

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

$$\Rightarrow -10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(14\lambda+100) + (5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(-\lambda-9)-52)-210)-1950$$

$$\Rightarrow -10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(14\lambda+100) + (5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(-\lambda-9)-52)-210) - 1950 = 0$$

$\lambda_1 = -21.12$, $\lambda_2 = -5.60$, $\lambda_3 = 2.67$, $\lambda_4 = 11.05$

eigenvector λ_4

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \\ -7.054 & 8 & -1 & -2 \\ 2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ 1 & -13 & -14 & -24.05 \end{bmatrix}$$

$$= \left(\begin{bmatrix} -0.07 \\ -0.02 \\ -1.69 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

Subtract column 2 multiplied by $\frac{1}{2} - \frac{\lambda}{10}$ from column 3

$$C_3 = C_3 - \left(\tfrac{1}{2} - \tfrac{\lambda}{10}\right) C_2$$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} = \begin{bmatrix} 4-\lambda & 8 & \frac{4\lambda}{5}-5 & -2 \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -4 \\ 0 & 10 & 0 & -10 \\ -1 & -13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-13 \end{bmatrix}$$

~~Add Column~~ $C_4 = C_4 + C_2$

$$\begin{vmatrix} 4-\lambda & 8 & \frac{4\lambda}{5}-5 & -2 \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -4 \\ 0 & 10 & 0 & -10 \\ -1 & -13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-13 \end{vmatrix} =$$

$$\begin{vmatrix} 4-\lambda & 8 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & 6 \\ -2 & -\lambda-9 & & -\lambda-13 \\ 0 & 10 & 0 & 0 \\ 1 & 13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{vmatrix}$$

row 3

$$\begin{vmatrix} 4-\lambda & 8 & \frac{4\lambda}{5}-5 & 6 \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -\lambda-13 \\ 0 & 10 & 0 & 0 \\ -1 & -13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{vmatrix} =$$