CS 440: Probabilistic Search

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0.1 Abstract

In this project, we demonstrated searching for a cell given probabilities of its location and using prior beliefs while iterating.

0.2 Academic Integrity

0.3 Problem 1

Assume:

- Belief$[C_i]_t$ is the belief that the target is in cell C_i at time t. It is calculated using $P(in(C_i)|O_t)$.
- $P(F(C_j))$ is the probability that the target is not found when searching cell C_j. It is calculated using $P(F(C_j)|in(C_j))*P(in(C_j))+P(!in(C_j))$, the probability that the target is not found in C_j if the target is in C_j added to the probability that the target is not in C_j.
- $P(in(C_i))$ is the probability that the target is in cell C_i, 1/the number of cells.
- O_t is Observations at time t.
- $N_{i,t}$ is the number of times cell C_i has been observed and resulted in a failure at time t.
- Belief$[C_i]_{t+1}$, is belief that the target is in cell C_i after applying a new observation, that the target is not at C_j. It is calculated by $P(in(C_i)|O_t \wedge F(C_j))$.

First, for the cell that was just observed as a failure, Belief$[C_j]_t$ will change to Belief$[C_j]_{t+1}$, or $P(in(C_i)|O_{t+1})$.

This can be converted to $P(O_{t+1}|in(C_j)) * P(in(C_j))/P(O_{t+1})$.

$P(O_{t+1}|in(C_j))$ converts to $P(F(C_j)|in(C_j))^{N_j}$ because given that the target is in C_j, the probabilities of not finding the target in all other cells becomes 1. Then, the probability of not finding the target in C_j is $P(F(C_j)|in(C_j))^{N_j}$

because the cell is searched N_j times and each had $P(F(C_j)|in(C_j))$ to fail.

$P(O_{t+1})$ converts to $P(O_t) * (P(in(C_j)) * P(F(C_j)|in(C_j)) + (1 - P(C_j)))$. $(P(in(C_j))*P(F(C_j)|in(C_j))+(1-P(C_j)))$ is the probability of either failing a search while the target is in C_j or of the target not being in C_j. $P(O_t)$ is multiplied by this probability to find the overall probability of the previous events AND the new observed event happening.

Dividing Belief$[C_j]_{t+1}$ by Belief$[C_j]_t$ will give the multiplicative difference between Belief$[C_j]_t$ and Belief$[C_j]_{t+1}$.
This is: $P(F(C_j)|in(C_j))^1/(P(in(C_j)) * P(F(C_j)|in(C_j)) + (1 - P(C_j)))$

So, multiplying the current belief by $P(F(C_j)|in(C_j))^1/(P(in(C_j))*P(F(C_j)|in(C_j))+(1 - P(C_j)))$ will update the belief for cell C_j.

This changes the total belief on the map by ΔBelief$[C_j]$, so dividing ΔBelief$[C_j]$ by the number of cells excluding C_j will be the belief all other cells will each increase by.

0.4 Problem 2

The probability that the target will be found in cell C_i given observations is:
Belief$[C_i] * (1 - P(\text{Target not found in Cell}_i|\text{Target is in Cell}_i))$

0.5 Problem 3

0.6 Problem 4