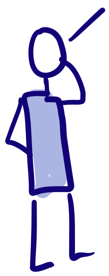
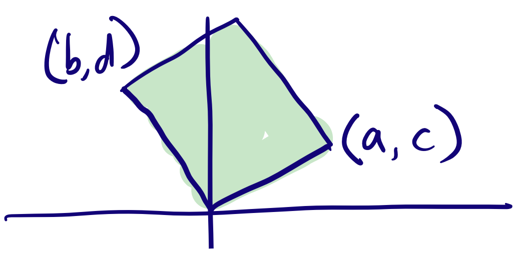
# A graphic linear Algebra refresher



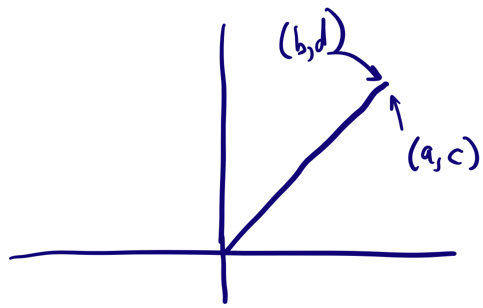
where to start...
How about the determinant

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det A = ad - bc$$

It is the <u>area</u> of a parallelogram defined by $(a, c)$ and $(b, d)$




non-zero determinant




zero-determinant

In 3D it is the <u>volume</u> of a parallelpiped defined by the column vectors.

$$A = \begin{array}{c} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\ \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \end{array} \qquad \det A = (a_1 \times a_2) \cdot a_3$$

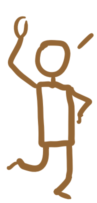
The determinant is related to how much space a matrix occupies, or how big it is in some sense.

In the linear algebra equation

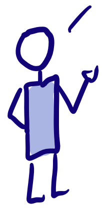
$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

$\underline{\underline{A}}$ must have a non-zero determinant and $\underline{b}$ cannot all be zero to have a solution.

ooo, then the solution is $\underline{x} = (\underline{\underline{A}}^T \underline{\underline{A}})^{-1} \underline{\underline{A}}^T \underline{b}$!



# Matrix Multiplication



The first thing to keep in mind about Matrix multiplication is that

**THE SHAPES MATTER!**

Let's start in 1D. When we say

$A \cdot B$ (Read as A dot B), it means

$$\begin{bmatrix} a_0 & \cdots & a_n \end{bmatrix} \begin{bmatrix} b_0 \\ \vdots \\ b_n \end{bmatrix}$$

we think of $\underline{A}$ as a row vector and $\underline{B}$ as a column vector, and Both have $\underline{n}$ elements.

$$\overset{\underline{A}}{(1 \times n)}\overset{\underline{B}}{(n \times 1)} \Rightarrow (1 \times 1)$$

these must be the same

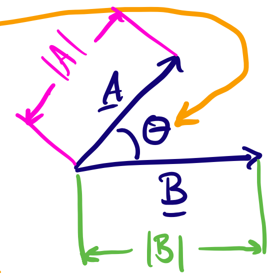
The dot product produces a scalar, i.e. one number.

If $\underline{A} \cdot \underline{B} = 0$ we say A is orthogonal to B.

why orthogonal?

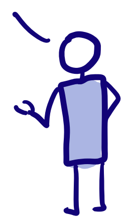
The dot product is also defined like this.

$$\underline{A}\cdot\underline{B} = \underbrace{|A|}_{\text{length of } A}\ \underbrace{|B|}_{\text{length of } B}\ \underbrace{\cos\theta}_{\text{Angle between } \underline{A} \text{ and } \underline{B}}$$



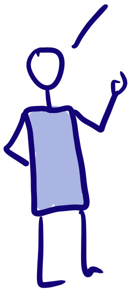

Professor, what about matrix multiplication?

OK, for a matrix times a vector, it is like many dot products



$$\begin{array}{c} \underline{a_0} \\ \underline{a_1} \\ \underline{a_2} \end{array}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\underline{b}} = \overbrace{\begin{bmatrix} \underline{a_0}\cdot\underline{b} \\ \underline{a_1}\cdot\underline{b} \\ \underline{a_2}\cdot\underline{b} \end{bmatrix}}^{\text{column vector dot products}} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$$

Shape analysis $(3\times \underbrace{3)\ \ (3}_{\text{these must be the same}}\times 1) \quad = \quad (3\times 1)$

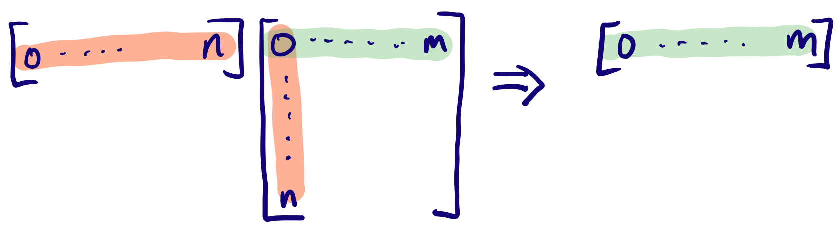


Here the matrix acts as a Transformer.
There's more than meets the eye.

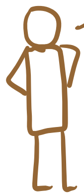
In $\underline{\underline{A}}\,\underline{x} = \underline{b}$ we say $\underline{\underline{A}}$ transforms $\underline{x}$ into $\underline{b}$

The transformation can be a scaling, or a rotation, or a combination of those.

What about a vector times a matrix?
Yes, if the shapes line up.




$$(1 \times n)\,(n \times m) \Rightarrow (1 \times m)$$

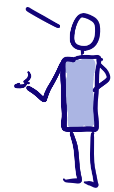


Tell me more about this transformation idea.

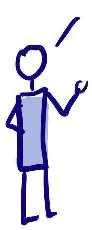
$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\underline{\underline{I}}} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \cdot a + 0 \cdot b \\ 0 \cdot a + 1 \cdot a \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

the identity matrix

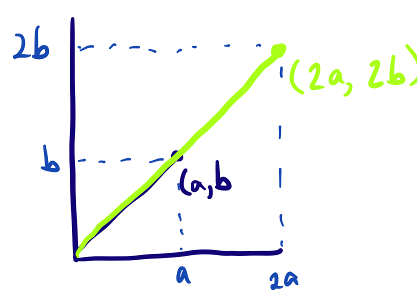
Not really a transform, or is it?



$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$



This matrix doubles the vector, it is a scaling matrix



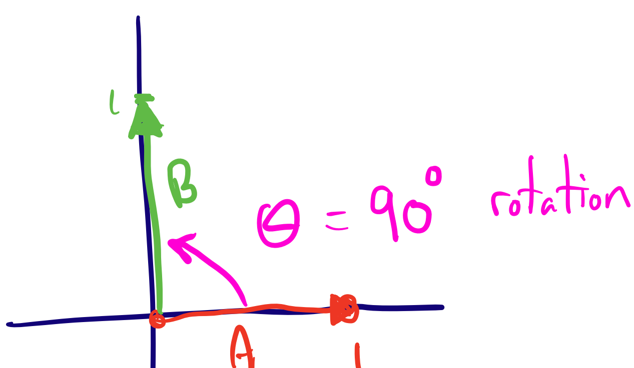
Here is a rotation.

$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix} = \text{a rotation matrix with}$$

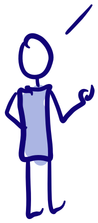
let $\theta = 90^\circ = \cos\theta = 0$

$\sin\theta = 1$

$$\begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}\overset{A}{\begin{bmatrix}1 \\ 0\end{bmatrix}} = \overset{B}{\begin{bmatrix}0 \\ 1\end{bmatrix}}\begin{array}{l}\leftarrow x \\ \leftarrow y\end{array}$$

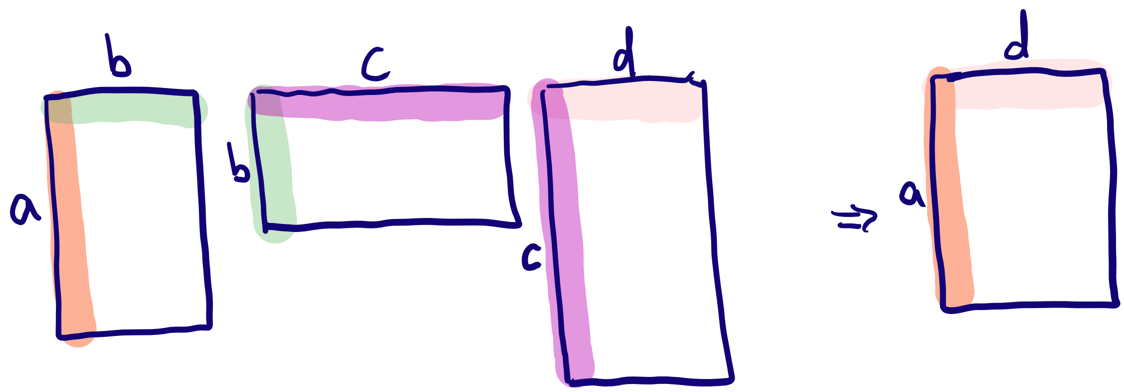



The length here does not change.



We can also do matrix muliplication on non-square arrays. AS LONG AS THE SHAPES ALIGN!

$$(a \times b) @ (b \times c) @ (c \times d) \Rightarrow (a \times d)$$




There is a lot more, but...
You probably need a break now!