

# Nonlinear Algebra



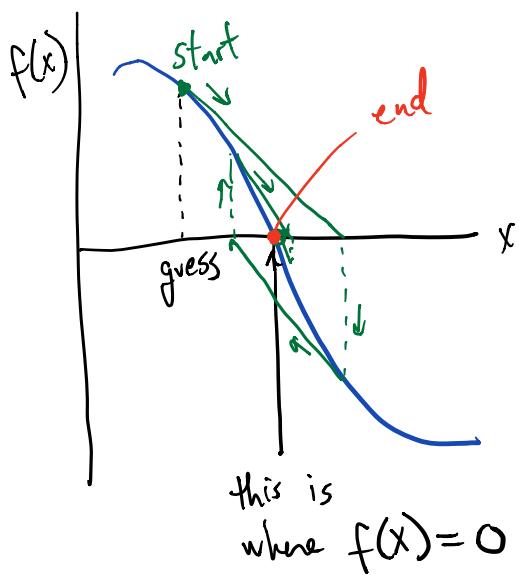
$$f(\underline{x}) = \underline{0}$$

↑                    ↑  
Variables            one zero for  
we want            each equation  
to solve  
for

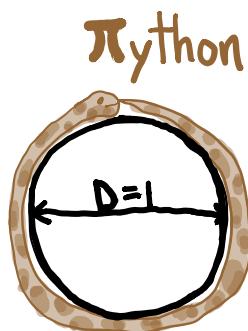
We need at least one  
independent equation for  
each variable

Solving nonlinear equations with `fsolve` is  
iterative and requires an initial guess  
to start.

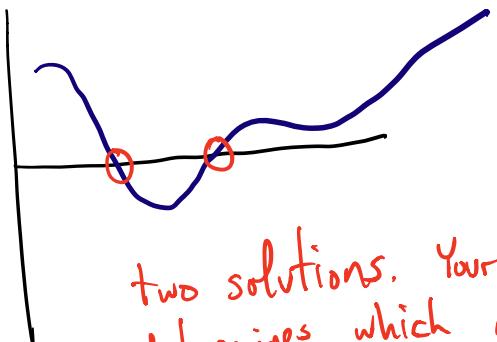
This is why we have to iterate.



A rarely seen



There can be more than one place  
where  $f(x)=0$ !



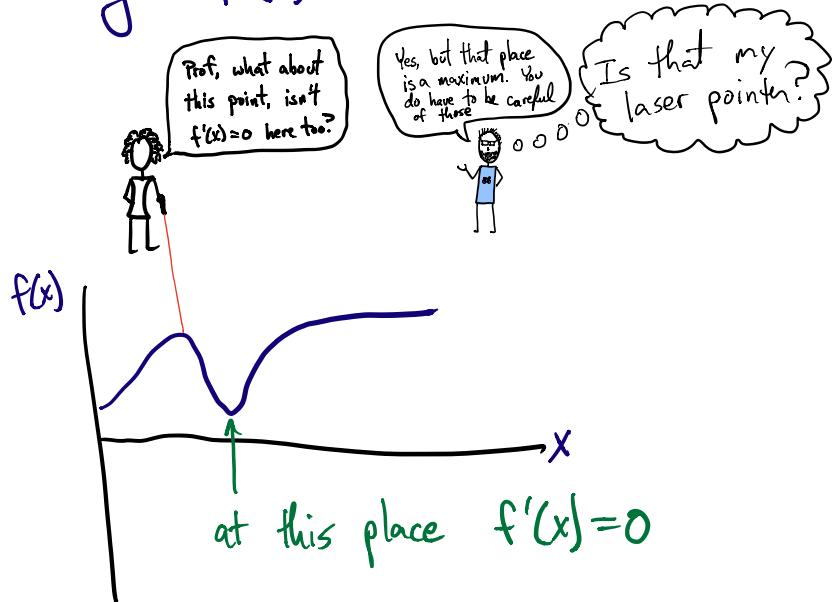
two solutions. Your initial guess  
determines which one you get.

It is your job as an engineer to say  
which one is right.

# Minimize



You can think of minimize as  
solving  $f'(x) = 0$ ,



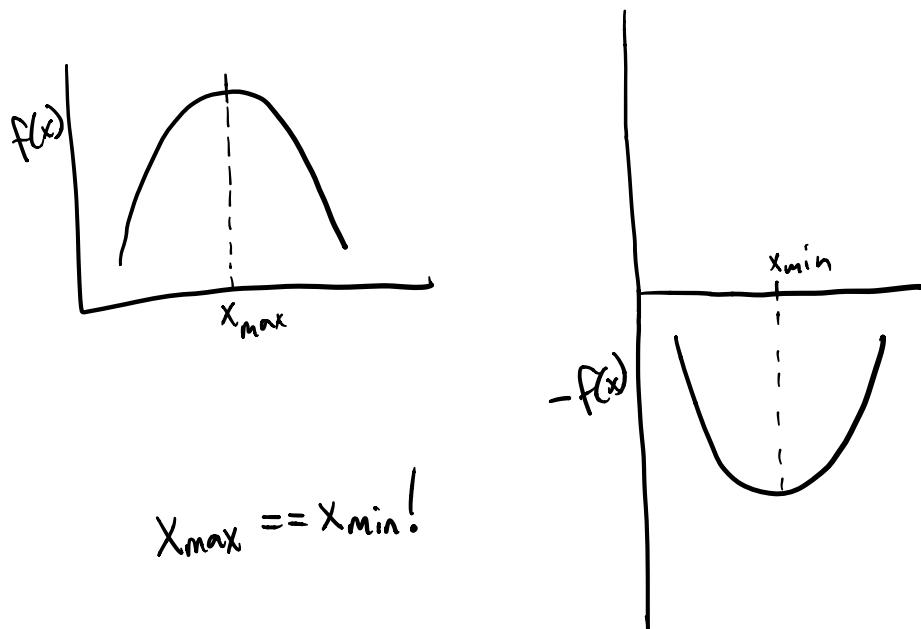
It is still an iterative process to solve,  
so you need an initial guess  
from `scipy.optimize import minimize`.

`sol = minimize(func, xo)`

This should return a number

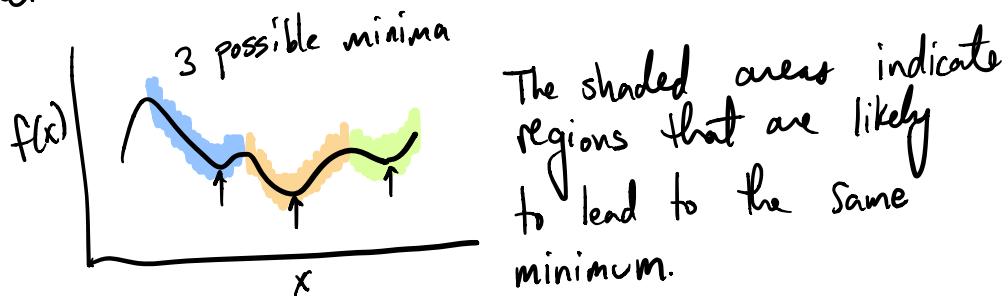
This can be an array of unknowns

You can also use minimize to MAXIMIZE!  
 The trick is to minimize the negative of the function!



$$x_{\max} = x_{\min}!$$

As with fsolve, if there are multiple minima, the initial guess you choose will determine which one is found.

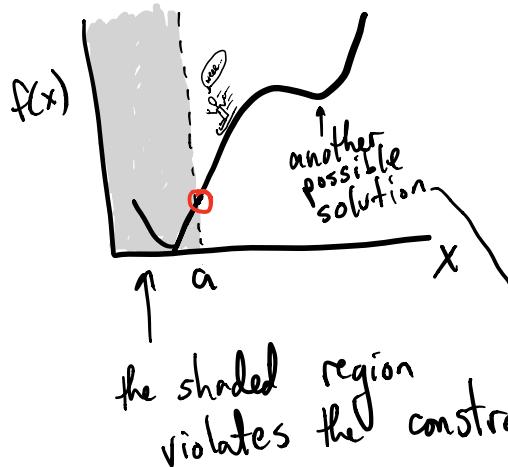


## Constrained optimization

With constraints, it will not always be the case that

1) there is a solution

2)  $f'(x) = 0$ .

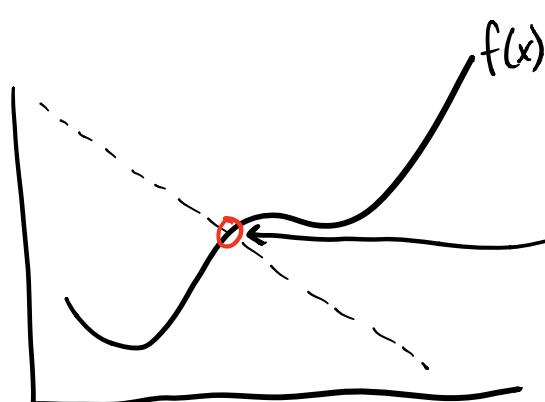


minimize  $f(x)$   
with  $x \geq a$ .

this is an  
inequality  
constraint.

$x = a$  is the solution,  
and  $f'(x) \neq 0$ .

but  $f(a)$  is smaller.



minimize  $f(x)$   
with  $x = -f(x)$

This is the solution.

I have \$10,  
Bagels cost \$2 each but are high  
in carbs. Croissants are \$1 each.  
How many of each should I buy  
to minimize carbs + make sure  
I don't overspend?



We define constraint functions

Equality:  $\text{constraint}(x) = 0$

(similar  
to fsolve)

At the solution  
these are true.

Inequality:  $\text{constraint}(x) \geq 0$

Then we tell minimize about the constraint(s)  
with a dictionary.



Curly brackets for dict → { 'type': 'eq' or 'ineq',  
'fun': constraint-function-name }  
keys                          values

this is not a string, it should be a function defined somewhere

Multiple constraints are defined as a list of  
dictionaries: [ { ... }, { ... }, { ... } ]  
three constraints.

You can mix equality and inequality constraints.