

# Integration in Python sketchnote

By John Kitchin

There are 3 main applications:

- ① Integrating Data
- ② Integrating Functions
- ③ Solving differential equations

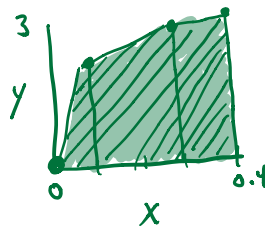
Follow the colored path to your application!

## Integrating data

Data is represented in arrays:

$x = [0, 0.1, 0.3, 0.4]$

$y = [0, 2.0, 2.5, 3.0]$



///  $\approx$  the area under the curve defined by the points. using the trapezoid rule

We use `numpy.trapz(y, x)`

or `scipy.integrate.simps(y, x)`

to approximate

$$\int_{x_0}^{x_f} y(x) dx$$

# → Integrating functions



If you know  $f(x)$  in analytical form, and can write a Python function for it, e.g.

```
def f(x):  
    return some function of x
```

Then we use `scipy.integrate.quad`

$$\int_a^b f(x) dx = \text{quad}(f, a, b)$$

$a, b$  must be numbers, or `numpy.inf` for infinity  
`quad` returns the integral and an error estimate.

```
I, err = quad(f, a, b)
```

This assumes  $f(x)$  is well-behaved on the interval  $(a, b)$



# Integrating differential equations

When we have a first order differential equation that looks like:

$$y' = f(x, y) \text{ with } y(x_0) = y_0$$

All the  $y'$  have to be on the left  
only functions of  $x, y$  on the right



These are not OK: not 1st order

$$y'' = f(x, y, y')$$

$$y' = f(x, y, y')$$

$y'$  on the right.

Then we use `scipy.integrate.solve_ivp` to get the solution  $y(x)$ . That solution looks like an array of  $x$  (or  $t$ ) values, and the corresponding  $y$ -values.

$$\text{sol} = \text{solve\_ivp}(f, \text{tspan}, y_0)$$

the  $f(x, y)$

$(x_0, x_f)$   
integration range

Array of initial conditions  
(can be a list)



Data structure

$\text{sol.t}$  = 1D array of  $t$  points we got the solution for

$\text{sol.y}$  = 2D array of  $y$  values, each row is a solution to the equations  
for 1 equation the  $y$  values are in  $\text{sol.y}[0]$

The solver chooses the  $t$ -points adaptively to get an accurate answer at the endpoint.  
To get the points we want use the  $t\_eval$  optional argument like this.

$tspan = x_0, x_f$

unpacks  $x_0$  to start  
 $x_f$  to end

$\text{sol} = \text{solve\_ivp}(f, tspan, y_0, t\_eval = \text{np.linspace}(*tspan))$

then  $\text{sol.t}$  will have all the points you put into  $t\_eval$  and corresponding  $y$ -values.



$\int_{\text{top}}^{\text{bottom}} \text{this } dx = \text{boring}$

Maybe next time will be better!