Concrete Shell Structures—Practice and Commentary

Reported by Joint ACI-ASCE Committee 334

Anton Tedesko
Chairman

Eric C. Molke
Vice Chairman

Alfred L. Parme
Secretary

Frank Baron, John V. Christiansen, Milo S. Ketchum, Jr., John B. Skilling, David P. Billington, Willhelm Flügge, James A. McCarthy, Bruno Thurlimann, Richard R. Bradshaw, Richard M. Gensert, Stefan J. Medwadowski, Robert Zaborowski, Felix Candela, Otto Gruenwald, Mario G. Salvadori

Committee members voting on the 1992 revisions:

Phillip L. Gould
Chairman

Stuart E. Swartz
Secretary

Jack Christiansen, Ajaya K. Gupta, Stefan J. Medwadowski, David B. South, John F. Abel, Robert B. Haber, Luis F. Meyer, Anton Tedesko, David P. Billington, Kye J. Han, Eldon F. Mockry, Bing-Yuan Ting, Arthur J. Boyt, Jr., Harry G. Harris, John K. Parsons, Daniel F. Tully, Eli W. Cohen, Mark A. Ketchum, William C. Schnobrich, Arnold Wilson, Morris N. Fialkow, Milo S. Ketchum, Alexander Scordelis

A report on the practical aspects of shell design including recommendations and a commentary for designers of thin concrete shells. General guidance based on current practice is given on analysis, proportioning, reinforcing, and construction. A selected bibliography on analytical methods featuring design tables and aids is included to assist the engineer.

Keywords: aggregate size; buckling; construction; design: double-curvature shell; edge beam; folded plate; formwork; model; prestressing; reinforcement; shell; single-curvature shell; splice; stiffening member; supporting member; thickness; thin shell.

CONTENTS

ACI 334.1R-92 replaces 334.1R-64 (Revised 1982)(Reapproved 1996) and became effective March 1, 1992. The 1992 revision included minor editorial corrections and the deletion of the year designation for ACI 318 to make the current code the applicable reference.

PREFACE

With the increased use of thin shells has come an increased understanding of their behavior through field observations, laboratory tests, and mathematical refinement of analytic procedures. However, because of the wide range of geometry possible with thin shells, the accumulated understanding is still limited. For some thin shell systems, such as cylindrical barrel shells, the design can be made with the same degree of accuracy as for conventional reinforced concrete construction. For other thin shell systems, such as those of double curvature, the design must be at times based on less-refined analyses in the same sense as the empirical design of flat plate floors. Therefore, it was felt desirable to divide this report into two parts.

The first part, entitled "Criteria," covers general design recommendations. The second part, entitled "Commentary," contains data of general interest to the designers of thin shells and reflects current practice. A "Selected Bibliography" is included as reference material on current methods of analysis.

The analysis, design, and construction of thin shell structures requires a thorough knowledge in this field. Therefore, the recommendations contained herein are not sufficient in themselves for the satisfactory execution of thin shell structures. Designers are referred to the many texts and technical papers that are readily available.

PART I—CRITERIA

CHAPTER 1—GENERAL

1.1—Definitions

1.1.1 *Thin shells*—Curved or folded slabs whose thicknesses are small compared to their other dimensions. They are characterized by their three-dimensional load-carrying behavior, which is determined by their geometrical shape, their boundary conditions, and the nature of the applied load. Thin shells are usually bounded by supporting members and edge members.

1.1.2 *Auxiliary members*—In a broad sense, any member located along the boundary of a shell or shell segment, with a capacity to stiffen the shell and distribute or carry load in composite action with the shell. They are classified as follows in accordance with established usage, although for certain shells a member may serve in a combination of capacities:

a) *Supporting members*—Beams, arches, trusses, diaphragms, etc., along the edges of thin shells that serve both to support and to stiffen the thin shell.
b) *Edge members*—Beams, trusses, etc., along the edges of thin shells that do not form part of the main supporting structure, but serve to stiffen and act integrally, that is, in composite action with the thin shell to carry loads to the supporting members.
c) *Stiffening members*—Ribs that serve only to stiffen the thin shell or to control local deformations.

1.1.3 *Elastic analysis*—Any structural analysis based on elastic behavior and involving assumptions that are suitable to approximations of three-dimensional elastic behavior. Analyses based on the results of elastic model tests, when conducted properly, are considered as valid elastic analyses.

1.2—Scope

1.2.1 These recommendations cover the design of thin shell concrete structures and only apply to the thin shell portions of such structures unless otherwise stated.

1.2.2 All applicable sections of the ACI Building Code (ACI 318), including the precast and prestressed concrete sections of the Code, should be followed in the design of shell structures except where they conflict with the following provisions.

CHAPTER 2—ANALYSIS

2.1—Assumptions

2.1.1 For an elastic analysis, concrete may be assumed uncracked, homogeneous, and isotropic.

2.1.2 Poisson's ratio may be assumed equal to zero.

2.2—Analysis of thin shells

2.2.1 Elastic behavior is the commonly accepted basis for determining stresses, displacements, and stability of thin shells.

2.2.2 The rigor and the necessary degree of accuracy required in the analysis of any specific thin shell structure depend on certain critical factors. These include: the configuration of the surface and the degree of curvature (geometry), the size of the structure, the type of boundary conditions, and the nature of the loading. Because of the complex interrelationship between these factors, specific recommendations regarding rigor and accuracy of analysis are not given.

2.2.3 Approximate methods of analysis that do not satisfy compatibility of strains or stresses in the shell may be used in cases where authoritative sources and experience have shown them to be applicable within the range employed.

2.2.4 Equilibrium checks of internal stresses and external loads are to be made to insure consistency of results.

2.2.5 An ultimate strength analysis may be used only as a check on the adequacy of the design. It is not to be used as a sole criterion for design, except where it can be proven to be applicable.

2.3—Analysis of supporting members

2.3.1 Supporting members shall be designed in accordance with a recognized elastic analysis.

2.3.2 A portion of the thin shell shall be assumed to act with the supporting member.

2.4—Model analysis

2.4.1 Models may be used as the basis for a design and/or to check the validity of assumptions involved in a mathematical analysis. When models are used, only those portions that significantly affect the items under study need be simulated. Every attempt must be made to ensure that these tests reveal the quantitative behavior of the prototype structure.

CHAPTER 3—STABILITY ANALYSIS

3.1—Buckling

3.1.1 When investigating shells for stability, consideration shall be given to the possible reduction in the value of the buckling load caused by large deflections, creep effects, and the deviation between the actual and theoretical shell surface.

CHAPTER 4—PROPORTIONING

4.1—Allowable stresses and load factors

4.1.1 Unless otherwise stated, concrete and steel stresses and load factors will be as specified in the Building Code (ACI 318).

4.1.2 Minimum standard cylinder strength f_c' shall be 3000 psi.

4.2—Shell thickness

4.2.1 Shell thickness is not always dictated by strength requirements but often by deformation of edge members, stability, and cover over reinforcing steel.

4.2.2 Stress concentrations due to abrupt changes in section shall be considered and, where necessary, the thin shell shall be gradually thickened.

4.3—Shell reinforcement

4.3.1 The stress in the reinforcement may be assumed at the allowable value independently of the strain in the concrete.

4.3.2 Where the tensile stresses vary greatly in magnitude over the shell, as in the case of cylindrical shells, the reinforcement capable of resisting the total tension may be concentrated in the region of maximum tensile stress. Where this is done, the percentage of crack control reinforcing in any 12 in. width of shell shall be not less than 0.35% throughout the tensile zone.

4.3.3 The principal tensile stresses shall be resisted entirely by reinforcement.

4.3.4 Reinforcement to resist the principal tensile stresses, assumed to act at the middle surface of the shell, may be placed either in the general direction of the lines of principal tensile stress (also referred to as parallel to the lines of principal tensile stress), or in two or three directions. In the regions of high tension it is advisable, based on experience, to place the reinforcing in the general direction of the principal stress.

4.3.5 The reinforcement may be considered parallel to the line of principal stress when its direction does not deviate from the direction of the principal stress more than 15 degrees. Variations in the direction of the principal stress over the cross section of a shell due to moments need not be considered for the determination of the maximum deviation. In areas where the stress in the reinforcing is less than the allowable stress, a deviation greater than 15 degrees can still be considered parallel placing; a stress decrease of 5% shall be considered to compensate for each additional degree of deviation above 15 degrees. Wherever possible, such reinforcing may run along lines considered most practical for construction, such as straight lines.

4.3.6 Where placed in more than one direction, the reinforcement shall resist the components of the principal tension force in each direction.

4.3.7 In those areas where the computed principal tensile stress in the concrete exceeds 300 psi, placement of at least one layer of the reinforcing shall be parallel to the principal tensile stress, unless it can be proven that a deviation of the reinforcing from the direction parallel to the lines of principal tensile stress is permissible because of the geometrical characteristics of a particular shell and because for reasons of geometry only insignificant and local cracking could develop.

4.3.8 Where the computed principal tensile stress (psi) in the concrete exceeds the value $2\sqrt{f_c'}$ (where f_c' is also in psi), the spacing of reinforcement shall not be greater than three times the thickness of the thin shell. Otherwise the reinforcement shall be spaced at not more than five times the thickness of the thin shell, nor more than 18 in.

4.3.9 Minimum reinforcement shall be provided as required in the Building Code (ACI 318) even where not required by analysis.

4.3.10 The percentage of reinforcement in any 12 in. width of shell shall not exceed $30f_c'/f_s$. However, the maximum percentage shall not exceed 6% if f_s = 20,000 psi, 5% if f_s = 25,000 psi, or 4% if f_s = 30,000 psi when the latter values are acceptable. If the deviation of the reinforcing from the lines of principal stress is greater than 10 deg, the maximum percentage shall be one-half of the above values.

4.3.11 Splices in principal tensile reinforcement shall be kept to a practical minimum. Where necessary they shall be staggered with not more than 1/3 of the bars spliced at any one cross section. Bars shall be lapped only within the same layer. The minimum lap for shell reinforcing bars, where draped, shall be 30 diameters with a 1 ft 6 in. minimum unless more is required by the Building Code (ACI 318), except that the minimum may be 12 in. for reinforcement not required by analysis. The minimum lap for welded wire fabric shall be 8 in. or one mesh, whichever is greater, except that Building Code requirements shall govern where the wire fabric at the splice must carry the full allowable stress.

4.3.12 The computed stress of the shell reinforcing at the junction of shell and supporting member or edge member shall be developed by anchorage within or beyond the width of the member.

4.3.13 Reinforcement to resist bending moments shall be proportioned and provided in the conventional manner with proper allowance for the direct forces.

4.4—Prestressing

4.4.1 Where prestressing tendons are draped within a thin shell, the resulting tendon profile not lying in one plane will exert a force on the shell which can be resolved into components. The design shall account for these force components.

4.4.2 Where prestressing tendons are anchored, special reinforcing shall be added to assure that no local overstressing occurs due to the application of these concentrated reactions.

4.5—Concrete cover over reinforcement

4.5.1 The concrete cover over reinforcement at surfaces protected from weather and not in contact with the ground shall be at least 1/2 in. for bars (3/8 in. when precast), 3/8 in. for welded wire fabric, and 1 in. for prestressed tendons. In no case shall the cover be less than the diameter of bar, prestressed tendon, or duct.

4.5.2 If greater concrete cover is required for fire protection, such cover requirements shall apply only to the principal tensile and moment reinforcement whose yielding would cause failure.

CHAPTER 5—CONSTRUCTION

5.1—Aggregate size

5.1.1 The maximum size aggregate shall not exceed 1/2 the shell thickness, nor the clear distance between reinforcement bars, nor 1/2 times the cover. Where formwork is required for two faces, the maximum size of aggregate shall not exceed 1/4 the minimum clear distance between the forms nor the cover over the reinforcement.

5.2—Forms

5.2.1 Removal of thin shell concrete forms shall be considered a matter of design and the form removal sequence shall be specified or approved by the engineer.

5.2.2 The minimum strength of concrete f_c', based on field-cured cylinders, at the time of decentering and of reshoring, when required, shall be designated by the engineer.

5.2.3 Where, in the opinion of the designing engineer, stability of short- or long-time deflections are important factors, the modulus of elasticity at the time of decentering, based on field-cured beams, shall be specified by the engineer. The proportions and loading of these specimens shall insure action that is primarily flexural.

5.2.4 The batter on vertical elements, or other elements, if desired or required for stripping, and the construction tolerances shall be designated by the engineer.

PART II—COMMENTARY

CHAPTER 1—GENERAL

This commentary contains data of general interest to the designers of thin shells and reflects current practice. The following sections are not to be considered design criteria for use in all cases, but discuss general methods of analysis and limitations that have been found helpful.

CHAPTER 2—ANALYSIS OF SHELL

Analytical procedures should be consistent with the complexity and importance of the structure. Judgment and experience may be guides as reliable as analytical procedures based on simplifying assumptions. Factors such as size, curvature, boundary conditions, configuration, and distribution of load must be considered collectively by the designer in the choice of analytical procedures. For the purpose of discussion, a few comments will be made on each individually.

The most important factor to be considered is size. Shells generally possess a large reserve strength. The compressive stresses in a shell are usually a fraction of the allowable stresses. This is due to the fact that shell thickness is usually dictated by construction and code requirements. Moreover, for the major portion of the shell, the reinforcement is determined by requirements for temperature and shrinkage. Therefore, the load-carrying capacity of a shell is generally greater than required. As spans increase or curvatures decrease, the compression stresses increase, whereas the construction or code requirements generally remain the same. Thus reserve capacity tends to decrease with larger sizes and flatter shells. Therefore, approximations that are valid for relatively small structures may not be at all valid when the same type of structure is increased in size.

Of almost equal importance to size is the manner in which the shell is supported. If proper support is provided, capable of taking the shell reaction without appreciable deformations, the shell may respond to loads in direct compressive or tensile stresses that, for the usual spans, will be less than the allowable stress. If the supports are flexible, the stresses in the shell can be higher than those that would prevail in a properly supported shell, and thus a more precise determination of stresses is needed.

While size and support conditions have an important bearing on the degree of accuracy needed in the analysis, the distribution of load has a less important effect on stresses. This is due to the fact that bending moments in the shell are more closely related to the boundary conditions than to the load. Hence, it is usually unnecessary to analyze a thin shell for partial live loads even though the supporting members must be analyzed for such partial loads. For this reason, snow load on thin shells may be assumed either uniformly distributed on the horizontal projection or uniformly distributed over the surface of the shell. On the other hand, local bending moments due to large concentrated loads on the shell must be considered.

Finally, the degree of analytical accuracy required is also dependent on the geometry of the shell, especially the type and amount of curvature. With the variety of possible shapes, generalized recommendations of the effect of shape cannot be made. A few pertinent remarks on some frequently used shapes follow.

2.1—Thin shells of single curvature

Examples of shells of single curvature are segments of cylinders and cones. These shell systems tend to act like beams in the longitudinal direction where the span is long in proportion to the radius. Their behavior departs from that of beams in proportions as the valley deflects relative to the crown. The smaller such a relative deflection is, the more nearly the direct longitudinal stresses will be distributed linearly as in a beam section. The range over which the beam analysis is applicable has been discussed.[5,9]

For simply supported cylindrical shells with parameters in a range where beam analysis does not give results of sufficient accuracy, a number of design tables have been presented.[7,8,10,11,13] The use of these tables is recommended.

No analysis comparable to that for simply supported cylindrical shells is available for the case of shells continuous over supports. For the latter cases, the effects of continuity may be estimated by first calculating the longitudinal stresses and transverse shearing stresses as for simply supported spans and then multiplying these by the ratio of the moment and shear coefficients for continuous to simple spans. It is pointed out, however, that the shear deformation in certain shells may have a strong influence on the distribution of longitudinal stresses (Reference 1, pp. 138-140).

For singly curved shells moderately tapered in plan, that is, those in which the chords at each end are different, experience indicates that an analysis based on a cylindrical shell with a chord width equal to the average is satisfactory. However, shear conditions at the smaller end should be investigated on the basis of the actual cross section. For shells tapered markedly, a more precise solution is available.[34]

2.2—Folded plates

Analysis of prismatic folded plates is based on the conventional flexural theory for longitudinal action and on a one-way continuous slab action for transverse behavior. A number of satisfactory analytic procedures, all based on the same fundamental approach, are available.[16] Attention is called to the fact that the iteration procedures that are available may not always converge.

Folded plates are more sensitive to longitudinal support conditions than curved thin shells. The transverse bending moments in the outer bays of multiple folded plate structures are apt to be much larger than those in barrel shells of the same size. For this reason, folded plates are often subjected to critical stresses during construction when certain plates may temporarily be shored or act as exterior plates.

Although a folded plate analysis is recommended to determine the effects produced in the two or three outer bays of a multiple folded plate structure, the behavior of the opposite interior plates is similar to that of a beam. Hence a rigorous folded plate analysis needs to be applied to only, at most, the exterior and first interior bays of a multiple folded plate system except when the depth is less than 1/15 of the span.[15]

2.3—Thin shells of double curvature

Shells of double curvature both the synclastic (surfaces in which the curvatures in the two principal directions have the same sign) and anticlastic (surfaces in which curvatures in the two principal directions have opposite signs) are inherently better suited to resist loads by direct forces than are shells of single curvature. The reason for this is obvious from the fact that this type of shell possesses arch action along both curvatures. In order that surfaces curved in two directions behave as a shell, however, it is important that proper support or edge members be provided.

For most shells of double curvature distributed loads on the shell can be transmitted to the supports by tangential shearing and normal forces acting along the periphery of the shell irrespective of the load distribution. When this can be done, the internal forces acting in the shell are all direct forces, generally of small magnitude except in the region of the column supports where often critical tensile forces and moments are developed.

The direct stresses throughout the major portion of the shell are usually of little significance except as they relate to buckling. A careful evaluation should be made of the bending moments produced in the vicinity of the edge members by the interaction of the edge member and the shell. For moderate-size shells, this effect usually is confined within a few feet of the vicinity of the edge member. For such determination, the formulas for cylindrical shells[4] have been used in certain cases. An exception to this are some anticlastic shells, like the hyperbolic paraboloid, wherein bending can prevail throughout a greater portion of the shell. To a limited extent, this also occurs in domes, when the supports do not provide a reaction tangent to the shell surface. In these cases, the bending moments may extend a significant distance into the shell.

Because of the popularity of the hyperbolic paraboloid stiffened only in the conventional way by unsupported edge beams, it becomes important to recognize that its ability to carry loads by direct force is restricted to uniform load. For other loadings, as for example the dead load produced by sharply tilted saddle-shape shells, the shell is subject to bending moments. The moments are not significant for conventional-size shells, but they may be very significant for large spans.

CHAPTER 3—ANALYSIS OF SUPPORTING MEMBERS

The analysis of the supporting members may be based on the consideration of either the total shell load or on the shearing forces acting between shell and supporting member. When the total load is used, the axial thrust and moment in the supporting member shall be adjusted by subtracting (algebraically) the direct thrust and moment in the thin shell. If total shell loads are used, the moments due to the eccentricity of the shell with respect to the centroidal axis of the supporting member shall be taken into account.[13]

Supporting members shall be designed for partial live loads when such loadings control the design.

CHAPTER 4—PRESTRESSING

Prestressing can often be employed advantageously on shells covering large spans to reduce the bending moments in the shell under service loads and to control deflection and cracking. As with conventional prestressed structures, the analysis must be made at design and at ultimate loads to ensure both proper behavior at design loads and an adequate overload capacity. With respect to overload capacity, it is important to determine the shell bending moments at ultimate loads because they may be several times the bending moments produced by the combination of design load and prestressing. In all cases, the shell must be designed for the shell bending moment produced by the design loads times the appropriate overload factor minus (algebraically) the bending moment produced by the prestressing force.

CHAPTER 5—STABILITY

In common with other structures, shells should be investigated for buckling. In some simple two-dimensional structures it is sufficient to determine only the lowest value of the load at which buckling commences. In the case of shells, however, it may be also necessary to investigate the postbuckling behavior because it has an important bearing on the magnitude of the failure load.

As a thin shell deforms under load, principal membrane forces develop. If one of these forces is tensile, it tends to return the shell back to its original position, thus enabling it to carry loads greater than the initial buckling load. If, on the other hand, both the principal membrane forces are compressive, they tend to increase with deformation of the shell. After the initial buckling, the shell can only transmit loads smaller than the initial buckling load. This is particularly true for concrete shells because of creep and deviation of the actual shape of the shell from the assumed theoretical surface.

The importance of the postbuckling behavior, which small deflection theory of buckling is not capable of predicting, was discovered as a consequence of attempts to correlate experimental results with analytical predictions.

Some surfaces subjected to various load combinations that have been studied analytically and experimentally[22,25,27] with respect to their buckling characteristics are as follows:

1. Circular cylinders (ribbed and unribbed);
2. Segments of circular cylinders;
3. Elliptical cylinders;
4. Cones;
5. Spherical domes; and
6. Translational surfaces.

Good correlation exists when one of the principal membrane forces is tensile; such cases include:

1. Cylindrical shells under torsional loading; and
2. Cylindrical shells under radial inward pressure.

As stated, poor correlation between the results of theory and experiment exists when both principal membrane forces are compressive, as in the case of:

1. Cylindrical shells under axial compressive load;
2. Cylindrical shells under distributed load normal to the surface, which causes bending; and
3. Domes under inward radial pressure.

In extreme cases, the buckling load obtained experimentally has been found to be as little as 10% of that predicted by the small deflection theory.

The effect of creep of concrete is similar to lowering E, the modulus of elasticity. The effect may be estimated as follows:

Assume a reduced value of E or, knowing the principal membrane forces at any point of a given shell, determine the tangent modulus of elasticity. Divide the tangent modulus of elasticity by a multiplier for long-term deflections. The multiplier shall not be less than 2. The use of a reduced E accomplishes the same purpose as a buckling load reduction factor.

A deviation of the shell dimensions from the geometry used in stress analysis is associated with a change in the principal radii of curvature of the shell surface. Elastic and creep deformations may also lead to a change of curvature. If the actual radii are larger (that is, the shell is flatter), the membrane forces are generally greater. Hence the value of the buckling load will be lower, possibly substantially lower.

In general, the value of the buckling load depends on shell geometry, type of restraint at boundary, material properties of shell, the location of reinforcing steel, and the type of load. With respect to shell geometry, the dominant factor is whether the surface is synclastic or anticlastic. For

a) Synclastic surfaces, such as most shells of revolution, cylinders, and cones, the buckling resistance usually lies somewhere between a sphere and a cylinder—the two shell forms that have been extensively studied. Hence an estimate of the critical stress may be sometimes obtained by comparing the given shell to a cylinder or a sphere.
b) Anticlastic surfaces, such as hyperbolic paraboloids, the buckling resistance is much greater than that offered by synclastic surfaces because their shape results in principal tensile forces in one direction. Hence it is often possible to use the linear buckling theory.

Resistance to buckling is increased when reinforcing steel is provided on both faces of the shell.

Some methods that may be used to estimate failure loads are as follows:

1. Energy methods, based on minimization of the energy of the system. This approach permits computation of the lower as well as the upper boundary of the buckling load, and is applicable to all types of shells.
2. Equilibrium and compatibility methods. This approach is based on the classical equations of equilibrium and compatibility.
3. Experimental methods yield very good estimates of buckling load for any type of shell. However, it should be noted that most experiments have been made with materials other than concrete.
4. Recommended procedures such as those advocated for cylindrical shells and domes with use of buckling

graphs.[1,4,26] Where the influences of creep, deformations, and decentering conditions are not expected to be unusual and are not separately evaluated, it is recommended to calculate buckling stability in two directions separately (with the use of graphs) by computing the buckling load or critical stress based on the unreduced value of the modulus of elasticity expected at the time of decentering. It is further recommended to calculate the apparent safety factors (F_1, F_2) for the two directions by dividing buckling load (or critical stress) by working load (or working stress) and to compute the combined apparent safety factor for buckling as

$$F = \frac{F_1 F_2}{F_1 + F_2}$$

This apparent safety factor must be greater than 5 for the above described conditions.

CHAPTER 6—PROPORTIONING

It is often desirable to gradually increase the thickness of a thin shell where it is connected to supporting members or edge members. In barrel shells and domes, this gradual thickening usually commences at a distance from the edge of about ten times the minimum thickness unless otherwise dictated by stress analysis. The increased thickness is usually twice the minimum thickness required at the center of the span unless otherwise determined by analysis.

The use of high-strength steel reinforcement with working stresses above 20,000 psi is acceptable in thin shell structural systems. However, in the shell portion it may be used only when the main reinforcement follows the direction of principal stress. The working stress should not exceed 50% of the yield point or the permissible working stress. Where such high-strength steel is used, careful attention must be paid to crack control and to deformations.

At the edges of thin shells, it is usually desirable to provide top and bottom reinforcement. The torsional resistance of the supporting members or edge members often governs the top and bottom reinforcement when the shell terminates with these members.

As the principal tensile stress in the shell increases, it is desirable to provide an increasing amount of steel in the direction of the principal stress or in two or three directions within the shell.

CHAPTER 7—CONSTRUCTION

7.1—Forms and decentering

For thin shell structures of large spans or unusual proportions, the engineer should prepare a decentering drawing before the method of shoring is determined by the contractor. In the preparation of the decentering drawing, the decentering procedure should be studied to avoid any temporary support including a concentrated reaction on the shell. Generally, decentering should begin at points of maximum deflections and should progress toward points of minimum deflection, with the decentering of edge members proceeding simultaneously with the adjoining shell. Where edge members are to be reshored, the spacing of the shoring should be specified by the engineer.

The supporting members should be decentered after the forms under the thin shell have been decentered. When this is done, the stresses created by the decentering procedure should be considered.

Where the time of decentering is based on the attainment of a prescribed modulus of elasticity of field-cured beams, it has been found satisfactory to use lightly reinforced beams, 4 in. x 6 in. x 6 ft long, for the field control beams.[28]

It is desirable to have the engineer specify the maximum deviations permissible from the true surface of a shell, such as, for example, 1/2 in. deviation for an arc of x ft in a shell of a radius of y ft.

When movable forms are used, a batter of 1/8 in. per ft minimum is recommended for vertical surfaces to permit easy removal.

7.2—Concrete placing

Concrete should be placed in a symmetrical pattern, otherwise the shoring must be braced for the effect of unbalanced loads on the forms. It is recommended that concreting commence at the low point or points of the shell and proceed upward to the high point. The concrete should be deposited as nearly as possible in its final position.

When deep beams or diaphragms are employed as supporting members, and when these members are to be cast integrally with the thin shell, care should be taken to use a rate of concreting that will permit consolidation of concrete in the deep beams or diaphragms and prevent cold joints in the shell.

Construction joints should be shown in detail on design drawings and should be located preferably in areas of compressive stresses.

If the slope of the shell is steeper than approximately 45 degrees, it is desirable to use formwork on both faces except when the thin shell is shotcreted or plastered.

Shells that cannot be cast in 1 day should be sectioned for two or more placements with 2 to 3 days between each cast to reduce shrinkage stresses.

7.3—Curing of concrete

Thin shells are susceptible to shrinkage cracking if improperly cured. In hot weather, the use of retarders, preliminary fog spray curing, and wet burlap or water curing is advisable. In cold weather, the use of accelerators and special precautions against freezing are usually required. In moderate weather (40 to 70°F), ordinary methods such as membrane curing compounds are usually satisfactory, although wet curing may produce better results.

CHAPTER 8—MODELS

It has been stated in the criteria that model analysis is acceptable as a basis for design or to check the validity of the assumptions involved in a mathematical analysis. Models should be looked upon as auxiliary tools to aid a rational analysis rather than a substitute for a theoretical analysis. It must be emphasized that although the model analysis technique is valuable for the design or analysis problems of complex shells, extreme care must be used in the planning, execution, and interpretation of the model experiments. Requirements of similitude should be followed, unless it can be demonstrated that the given departures from similitude cause insignificant changes in results, or changes that can be accurately predicted and accounted for in the interpretation. The model experiment should be so arranged that as many checks on results can be made as is reasonably possible. For example:

1. Can equilibrium of forces in the model be checked?
2. For symmetrical loads on a symmetrical structure, are symmetric readings equal?
3. Are environmental conditions affecting the model results?
4. Are the loads of sufficient magnitude to induce possible secondary effects due to deflections?

Many possible errors may enter into a model analysis, grossly affecting the stress readings. As a matter of good practice, after strain measurements are completed it is recommended that the model be subjected to overloads whose effects approach the anticipated capacity of the models or several times the service load. Such high loading may reveal weaknesses not apparent from a limited number of strain gauges.

CHAPTER 9—STANDARDS AND ACI DOCUMENTS CITED IN THIS REPORT

The document of the standards-producing organization referred to in this document is listed below with its serial designation.

American Concrete Institute
318 Building Code Requirements for Reinforced Concrete

The above publication may be obtained from the following organization:

American Concrete Institute
P.O. Box 9094
Farmington Hills, MI 48333-9094

PART III—SELECTED BIBLIOGRAPHY

General considerations for analysis

1a. Flügge, W., *Stresses in Shells*, Springer-Verlag, Berlin, 1962, 499 pp. (A practical textbook devoted exclusively to shells, with numerous examples.)

1b. Flügge, W., *Statik lind Dynanlik der Scllalen,* Springer-Verlag, Berlin, 1962. (A practical textbook devoted exclusively to shells.)

2. Girknlann, K., *Flächentragwerke*, Springer-Verlag, Vienna, 5th edition, 1959. (A textbook on practical methods of analysis of plate and shell structures.)

3. Pfluger, A., *Elementary Statics of Shells,* F.W. Dodge Corp., New York, 2nd edition, 1961, 122 pp.

4. Timoshenko, S. P., and Woinowsky-Krieger, S., *Theory of Plates and Shells*, 2nd edition, McGraw-Hill Book Co., Inc., New York, 1959. (One of the classics in the application of theory of elasticity.)

Cylindrical shell roofs

5. Chinn, J., "Cylindrical Shell Analysis Simplified by Beam Method," ACI Journal, *Proceedings* V. 55, No. 10, May 1959, pp. 1183-1192. Discussion by M. S. Ketchum, A.L. Parme, and H. W. Conner; A. Siev, A. Tedesko, and A. Zweig, ACI Journal, *Proceedings* V. 55, Dec. 1959, Part 2, pp. 1583-1604.

6. Dabrowski, R., "Analysis of Prestressed Cylindrical Shell Roofs," *Proceedings*, ASCE, V. 89, No. ST5, Oct. 1963, pp. 91-116.

7. Gibson, J. E., *Computer Analysis of Cylindrical Shells,* Spon Limited, London, 1961. (A series of tables giving stress resultants and moments in circular cylindrical shells.)

8. Holand, I., "Design of Circular Cylindrical Shells," *Series* 2, No. 3, Norges Tekniske Vitenskapsakademi, 1957. (A comparison of the equations used for cylindrical shell analysis.)

9. Lundgren, H., *Cylindrical Shells, V. I—Cylindrical Roofs,* The Institution of Danish Civil Engineers, The Danish Technical Press, Copenhagen, 1951. (Application of the theory of elasticity and of rupture to the design of cylindrical roofs, and other thin-walled cylindrical structures.)

10. Parme, A. L., and Conner, H. W., "Design Constants for Interior Cylindrical Concrete Shells," ACI Journal, *Proceedings* V. 58, No. 1, July 1961, pp. 83-106. (Compact design tables, limited in scope.)

11. Ruediger, D., and Urban, J., *Circular Cylindrical Shells*, B. G. Teubner Verlagsgesellschaft, Leipzig, 1955. (Derivation of the shallow shell theory for cylindrical shells with tables for a wide range of shell dimensions.)

12. Thurlimann, B., and Johnston, B. G., "Analysis and Tests of a Cylindrical Shell Roof Model," *Proceedings,* ASCE, V. 80, Separate No. 434, 1954.

13. "Design of Cylindrical Concrete Shell Roofs," *Manual* No. 31, American Society of Civil Engineers, 1952. (Comprehensive series of design tables with mathematical derivations as background.)

Folded plates

14. Yitzhaki, D., *The Design of Prismatic and Cylindrical Shell Roofs,* Haifa Science Publishers, Haifa, Israel, 1958, 253 pp. (Thorough treatment of roofs of simple and continuous spans.)

15. "Direct Solution of Folded Plate Concrete Roofs," *Advanced Engineering Bulletin* No. 3, Portland Cement Association, Chicago, 1960. (Concise design tables.)

16. "Phase I—Report on Folded Plate Construction," *Proceedings,* ASCE, V. 89, ST6, Dec. 1963, pp. 365-406. (A complete bibliography on folded plates is included with this report.)

Thin shells of double curvature

17. Apeland, K., "Stress Analysis of Translational Shells," *Proceedings*, ASCE, V. 87, EM1, Feb. 1961, pp. 111-139.

18. Apeland, K., and Popov, E. P., "Design Tables for Translational Shells," *Reports* 63-1, Structures and Materials Research, University of California, Berkeley, 1963.

19. Bouma, A. L., "Some Applications of the Bending Theory Regarding Doubly Curved Shells," *Proceedings,* Symposium on the Theory of Thin Elastic Shells, Delft, 1959. (A discussion showing the effects of the type of shell curvature on the shell behavior, based on shallow shell theory.) (See Reference 32.)

20. Candela, F., "General Formulas for Membrane Stresses in Hyperbolic Paraboloical Shells," ACI JOURNAL, *Proceedings* V. 57, No.4, Oct. 1950, pp. 353-372.

21. Murray, N. W., "A General Design Method for Axisymmetric Shell Roofs," *Proceedings,* Institution of Civil Engineers (London), V. 20, Sept. 1961, pp. 151-162. (A general numerical solution for shells of revolution with varying thicknesses.)

Stability

22. ACI-ASCE Committee 334, *Concrete Shell Buckling,* E. P. Popov and S. J. Medwadowski, eds., SP-67, American Concrete Institute, Detroit, 1981, 240 pp.

23. Gerard, G., and Becker, H., *Handbook of Structural Stability:* Part III, NACA N 3783 (1957); and Part VI, NACA N 3786 (1958); National Aeronautics and Space Administration, Washington, D.C.

24. Nash, W. A., "Recent Advances in Buckling of Thin Shells," *Applied Mechanics Reviews*, V. 13, Mar. 1960, pp. 161-164.

25. Schmidt, H., "Ergebnisse von Beulversuchen mit doppelt gekrummten Schalenmodellen aus Aluminum," *Proceedings*, Symposium on Shell Research, Interscience, John Wiley & Sons, Inc., New York, and North-Holland Publishing Co., Amsterdam, 1962, pp. 159-181. (A summary of past work on thin shell buckling along with recommendations for reinforced concrete design.)

26. Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, 2nd edition, McGraw-Hill Book Co., Inc., New York, 1961.

27. *Collected Papers on Instability of Shell Structures*, 1N-0-1510, National Aeronautics and Space Administration Washington, D.C., Dec. 1962.

Construction

28. Tedesko, A., "Construction Aspects of Thin Shell Structures," ACI JOURNAL, *Proceedings* V. 49, No. 6, Feb. 1953, pp. 505-520.

Symposia

29. *Proceedings of the Symposium on Concrete Shell Roof Construction*, Cement and Concrete Association, London, July 1954.

30. *Proceedings of the Conference on Thin Concrete Shells*, Massachusetts Institute of Technology, Cambridge, MA, 1954, 133 pp.

31. *Proceedings of the Second Symposium on Concrete Shell Roof Construction*, Teknisk Ukeblad, Oslo, Norway, 1958.

32. *Proceedings of the Symposium on the Theory of Thin Elastic Shells* (Delft, 1959), Interscience, John Wiley & Sons, Inc., New York, and North-Holland Publishing Co., Amsterdam, 1960, 496 pp.

33. *Proceedings of the Symposium on Shell Research,* (Delft, 1961), Interscience, John Wiley & Sons, Inc., New York, and North-Holland Publishing Co., Amsterdam, 1962, 364 pp.

34. *Proceedings, Colloquium on Simplified Calculation Methods of Shell Structures* (Brussels, 1961), Interscience, John Wiley & Sons, Inc., New York, and North-Holland Publishing Co., Amsterdam, 1962, 529 pp.

35. *Proceedings of the World Conference on Shell Structures* (San Francisco, 1962), Publication No. 1187, National Academy of Sciences-National Science Foundation, Washington, D.C., 1964, 690 pp.