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## *Steel Design Guide Series*

# *Floor Vibrations*
## *Due to Human Activity*

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# Chapter 1
# INTRODUCTION

## 1.1 Objectives of the Design Guide

The primary objective of this Design Guide is to provide basic principles and simple analytical tools to evaluate steel framed floor systems and footbridges for vibration serviceability due to human activities. Both human comfort and the need to control movement for sensitive equipment are considered. The secondary objective is to provide guidance on developing remedial measures for problem floors.

## 1.2 Road Map

This Design Guide is organized for the reader to move from basic principles of floor vibration and the associated terminology in Chapter 1, to serviceability criteria for evaluation and design in Chapter 2, to estimation of natural floor frequency (the most important floor vibration property) in Chapter 3, to applications of the criteria in Chapters 4, 5 and 6, and finally to possible remedial measures in Chapter 7. Chapter 4 covers walking-induced vibration, a topic of widespread importance in structural design practice. Chapter 5 concerns vibrations due to rhythmic activities such as aerobics and Chapter 6 provides guidance on the design of floor systems which support sensitive equipment, topics requiring increased specialization. Because many floor vibrations problems occur in practice, Chapter 7 provides guidance on their evaluation and the choice of remedial measures. The Appendix contains a short historical development of the various floor vibration criteria used in North America.

## 1.3 Background

For floor serviceability, stiffness and resonance are dominant considerations in the design of steel floor structures and footbridges. The first known stiffness criterion appeared nearly 170 years ago. Tredgold (1828) wrote that girders over long spans should be "made deep to avoid the inconvenience of not being able to move on the floor without shaking everything in the room". Traditionally, soldiers "break step" when marching across bridges to avoid large, potentially dangerous, resonant vibration.

A traditional stiffness criterion for steel floors limits the live load deflection of beams or girders supporting "plastered ceilings" to span/360. This limitation, along with restricting member span-to-depth rations to 24 or less, have been widely applied to steel framed floor systems in an attempt to control vibrations, but with limited success.

Resonance has been ignored in the design of floors and footbridges until recently. Approximately 30 years ago, prob-lems arose with vibrations induced by walking on steel-joist supported floors that satisfied traditional stiffness criteria. Since that time much has been learned about the loading function due to walking and the potential for resonance.

More recently, rhythmic activities, such as aerobics and high-impact dancing, have caused serious floor vibration problems due to resonance.

A number of analytical procedures have been developed which allow a structural designer to assess the floor structure for occupant comfort for a specific activity and for suitability for sensitive equipment. Generally, these analytical tools require the calculation of the first natural frequency of the floor system and the maximum amplitude of acceleration, velocity or displacement for a reference excitation. An estimate of damping in the floor is also required in some instances. A human comfort scale or sensitive equipment criterion is then used to determine whether the floor system meets serviceability requirements. Some of the analytical tools incorporate limits on acceleration into a single design formula whose parameters are estimated by the designer.

## 1.4 Basic Vibration Terminology

The purpose of this section is to introduce the reader to terminology and basic concepts used in this Design Guide.

**Dynamic Loadings.** *Dynamic loadings* can be classified as harmonic, periodic, transient, and impulsive as shown in Figure 1.1. *Harmonic or sinusoidal loads* are usually associated with rotating machinery. *Periodic loads* are caused by rhythmic human activities such as dancing and aerobics and by impactive machinery. *Transient loads* occur from the movement of people and include walking and running. Single jumps and heel-drop impacts are examples of *impulsive loads.*

**Period and Frequency.** *Period* is the time, usually in seconds, between successive peak excursions in repeating events. *Period* is associated with harmonic (or sinusoidal) and repetitive time functions as shown in Figure 1.1. *Frequency* is the reciprocal of period and is usually expressed in Hertz (cycles per second, Hz).

**Steady State and Transient Motion.** If a structural system is subjected to a continuous harmonic driving force (see Figure 1.1a), the resulting motion will have a constant frequency and constant maximum amplitude and is referred to as *steady state motion.* If a real structural system is subjected to a single impulse, damping in the system will cause the

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motion to subside, as illustrated in Figure 1.2. This is one type of *transient motion.*

**Natural Frequency and Free Vibration.** *Natural frequency* is the frequency at which a body or structure will vibrate when displaced and then quickly released. This state of vibration is referred to as *free vibration.* All structures have a large number of natural frequencies; the lowest or "fundamental" natural frequency is of most concern.

**Damping and Critical Damping.** Damping refers to the loss of mechanical energy in a vibrating system. Damping is usually expressed as the percent of *critical damping* or as the ratio of actual damping (assumed to be viscous) to critical damping. Critical damping is the smallest amount of viscous damping for which a free vibrating system that is displaced from equilibrium and released comes to rest without oscillation. "Viscous" damping is associated with a retarding force that is proportional to velocity. For damping that is smaller than critical, the system oscillates freely as shown in Figure 1.2.

Until recently, damping in floor systems was generally determined from the decay of vibration following an impact (usually a heel-drop), using vibration signals from which vibration beyond 1.5 to 2 times the fundamental frequency has been removed by filtering. This technique resulted in damping ratios of 4 to 12 percent for typical office buildings. It has been found that this measurement overestimates the damping because it measures not only energy dissipation (the true damping) but also the transmission of vibrational energy to other structural components (usually referred to as geometric dispersion). To determine modal damping all modes of vibration except one must be filtered from the record of vibration decay. Alternatively, the modal damping ratio can be determined from the Fourier spectrum of the response to impact. These techniques result in damping ratios of 3 to 5 percent for typical office buildings.

**Resonance.** If a frequency component of an exciting force is equal to a natural frequency of the structure, *resonance* will occur. At resonance, the amplitude of the motion tends to become large to very large, as shown in Figure 1.3.

**Step Frequency.** *Step frequency* is the frequency of application of a foot or feet to the floor, e.g. in walking, dancing or aerobics.

**Harmonic.** A *harmonic multiple* is an integer multiple of frequency of application of a repetitive force, e.g. multiple of step frequency for human activities, or multiple of rotational frequency of reciprocating machinery. (Note: Harmonics can also refer to natural frequencies, e.g. of strings or pipes.)

**Mode Shape.** When a floor structure vibrates freely in a particular mode, it moves up and down with a certain configuration or *mode shape.* Each natural frequency has a mode shape associated with it. Figure 1.4 shows typical mode shapes for a simple beam and for a slab/beam/girder floor system.

**Modal Analysis.** *Modal analysis* refers to a computational, analytical or experimental method for determining the natural frequencies and mode shapes of a structure, as well as the responses of individual modes to a given excitation. (The responses of the modes can then be superimposed to obtain a total system response.)
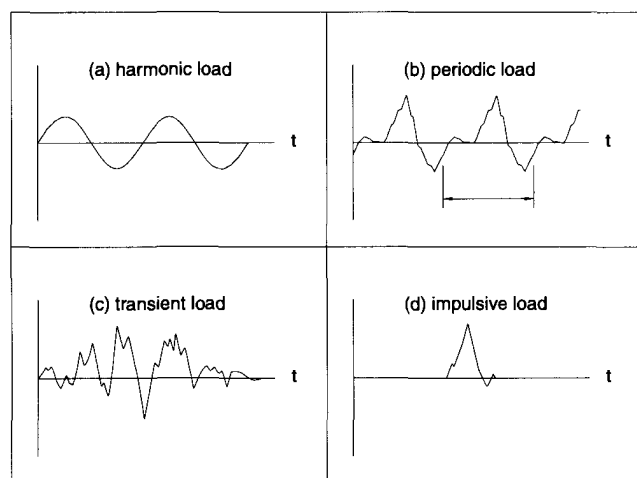


*Fig. 1.1 Types of dynamic loading.*
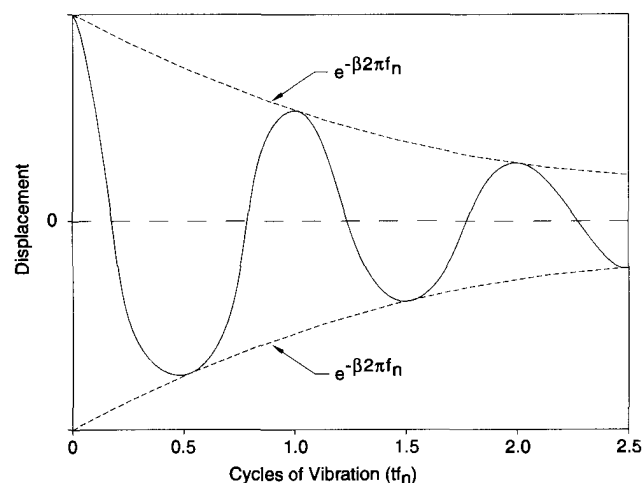


*Fig. 1.2 Decaying vibration with viscous damping.*

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**Spectrum.** A *spectrum* shows the variation of relative amplitude with frequency of the vibration components that contribute to the load or motion. Figure 1.5 is an example of a frequency spectrum.

**Fourier Transformation.** The mathematical procedure to transform a time record into a complex frequency spectrum (Fourier spectrum) without loss of information is called a *Fourier Transformation.*

**Acceleration Ratio.** The acceleration of a system divided by the acceleration of gravity is referred to as the *acceleration ratio.* Usually the peak acceleration of the system is used.

**Floor Panel.** A rectangular plan portion of a floor encompassed by the span and an effective width is defined as a *floor panel.*

**Bay.** A rectangular plan portion of a floor defined by four column locations.

## 1.5 Floor Vibration Principles

Although human annoyance criteria for vibration have been known for many years, it has only recently become practical to apply such criteria to the design of floor structures. The reason for this is that the problem is complex—the loading is complex and the response complicated, involving a large number of modes of vibration. Experience and research have shown, however, that the problem can be simplified sufficiently to provide practical design criteria.

Most floor vibration problems involve repeated forces caused by machinery or by human activities such as dancing, aerobics or walking, although walking is a little more complicated than the others because the forces change location with each step. In some cases, the applied force is sinusoidal or nearly so. In general, a repeated force can be represented by a combination of sinusoidal forces whose frequencies, $f$, are multiples or harmonics of the basic frequency of the force repetition, e.g. step frequency, $f_{step}$, for human activities. The time-dependent repeated force can be represented by the Fourier series

$$F = P\,[1 + \Sigma\alpha_i\cos(2\pi i f_{step} t + \phi_i)] \qquad (1.1)$$

where

$P$ = person's weight

a) Beam
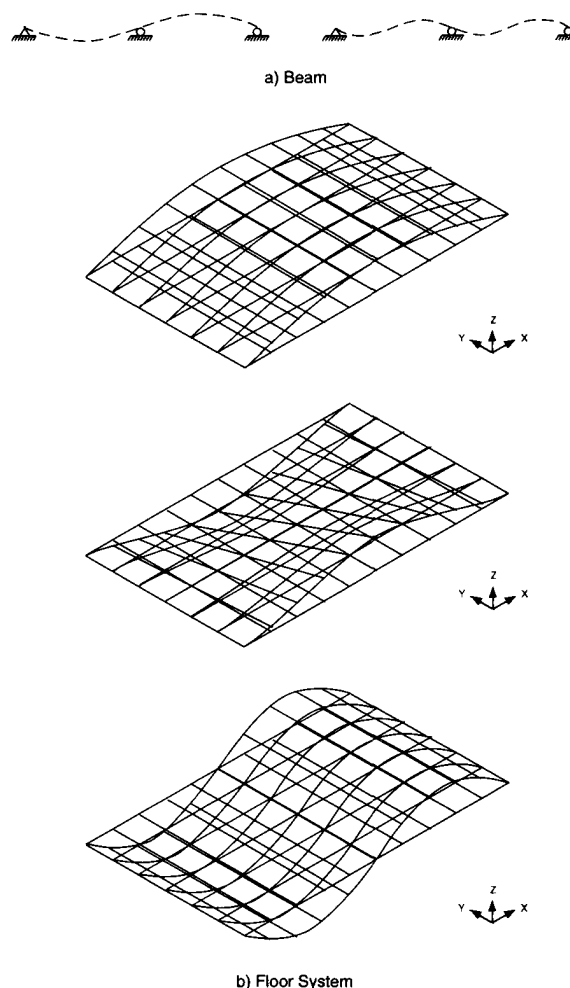
b) Floor System

*Fig. 1.4  Typical beam and floor system mode shapes.*

*Fig. 1.3  Response to sinusoidal force.*

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$\alpha_i$   = dynamic coefficient for the harmonic force

$i$   = harmonic multiple (1, 2, 3,...)

$f_{step}$ = step frequency of the activity

$t$   = time

$\phi_i$   = phase angle for the harmonic

As a general rule, the magnitude of the dynamic coefficient decreases with increasing harmonic, for instance, the dynamic coefficients associated with the first four harmonics of walking are 0.5, 0.2, 0.1 and 0.05, respectively. In theory, if any frequency associated with the sinusoidal forces matches the natural frequency of a vibration mode, then resonance will occur, causing severe vibration amplification.

The effect of resonance is shown in Figure 1.3. For this figure, the floor structure is modeled as a simple mass connected to the ground by a spring and viscous damper. A person or machine exerts a vertical sinusoidal force on the mass. Because the natural frequency of almost all concrete slab-structural steel supported floors can be close to or can match a harmonic forcing frequency of human activities, resonance amplification is associated with most of the vibration problems that occur in buildings using structural steel.

Figure 1.3 shows sinusoidal response if there is only one mode of vibration. In fact, there may be many in a floor system. Each mode of vibration has its own displacement configuration or "mode shape" and associated natural frequency. A typical mode shape may be visualized by considering the floor as divided into an array of panels, with adjacent panels moving in opposite directions. Typical mode shapes for a bay are shown in Figure 1.4(b). The panels are large for low-frequency modes (panel length usually corresponding to

a floor span) and small for high frequency modes. In practice, the vibrational motion of building floors are localized to one or two panels, because of the constraining effect of multiple column/wall supports and non-structural components, such as partitions.

For vibration caused by machinery, any mode of vibration must be considered, high frequency, as well as, low frequency. For vibration due to human activities such as dancing or aerobics, a higher mode is more difficult to excite because people are spread out over a relatively large area and tend to force all panels in the same direction simultaneously, whereas adjacent panels must move in opposite directions for higher modal response. Walking generates a concentrated force and therefore may excite a higher mode. Higher modes, however, are generally excited only by relatively small harmonic walking force components as compared to those associated with the lowest modes of vibration. Thus, in practice it is usually only the lowest modes of vibration that are of concern for human activities.

The basic model of Figure 1.3 may be represented by:

$$\text{Sinusoidal Acceleration} = \left(\frac{\text{Force}}{\text{Mass}}\right) \times \text{Response Factor} \quad (1.2)$$

where the response factor depends strongly on the ratio of natural frequency to forcing frequency ($f_n / f$) and, for vibration at or close to resonance, on the damping ratio ($\beta$). It is these parameters that control the vibration serviceability design of most steel floor structures.

It is possible to control the acceleration at resonance by increasing damping or mass since acceleration = force divided by damping times mass (see Figure 1.3). The control is most effective where the sinusoidal forces are small, as they are for walking. Natural frequency also always plays a role, because sinusoidal forces generally decrease with increasing harmonic—the higher the natural frequency, the lower the force. The design criterion for walking vibration in Chapter 4 is based on these principles.

Where the dynamic forces are large, as they are for aerobics, resonant vibration is generally too great to be controlled practically by increasing damping or mass. In this case, the natural frequency of any vibration mode significantly affected by the dynamic force (i.e. a low frequency mode) must be kept away from the forcing frequency. This generally means that the fundamental natural frequency must be made greater than the forcing frequency of the highest harmonic force that can cause large resonant vibration. For aerobics or dancing, attention should be paid to the possibility of transmission of vibrations to sensitive occupancies in other parts of the floor and other parts of the building. This requires the consideration of vibration transfer through supports, such as columns, particularly to parts of the building which may be in resonance with the harmonic force. The design criterion for rhythmic activities in Chapter 5 takes this into account.
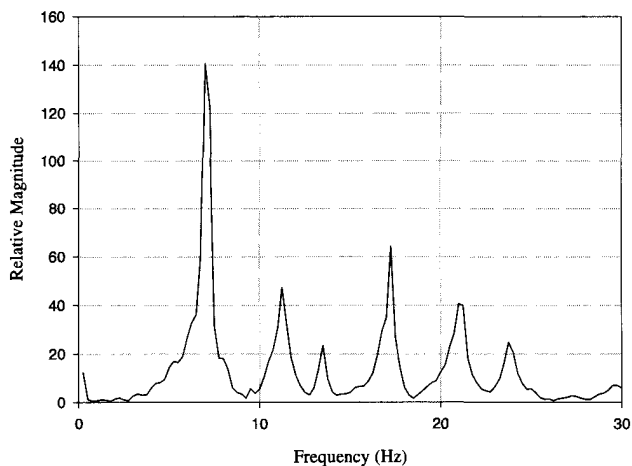


*Fig. 1.5 Frequency spectrum.*

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Where the natural frequency of the floor exceeds 9-10 Hz or where the floors are light, as for example wood deck on light metal joists, resonance becomes less important for human induced vibration, and other criteria related to the response of the floor to footstep forces should be used. When floors are very light, response includes time variation of static deflection due to a moving repeated load (see Figure 1.6), as well as decaying natural vibrations due to footstep impulses (see Figure 1.7). A point load stiffness criterion is appropriate for the static deflection component and a criterion based on footstep impulse vibration is appropriate for the footstep impulses.
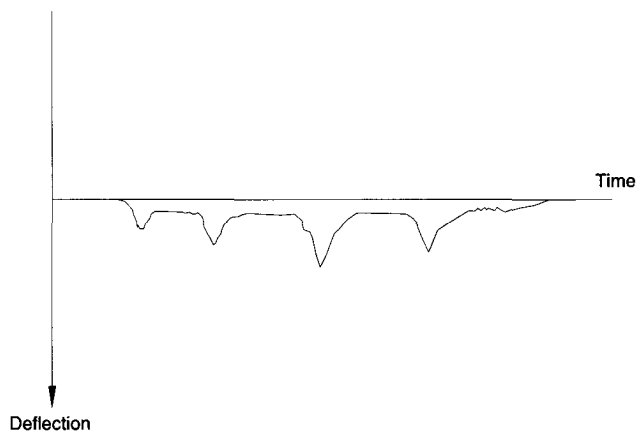


*Fig. 1.6 Quasi-static deflection of a point on a floor due to a person walking across the floor.*
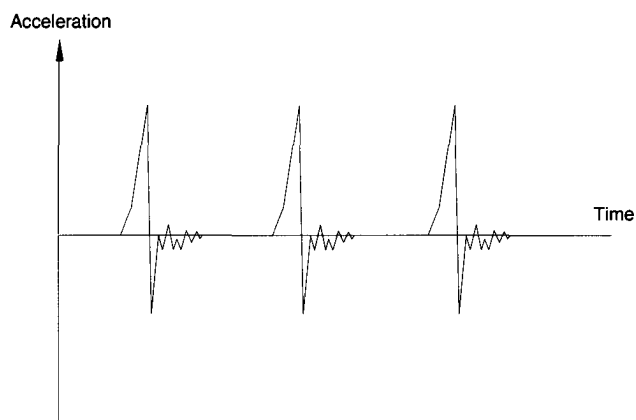


*Fig. 1.7 Floor vibration due to footstep impulses during walking.*

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# Chapter 2
# ACCEPTANCE CRITERIA FOR HUMAN COMFORT

## 2.1  Human Response to Floor Motion

Human response to floor motion is a very complex phenomenon, involving the magnitude of the motion, the environment surrounding the sensor, and the human sensor. A continuous motion (steady-state) can be more annoying than motion caused by an infrequent impact (transient). The threshold of perception of floor motion in a busy workplace can be higher than in a quiet apartment. The reaction of a senior citizen living on the fiftieth floor can be considerably different from that of a young adult living on the second floor of an apartment complex, if both are subjected to the same motion.

The reaction of people who feel vibration depends very strongly on what they are doing. People in offices or residences do not like "distinctly perceptible" vibration (peak acceleration of about 0.5 percent of the acceleration of gravity, $g$), whereas people taking part in an activity will accept vibrations approximately 10 times greater (5 percent $g$ or more). People dining beside a dance floor, lifting weights beside an aerobics gym, or standing in a shopping mall, will accept something in between (about 1.5 percent $g$). Sensitivity within each occupancy also varies with duration of vibration and remoteness of source. The above limits are for vibration frequencies between 4 Hz and 8 Hz. Outside this frequency range, people accept higher vibration accelerations as shown in Figure 2.1.

## 2.2  Recommended Criteria for Structural Design

Many criteria for human comfort have been proposed over the years. The Appendix includes a short historical development of criteria used in North American and Europe. Following are recommended design criteria for walking and rhythmic excitations. The recommended walking excitation criterion, methods for estimating the required floor properties, and design procedures were first proposed by Allen and Murray (1993). The criterion differs considerably from previous "heel-drop" based approaches. Although the proposed criterion for walking excitation is somewhat more complex than previous criteria, it has a wider range of applicability and results in more economical, but acceptable, floor systems.

### 2.2.1  Walking Excitation

As part of the effort to develop this Design Guide, a new criterion for vibrations caused by walking was developed with broader applicability than the criteria currently used in North America. The criterion is based on the dynamic response of steel beam- or joist-supported floor systems to walking forces, and can be used to evaluate structural systems supporting offices, shopping malls, footbridges, and similar occupancies (Allen and Murray 1993). Its development is explained in the following paragraphs and its application is shown in Chapter 4.

The criterion was developed using the following:

- Acceleration limits as recommended by the International Standards Organization (International Standard ISO 2631-2, 1989), adjusted for intended occupancy. The ISO Standard suggests limits in terms of rms acceleration as a multiple of the baseline line curve shown in Figure 2.1. The multipliers for the proposed criterion, which is expressed in terms of peak acceleration, are 10 for offices, 30 for shopping malls and indoor footbridges, and 100 for outdoor footbridges. For design purposes, the limits can be assumed to range between 0.8 and 1.5 times the recommended values depending on
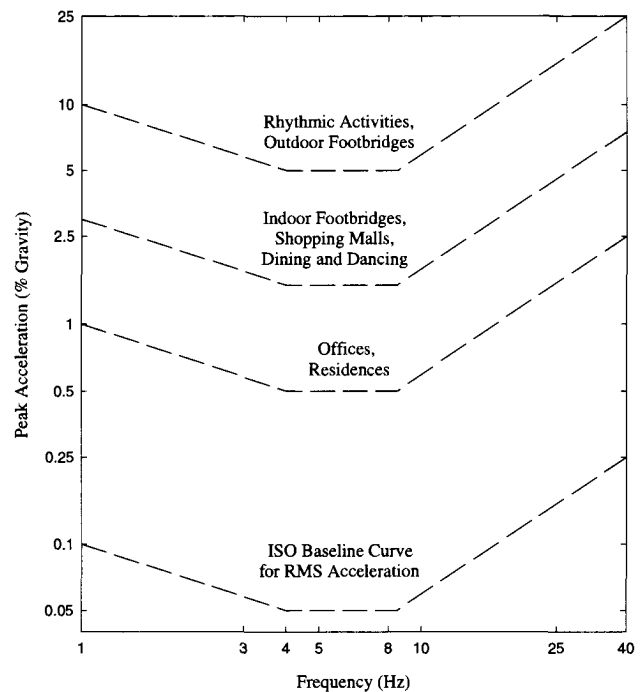


*Fig. 2.1  Recommended peak acceleration for human comfort for vibrations due to human activities (Allen and Murray, 1993; ISO 2631-2:  1989).*

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the duration of vibration and the frequency of vibration events.

- A time dependent harmonic force component which matches the fundamental frequency of the floor:

$$F_i = P\alpha_i \cos(2\pi i f_{step} t) \qquad (2.1)$$

where

$P$ = person's weight, taken as 0.7 kN (157 pounds) for design

$\alpha_i$ = dynamic coefficient for the $i$th harmonic force component

$i$ = harmonic multiple of the step frequency

$f_{step}$ = step frequency

Recommended values for $\alpha_i$ are given in Table 2.1. (Only one harmonic component of Equation (1.1) is used since all other harmonic vibrations are small in comparison to the harmonic associated with resonance.)

- A resonance response function of the form:

$$\frac{a}{g} = \frac{R\alpha_i P}{\beta W} \cdot \cos(2\pi i f_{step} t) \qquad (2.2)$$

where

$a/g$ = ratio of the floor acceleration to the acceleration of gravity

$R$ = reduction factor

$\beta$ = modal damping ratio

$W$ = effective weight of the floor

The reduction factor $R$ takes into account the fact that full steady-state resonant motion is not achieved for walking and that the walking person and the person annoyed are not simultaneously at the location of maximum modal displacement. It is recommended that $R$ be taken as 0.7 for footbridges and 0.5 for floor structures with two-way mode shape configurations.

For evaluation, the peak acceleration due to walking can be estimated from Equation (2.2) by selecting the lowest harmonic, $i$, for which the forcing frequency, $f = i \cdot f_{step}$, can match a natural frequency of the floor structure. The peak acceleration is then compared with the appropriate limit in Figure 2.1. For design, Equation (2.2) can be simplified by approximating the step relationship between the dynamic coefficient, $\alpha_i$, and frequency, $f$, shown in Figure 2.2 by the formula $\alpha = 0.83 exp(-0.35f)$. With this substitution, the following simplified design criterion is obtained:

$$\frac{a_P}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} \le \frac{a_o}{g} \qquad (2.3)$$

where

$a_p / g$ = estimated peak acceleration (in units of $g$)

$a_o / g$ = acceleration limit from Figure 2.1

$f_n$ = natural frequency of floor structure

$P_o$ = constant force equal to 0.29 kN (65 lb.) for floors and 0.41 kN (92 lb.) for footbridges

The numerator $P_o exp(-0.35f_n)$ in Inequality (2.3) represents an effective harmonic force due to walking which results in resonance response at the natural floor frequency $f_n$. Inequality (2.3) is the same design criterion as that proposed by Allen and Murray (1993); only the format is different.

Motion due to quasi-static deflection (Figure 1.6) and footstep impulse vibration (Figure 1.7) can become more critical than resonance if the fundamental frequency of a floor is greater than about 8 Hz. To account approximately for footstep impulse vibration, the acceleration limit $a_o / g$ is not increased with frequency above 8 Hz, as it would be if

| Table 2.1 Common Forcing Frequencies ($f$) and Dynamic Coefficients* ($\alpha_i$) | | | | | | |
|---|---|---|---|---|---|---|
| Harmonic $i$ | Person Walking | | Aerobics Class | | Group Dancing | |
| | f, Hz | $\alpha_i$ | f, Hz | $\alpha_i$ | f, Hz | $\alpha_i$ |
| 1 | 1.6–2.2 | 0.5 | 2-2.75 | 1.5 | 1.5-3 | 0.5 |
| 2 | 3.2–4.4 | 0.2 | 4-5.5 | 0.6 | — | — |
| 3 | 4.8–6.6 | 0.1 | 6-8.25 | 0.1 | — | — |
| 4 | 6.4–8.8 | 0.05 | — | — | — | — |

*dynamic coefficient = peak sinusoidal force/weight of person(s).
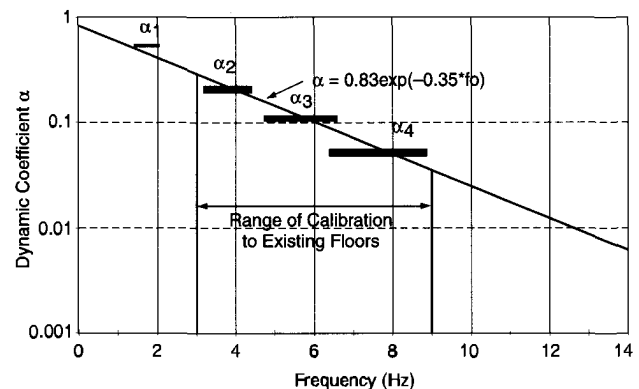


Fig. 2.2 Dynamic coefficient, $\alpha$, versus frequency.

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Figure 2.1 were used. That is, the horizontal portion of the curves between 4 Hz and 8 Hz in Figure 2.1 are extended to the right beyond 8 Hz. To account for motion due to varying static deflection, a minimum static stiffness of 1 kN/mm (5.7 kips/inch) under concentrated load is introduced as an additional check if the natural frequency is greater than 9-10 Hz. More severe criteria for static stiffness under concentrated load are used for sensitive equipment as described in Chapter 6.

Guidelines for the estimation of the parameters used in the above design criterion for walking vibration and application examples are found in Chapter 4.

## 2.2.2 Rhythmic Excitation

Criteria have recently been developed for the design of floor structures for rhythmic exercises (Allen 1990, 1990a; NBC 1990). The criteria are based on the dynamic response of structural systems to rhythmic exercise forces distributed over all or part of the floor. The criteria can be used to evaluate structural systems supporting aerobics, dancing, audience participation and similar events, provided the loading function is known. As an example, Figure 2.3 shows a time record of the dynamic loading function and an associated spectrum for eight people jumping at 2.1 Hz. Table 2.1 gives common forcing frequencies and dynamic coefficients for rhythmic activities.

The peak acceleration of the floor due to a harmonic rhythmic force is obtained from the classical solution by assuming that the floor structure has only one mode of vibration (Allen 1990):

$$\frac{a_p}{g} = \frac{1.3\alpha_i w_p / w_t}{\sqrt{\left[\left(\frac{f_n}{f}\right)^2 - 1\right]^2 + \left[\frac{2\beta f_n}{f}\right]^2}} \qquad (2.4)$$

where

$a_p / g$ = peak acceleration as a fraction of the acceleration due to gravity
$\alpha_i$ = dynamic coefficient (see Table 2.1)
$w_p$ = effective weight per unit area of participants distributed over floor panel
$w_t$ = effective distributed weight per unit area of floor panel, including occupants
$f_n$ = natural frequency of floor structure
$f$ = forcing frequency
= $i \cdot f_{step}$ where $f_{step}$ is the step frequency
$\beta$ = damping ratio

Equation (2.4) can be simplified as follows:

At resonance ($f_n = f$):

$$\frac{a_p}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha_i w_p}{w_t} \qquad (2.5a)$$

Above resonance ($f_n > 1.2f$):

$$\frac{a_p}{g} = \frac{1.3}{(f_n / f)^2 - 1} \cdot \frac{\alpha_i w_p}{w_t} \qquad (2.5b)$$

Most problems occur if a harmonic forcing frequency, $f = i \cdot f_{step}$, is equal to or close to the natural frequency, $f_n$, for which case the acceleration is determined from Equation (2.5a). Vibration from lower harmonics (first or second), however, may still be substantial, and the acceleration for a lower harmonic is determined from Equation (2.5b). The effective maximum acceleration, accounting for all harmonics, can then be estimated from the combination rule (Allen 1990a):

$$a_m = [\Sigma a_i^{1.5}]^{1/1.5} \qquad (2.6)$$

where

$a_i$ = peak acceleration for the i'th harmonic.
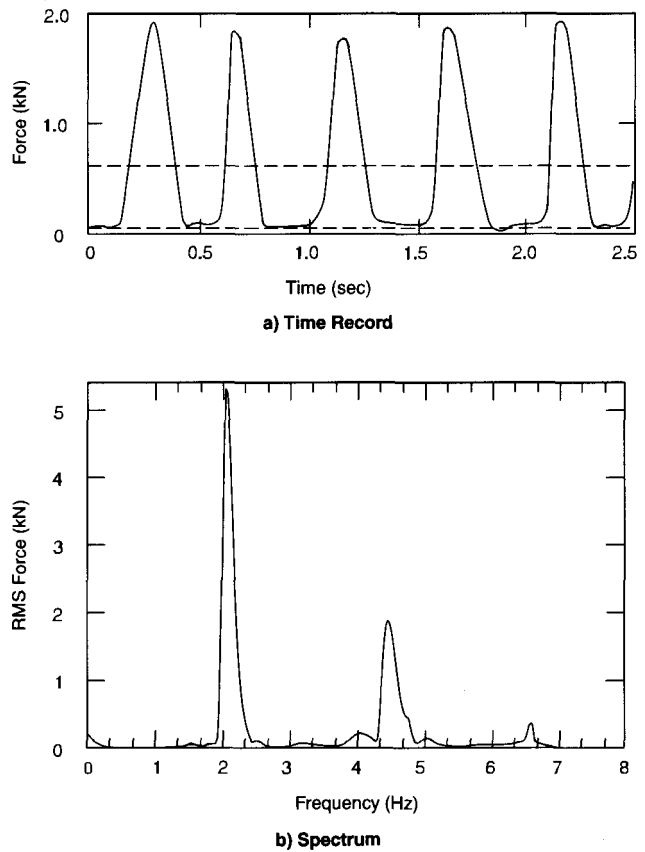


Fig. 2.3 Example loading function and spectrum from rhythmic activity.

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The effective maximum acceleration determined from Equation (2.6) can then be compared to the acceleration limit for people participating in the rhythmic activity (approximately 5 percent gravity from Figure 2.1). Experience shows, however, that many problems with building vibrations due to rhythmic exercises concern more sensitive occupancies in the building, especially for those located near the exercising area. For these other occupancies, the effective maximum acceleration, $a_m$, calculated for the exercise floor should be reduced in accordance with the vibration mode shape for the structural system, before comparing it to the acceleration limit for the sensitive occupancy from Figure 2.1.

The dynamic forces for rhythmic activities tend to be large and resonant vibration is generally too great to be reduced practically by increasing the damping and or mass. This means that for design purposes, the natural frequency of the structural system, $f_n$, must be made greater than the forcing frequency, f, of the highest harmonic that can cause large resonant vibration. Equation (2.5b) can be inverted to provide the following design criterion (Allen 1990a):

$$f_n \geq f \sqrt{1 + \frac{k}{(a_o/g)} \frac{\alpha_i w_p}{w_t}} \tag{2.7}$$

where

$k$  = constant (1.3 for dancing, 1.7 for lively concert or sports event, 2.0 for aerobics)

$a_o/g$ = acceleration limit (0.05, or less, if sensitive occupancies are affected)

and the other parameters are defined above. Careful analysis by use of Equations (2.5) and (2.6) can provide better guidance than Equation (2.7), as for example if resonance with the highest harmonic is acceptable because of sufficient mass or partial loading of the floor panel.

Guidance on the estimation of parameters, including building vibration mode shape, and examples of the application of Equations (2.5) to (2.7) are given in Chapter 5.

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# Chapter 3
# NATURAL FREQUENCY OF STEEL FRAMED FLOOR SYSTEMS

The most important parameter for the vibration serviceability design and evaluation of floor framing systems is natural frequency. This chapter gives guidance for estimating the natural frequency of steel beam and steel joist supported floor systems, including the effects of continuity.

## 3.1. Fundamental Relationships

Steel framed floors generally are two-way systems which may have several vibration modes with closely spaced frequencies. The natural frequency of a critical mode in resonance with a harmonic of step frequency may therefore be difficult to assess. Modal analysis of the floor structure can be used to determine the critical modal properties, but there are factors that are difficult to incorporate into the structural model (composite action, boundary and discontinuity conditions, partitions, other non-structural components, etc). An unfinished floor with uniform bays can have a variety of modal pattern configurations extending over the whole floor area, but partitions and other non-structural components tend to constrain significant dynamic motions to local areas in such a way that the floor vibrates locally like a single two-way panel. The following simplified procedures for determining the first natural frequency of vertical vibration are recommended.

The floor is assumed to consist of a concrete slab (or deck) supported on steel beams or joists which are supported on walls or steel girders between columns. The natural frequency, $f_n$, of a critical mode is estimated by first considering a "beam or joist panel" mode and a "girder panel" mode separately and then combining them. Alternatively, the natural frequency can be estimated by finite element analyses.

Beam or joist and girder panel mode natural frequencies can be estimated from the fundamental natural frequency equation of a uniformly loaded, simply-supported, beam:

$$f_n = \frac{\pi}{2} \left[ \frac{g E_s I_t}{w L^4} \right]^{1/2} \tag{3.1}$$

where

$f_n$ = fundamental natural frequency, Hz
$g$ = acceleration of gravity, 9.86 $m/s^2$ or 386 $in/sec^2$
$E_s$ = modulus of elasticity of steel
$I_t$ = transformed moment of inertia; effective transformed moment of inertia, if shear deformations are included
$w$ = uniformly distributed weight per unit length (actual, not design, live and dead loads) supported by the member
$L$ = member span

The combined mode or system frequency, can be estimated using the Dunkerley relationship:

$$\frac{1}{f_n^2} = \frac{1}{f_j^2} + \frac{1}{f_g^2} \tag{3.2}$$

where

$f_j$ = beam or joist panel mode frequency
$f_g$ = girder panel mode frequency

Equation (3.1) can be rewritten as

$$f_n = 0.18 \sqrt{\frac{g}{\Delta}} \tag{3.3}$$

where

$\Delta$ = midspan deflection of the member relative to its supports due to the weight supported (i.e. $\Delta = 5wL^4/(384 E_s I_t)$)

Sometimes, as described later in this Design Guide, shear deformations must also be included in determining $\Delta$.

For the combined mode, if both the beam or joist and girder are assumed simply supported, the Dunkerley relationship can be rewritten as

$$f_n = 0.18 \sqrt{\frac{g}{(\Delta_j + \Delta_g)}} \tag{3.4}$$

where

$\Delta_j$ and $\Delta_g$ = beam or joist and girder deflections due to the weight supported, respectively.

Tall buildings can have vertical column frequencies low enough to create serious resonance problems with rhythmic activity. For these cases, Equation (3.4) is modified to include the column effect:

$$f_n = 0.18 \sqrt{\frac{g}{(\Delta_j + \Delta_g + \Delta_c)}} \tag{3.5}$$

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where

$\Delta_c$ = axial shortening of the column due to the weight supported

Further guidance on the estimation of deflection of joists, beams and girders due to flexural and shear deformation is found in the following sections.

## 3.2 Composite Action

In calculating the fundamental natural frequency using the relationships in Section 3.1, the transformed moment of inertia is to be used if the slab (or deck) is attached to the supporting member. This assumption is to be applied even if structural shear connectors are not used, because the shear forces at the slab/member interface are resisted by deck-to-



a) Two Span



b) Three Span

*Fig. 3.1 Modal flexural deflections, $\Delta$, for beams continuous over supports.*

member spot welds or by friction between the concrete and metal surfaces.

If the supporting member is separated from the slab (for example, the case of overhanging beams which pass over a supporting girder) composite behavior should not be assumed. For such cases, the fundamental natural frequency of the girder can be increased by providing shear connection between the slab and girder flange (see Section 7.2).

To take account of the greater stiffness of concrete on metal deck under dynamic as compared to static loading, it is recommended that the concrete modulus of elasticity be taken equal to 1.35 times that specified in current structural standards for calculation of the transformed moment of inertia. Also for determining the transformed moment of inertia of typical beams or joists and girders, it is recommended that the effective width of the concrete slab be taken as the member spacing, but not more than 0.4 times the member span. For edge or spandrel members, the effective slab width is to be taken as one-half the member spacing but not more than 0.2 times the member span plus the projection of the free edge of the slab beyond the member Centerline. If the concrete side of the member is in compression, the concrete can be assumed to be solid, uncracked.

See Section 3.5 and for special considerations needed for trusses and open web joist framing.

## 3.3 Distributed Weight

The supported weight, w, used in the above equations must be estimated carefully. The actual dead and live loads, not the design dead and live loads, should be used in the calculations. For office floors, it is suggested that the live load be taken as $0.5 kN/m^2$ (11 psf). This suggested live load is for typical office areas with desks, file cabinets, bookcases, etc. A lower value should be used if these items are not present. For residential floors, it is suggested that the live load be taken as $0.25\ kN/m^2$ (6 psf). For footbridges, and gymnasium and shopping center floors, it is suggested that the live load be taken as zero, or at least nearly so.

Equations (3.1) and (3.3) are based on the assumption of a simply-supported beam, uniformly loaded. Joists, beams or girders usually are uniformly loaded, or nearly so, with the exception of girders that support joists or beams at mid-span only, in which case the calculated deflection should be multiplied by $4/\pi$ ($\approx 1.3$) to take into account the difference between the frequency for a simply-supported beam with distributed mass and that with a concentrated mass at mid-span.

## 3.4 Deflection Due to Flexure: Continuity

*Continuous Joists, Beams or Girders*

Equations (3.3) through (3.5) also apply approximately for continuous beams over supports (such as beams shear-connected through girders or joists connected through girders at
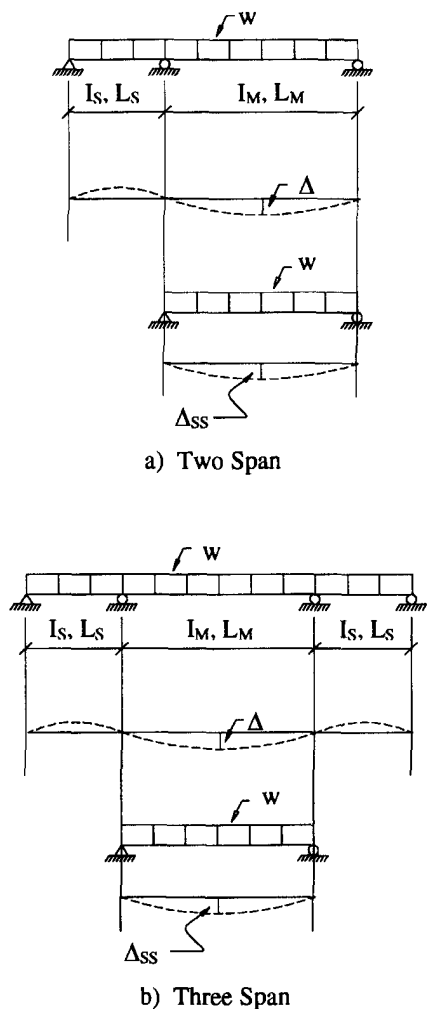
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top and bottom chords) for the situation where the distributed weight acts in the direction of modal displacement, i.e. down where the modal displacement is down, and up where it is up (opposite to gravity). Adjacent spans displace in opposite directions and, therefore, for a continuous beam with equal spans, the fundamental frequency is equal to the natural frequency of a single simply-supported span.

Where the spans are not equal, the following relations can be used for estimating the flexural deflection of a continuous member from the simply supported flexural deflection, $\Delta_{ss}$, of the main (larger) span, $L_m$ due to the weight supported. For two continuous spans:

$$\Delta = \left[\frac{0.4 + \dfrac{k_m}{k_s}\left(1 + 0.6\dfrac{L_S^2}{L_M^2}\right)}{1 + \dfrac{k_m}{k_s}}\right]\Delta_{SS} \qquad (3.6)$$

For three continuous spans

$$\Delta = \left[\frac{0.6 + 2\dfrac{k_m}{k_s}\left(1 + 1.2\dfrac{L_S^2}{L_M^2}\right)}{3 + 2\dfrac{k_m}{k_s}}\right]\Delta_{SS} \qquad (3.7)$$

where

$k_m = I_M / L_M$
$k_s = I_S / L_S$
$I$ = moment of inertia
$L_M$ and $L_S$ are defined in Figure 3.1.

*Members Continuous with Columns*

The natural frequency of a girder or beam moment-connected to columns is increased because of the flexural restraint of the
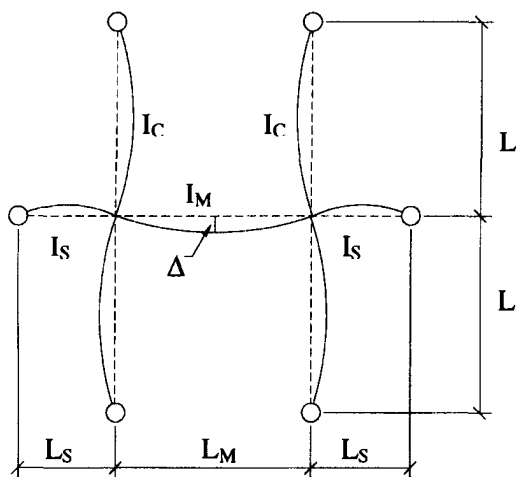


*Fig. 3.2 Modal flexural deflections, Δ, for beams or girders continuous with columns.*

columns. This is important for tall buildings with large columns. The following relationship can be used for estimating the flexural deflection of a girder or beam moment connected to columns in the configuration shown in Figure 3.2.

$$\Delta = \left[\frac{0.6 + 2\dfrac{k_m}{k_s}(1 + 1.2\lambda) + \boxed{1.2}\,n_c\dfrac{k_c}{k_s}}{3 + 2\dfrac{k_m}{k_s} + \boxed{6}\,n_c\dfrac{k_c}{k_s}}\right]\Delta_{SS} \qquad (3.8)$$

where

$k_m = I_M / L_M$
$k_s = I_S / L_S$
$k_c = I_C / L_C$
$L_M$, $L_S$ and $L_C$ are defined in Figure 3.2
$\Delta_{SS}$ = flexural deflection of the simply supported main (longer) beam or girder of span $L_M$ due to the weight supported
$\lambda = (L_S / L_M)^2$

*Cantilevers*

The natural frequency of a fixed cantilever can be estimated using Equation (3.3) through (3.5), with the following used to calculate $\Delta$. For uniformly distributed mass

$$\Delta_f = \frac{wL^4}{8EI} \qquad (3.9)$$

and for a mass concentrated at the tip

$$\Delta_f = \frac{WL^3}{3EI} \qquad (3.10)$$

Cantilevers, however, are rarely fully fixed at their supports. The following equations can be used to estimate the flexural deflection of a cantilever/backspan/column condition shown in Figure 3.3. If the cantilever deflection, $\Delta_T$, exceeds the deflection of the backspan, $\Delta_B$, then

$$\Delta = \Delta_T = C_m\left[1 + \frac{4}{3}\frac{L_B}{L_t} \cdot \frac{1 + 0.25 L_B^2 / L_T^2}{1 + n_c k_c / k_b}\right]\Delta_F \qquad (3.11)$$

If the opposite is true, then

$$\Delta = \Delta_B = \left[1 + 2.4\left(\frac{L_t^2 / L_B^2 - 0.5 k_c / k_b}{1 + n_c k_c / k_b}\right)\right]\Delta_{SS} \qquad (3.12)$$

where $L_C$, $L_B$ and $L_T$ are defined in Figure 3.3 and

$k_b = I_B / L_B$
$k_c = I_C / L_C$
$C_m$ = 0.81 for distributed mass and 1.06 for mass concentrated at the tip
$n_c$ = 2 if columns occur above and below, 1 if only above or below
$\Delta_f$ = flexural deflection of a fixed cantilever, due to the weight supported

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$\Delta_{SS}$ = flexural deflection of backspan, assumed simply supported

If the cantilever/backspan beam is supported by a girder, $k_c = 0$ in Equations (3.11) and (3.12).

## 3.5 Deflection Due to Shear in Beams and Trusses

Sometimes shear may contribute substantially to the deflection of the member. Two types of shear may occur:

- Direct shear due to shear strain in the web of a beam or girder, or due to length changes of the web members of a truss;
- Indirect shear in trusses as a result of eccentricity of member forces through joints.

For wide flange members, the shear deflection is simply equal to the accumulated shear strain in the web from the

a) $\Delta_T$ Greater Than $\Delta_B$

b) $\Delta_B$ Greater Than $\Delta_T$

*Fig. 3.3 Modal flexural deflections, Δ, for cantilever/backspan/columns.*

support to mid-span. For rolled shapes, shear deflection is usually small relative to flexural deflection and can be neglected.

For simply supported trusses, the shear deformation effect can usually be taken into account using:

$$I_{eff} = \frac{I_{comp}}{1 + 0.15 I_{comp} / I_{chords}} \qquad (3.13)$$

where

$I_{eff}$ = the "effective" transformed moment of inertia which accounts for shear deformation
$I_{comp}$ = the fully composite moment inertia
$I_{chords}$ = the moment of inertia of the joist chords alone

Equation (3.13) is applicable only to simply supported trusses with span-to-depth ratios greater than approximately 12.

For deep long-span trusses the shear strain can be considerable, substantially reducing the estimated natural frequency from that based on flexural deflection (Allen 1990a). The following method may be used for estimating such shear deflection assuming no eccentricity at the joints:

1. Determine web member forces, due to the weight supported.
2. Determine web member length changes $\delta_i = F_i L_i / A_i E_s$, where for the $i$th member, $F_i$ is the axial force due to the real loads, $L_i$ is the length, and $A_i$ is the cross-section area.
3. Determine shear increments, $v_i = \delta_i / \cos\gamma_i$, where $\gamma_i$ is the angle of the web member to vertical.
4. Sum the shear increments for each web member from the support to mid-span.

The total deflection, $\Delta$, is then the sum of flexural and shear deflections, generally at mid-span.

## 3.6 Special Considerations for Open Web Joists and Joist Girders

The effects of joist seats, web shear deformation, and eccentricity of joints must be considered in calculating the natural frequency of open web joist and hot-rolled girder or joist-girder framed floor systems.

For the case of a girder or joist girder supporting standard open web joists, it has been found that the joist seats are not sufficiently stiff to justify the full transformed moment of inertia assumption for the girder or joist girder. It is recommended that the effective moment of inertia of girders supporting joist seats be determined from

$$I_g = I_{nc} + (I_c - I_{nc}) / 4 \qquad (3.14)$$

where

$I_{nc}$ and $I_c$ = non-composite and fully composite moments of inertia, respectively.
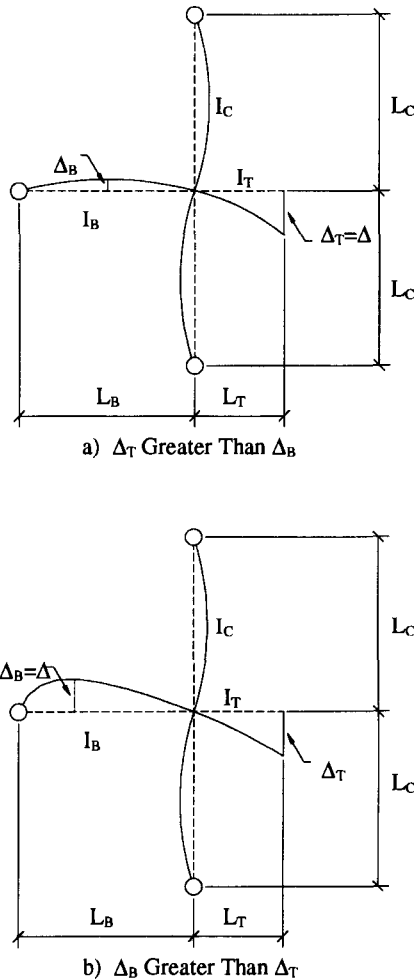
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The effective moment of inertia of joists and joist girders that is used to calculate the limiting span/360 load in Steel Joist Institute (SJI) load tables is 0.85 times the moment of inertia of the chord members. This factor accounts for web shear deformation. It has recently been reported (Band and Murray 1996) that the 0.85 coefficient can be increased to 0.90 if the span-to-depth ratio of the joist or joist-girder is not less than about 20. For smaller span-to-depth ratios, the effective moment of inertia of the joist or joist-girder can be as low as 0.50 times the moment of inertia of the chords. Barry and Murray (1996) proposed the following method to estimate the effective moment of inertia of joists or joist girders:

$$I_{mod} = C_r I_{chords} \qquad (3.15)$$

where, for joists or joist girders with single or double angle web members,

$$C_r = 0.90(1 - e^{-0.28(L/D)})^{2.8} \qquad (3.16)$$

for $6 \leq L/D \leq 24$, $L$ = span length, and $D$ = nominal depth of the joist and for joists with continuous round rod web members

$$C_r = 0.721 + 0.00725(L/D) \qquad (3.17)$$

for $10 \leq L/D \leq 24$.

The effective transformed moment of inertia of joist supported tee-beams can then be calculated using

$$I_{eff} = \cfrac{1}{\cfrac{\gamma}{I_{chords}} + \cfrac{1}{I_{comp}}} \qquad (3.18)$$

where

$$\gamma = \frac{1}{C_r} - 1 \qquad (3.19)$$

and

$I_{comp}$ = the transformed moment of inertia using the actual chord areas. (See Examples 4.5 and 4.6 in Section 4.4.2).

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# Chapter 4
# DESIGN FOR WALKING EXCITATION

## 4.1 Recommended Criterion

Existing North American floor vibration design criteria are generally based on a reference impact such as a heel-drop and were calibrated using floors constructed 20-30 years ago. Annoying floors of this vintage generally had natural frequencies between 5 and 8 hz because of traditional design rules, such as live load deflection less than span/360, and common construction practice. With the advent of limit states design and the more common use of lightweight concrete, floor systems have become lighter, resulting in higher natural frequencies for the same structural steel layout. However, beam and girder spans have increased, sometimes resulting in frequencies lower than 5 hz. Most existing design criteria do not properly evaluate systems with frequencies below 5 hz and above 8 hz.

The design criterion for walking excitations recommended in Section 2.2.1 has broader applications than commonly used criteria. The recommended criterion is based on the dynamic response of steel beam and joist supported floor systems to walking forces. The criterion can be used to evaluate concrete/steel framed structural systems supporting footbridges, residences, offices, and shopping malls.

The criterion states that the floor system is satisfactory if the peak acceleration, $a_p$, due to walking excitation as a fraction of the acceleration of gravity, $g$, determined from

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} \qquad (4.1)$$

does not exceed the acceleration limit, $a_o / g$, for the appropriate occupancy. In Equation (4.1),

$P_o$ = a constant force representing the excitation,
$f_n$ = fundamental natural frequency of a beam or joist panel, a girder panel, or a combined panel, as applicable,
$\beta$ = modal damping ratio, and
$W$ = effective weight supported by the beam or joist panel, girder panel or combined panel, as applicable.

Recommended values of $P_o$ and $\beta$, as well as $a_o / g$ limits for several occupancies, are given in Table 4.1. Figure 2.1 can also be used to evaluate a floor system if the original ISO plateau between 4 Hz and approximately 8 Hz is extended from 3 Hz to 20 Hz as discussed in Section 2.2.1.

If the natural frequency of a floor is greater than 9-10 Hz, significant resonance with walking harmonics does not occur, but walking vibration can still be annoying. Experience indi-

cates that a minimum stiffness of the floor to a concentrated load of 1 kN per mm (5.7 kips per in.) is required for office and residential occupancies. To ensure satisfactory performance of office or residential floors with frequencies greater than 9-10 Hz, this stiffness criterion should be used in addition to the walking excitation criterion, Equation (4.1) or Figure 2.1. Floor systems with fundamental frequencies less than 3 Hz should generally be avoided, because they are liable to be subjected to "rogue jumping" (see Chapter 5).

The following section, based on Allen and Murray (1993), provides guidance for estimating the required floor properties for application of the recommended criterion.

## 4.2 Estimation Of Required Parameters

The parameters in Equation (4.1) are obtained or estimated from Table 4.1 ($P_o$, $\beta$, and $a_o / g$) and Chapter 3 ($f_n$). For simply supported footbridges $f_n$ is estimated using Equation (3.1) or (3.3) and $W$ is equal to the weight of the footbridge. For floors, the fundamental natural frequency, $f_n$, and effective panel weight, $W$, for a critical mode are estimated by first considering the 'beam or joist panel' and 'girder panel' modes separately and then combining them as explained in Chapter 3.

*Effective Panel Weight, W*

The effective panel weights for the beam or joist and girder panel modes are estimated from

$$W = wBL \qquad (4.2)$$

where

$w$ = supported weight per unit area
$L$ = member span
$B$ = effective width

For the beam or joist panel mode, the effective width is

$$B_j = C_j (D_s / D_j)^{1/4} L_j \qquad (4.3a)$$

but not greater than ⅔ × floor width

where

$C_j$ = 2.0 for joists or beams in most areas
   = 1.0 for joists or beams parallel to an interior edge
$D_s$ = transformed slab moment of inertia per unit width
   = $d_e^3 / (12n)$ mm³ or $12d_e^3 / (12n)$ in⁴/ ft
$d_e$ = effective depth of the concrete slab, usually taken as

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| | Constant Force $P_o$ | Damping Ratio $\beta$ | Acceleration Limit $a_o / g \times 100\%$ |
|---|---|---|---|
| **Table 4.1**<br>**Recommended Values of Parameters in**<br>**Equation (4.1) and $a_o / g$ Limits** | | | |
| Offices, Residences, Churches | 0.29 kN (65 lb) | 0.02–0.05* | 0.5% |
| Shopping Malls | 0.29 kN (65 lb) | 0.02 | 1.5% |
| Footbridges—Indoor | 0.41 kN (92 lb) | 0.01 | 1.5% |
| Footbridges—Outdoor | 0.41 kN (92 lb) | 0.01 | 5.0% |

\* 0.02 for floors with few non-structural components (ceilings, ducts, partitions, etc.) as can occur in open work areas and churches,
0.03 for floors with non-structural components and furnishings, but with only small demountable partitions, typical of many modular office areas,
0.05 for full height partitions between floors.

the depth of the concrete above the form deck plus one-half the depth of the form deck

$n$ = dynamic modular ratio = $E_s / 1.35 E_c$
$E_s$ = modulus of elasticity of steel
$E_c$ = modulus of elasticity of concrete
$D_j$ = joist or beam transformed moment of inertia per unit width
$I_t / S$, mm$^3$ (in.$^4$/ft)
$I_t$ = effective moment of inertia of the tee-beam
$S$ = joist or beam spacing
$L_j$ = joist or beam span.

For the girder panel mode, the effective width is

$$B_g = C_g (D_j / D_g)^{1/4} L_g \qquad (4.3b)$$

but not greater than $\tfrac{2}{3} \times$ floor length

where

$C_g$ = 1.6 for girders supporting joists connected to the girder flange (e.g. joist seats)
   = 1.8 for girders supporting beams connected to the girder web
$D_g$ = girder transformed moment of inertia per unit width
   = $I_g / L_j$ for all but edge girders
   = $\boxed{2I_g / L_j}$ for edge girders
$L_g$ = girder span.

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Where beams, joists or girders are continuous over their supports and an adjacent span is greater than 0.7 times the span under consideration, the effective panel weight, $W_j$ or $W_g$, can be increased by 50 percent. This liberalization also applies to rolled sections shear-connected to girder webs, but not to joists connected only at their top chord. Since continuity effects are not generally realized when girders frame directly into columns, this liberalization does not apply to such girders.

For the combined mode, the equivalent panel weight is approximated using

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \qquad (4.4)$$

where

$\Delta_j$ and $\Delta_g$ = maximum deflections of the beam or joist and girder, respectively, due to the weight supported by the member
$W_j$ and $W_g$ = effective panel weights from Equation (4.2) for the beam or joist and girder panels, respectively

Composite action with the concrete deck is normally assumed when calculating $\Delta_j$ and $\Delta_g$, provided there is sufficient shear connection between the slab/deck and the member. See Sections 3.2, 3.4 and 3.5 for more details.

If the girder span, $L_g$, is less than the joist panel width, $B_j$, the combined mode is restricted and the system is effectively stiffened. This can be accounted for by reducing the deflection, $\Delta g$, used in Equation (4.4) to

$$\Delta_g' = \frac{L_g}{B_j} (\Delta_g) \qquad (4\text{-}5)$$

where $L_g / B_j$ is taken as not less than 0.5 nor greater than 1.0 for calculation purposes, i.e. $0.5 \le L_g / B_j \le 1.0$.

If the beam or joist span is less than one-half the girder span, the beam or joist panel mode and the combined mode should be checked separately.

*Damping*

The damping associated with floor systems depends primarily on non-structural components, furnishings, and occupants. Table 4.1 recommends values of the modal damping ratio, $\beta$. Recommended modal damping ratios range from 0.01 to 0.05. The value 0.01 is suitable for footbridges or floors with

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no non-structural components or furnishings and few occupants. The value 0.02 is suitable for floors with very few non-structural components or furnishings, such as floors found in shopping malls, open work areas or churches. The value 0.03 is suitable for floors with non-structural components and furnishings, but with only small demountable partitions, typical of many modular office areas. The value 0.05 is suitable for offices and residences with full-height room partitions between floors. These recommended modal damping ratios are approximately half the damping ratios recommended in previous criteria (Murray 1991, CSA S16.1-M89) because modal damping excludes vibration transmission, whereas dispersion effects, due to vibration transmission are included in earlier heel drop test data.

*Floor Stiffness*

For floor systems having a natural frequency greater than 9-10 Hz., the floor should have a minimum stiffness under a concentrated force of 1 kN per mm (5.7 kips per in.). The following procedure is recommended for calculating the stiffness of a floor. The deflection of the joist panel under concentrated force, $\Delta_{jP}$, is first estimated using

$$\Delta_{jP} = \frac{\Delta_{oj}}{N_{eff}} \qquad (4.6)$$

where

$\Delta_{oj}$ = the static deflection of a single, simply supported, tee-beam due to a 1 kN (0.225 kips) concentrated force calculated using the same effective moment of inertia as was used for the frequency calculation

$N_{eff}$ = number of effective beams or joists. The concentrated load is to be placed so as to produce the maximum possible deflection of the tee-beam. The effective number of tee-beams can be estimated from

$$N_{eff} = 0.49 + 34.2 \frac{d_e}{S} + (9.0 \times 10^{-9}) \frac{L_j^4}{I_t}$$

$$- 0.00059 \left(\frac{L_j}{S}\right)^2 \geq 1.0 \qquad (4.7)$$

for

$$0.018 \leq \frac{d_e}{S} \leq 0.208$$

$$4.5 \times 10^6 \leq \frac{L_j^4}{I_t} \leq 257 \times 10^6$$

$$2 \leq \frac{L_j}{S} \leq 30$$

where

$d_e$ = effective slab depth,
$S$ = joist or beam spacing,
$L_j$ = joist or beam span, and
$I_t$ = transformed moment of inertia of the tee-beam.

Equation (4.7) was developed by Kittennan and Murray (1994) and replaces two traditionally used equations, one developed for open web joist supported floor systems and the other for hot-rolled beam supported floor systems; see Murray (1991).

The total floor deflection, $\Delta_P$, is then estimated using

$$\Delta_P = \Delta_{jP} + \Delta_{gP} / 2 \qquad (4.8)$$

where

$\Delta_{gP}$ = maximum deflection of the more flexible girder due to a 1 kN (0.225 kips) concentrated load, $P_1$, using the same effective moment of inertia as used in the frequency calculation.

The deflections $\boxed{\Delta_{oj}}$ and $\Delta_{gP}$ are usually estimated using

$$\Delta = \frac{P_1 L^3}{48 E I_t} \qquad (4.9)$$

which assumes simple span conditions. To account for rotational restraint provided by beam and girder web framing connections, the coefficient 1/48 may be reduced to 1/96, which is the geometric mean of 1/48 (for simple span beams) and 1/192 (for beams with built-in ends). This reduction is commonly used when evaluating floors for sensitive equipment use, but is not generally used when evaluating floors for human comfort.

## 4.3 Application Of Criterion

*General*

Application of the criterion requires careful consideration by the structural engineer. For example, the acceleration limit for outdoor footbridges is meant for traffic and not for quiet areas like crossovers in hotel or office building atria.

Designers of footbridges are cautioned to pay particular attention to the location of the concrete slab relative to the beam height. The concrete slab may be located between the beams (because of clearance considerations); then the footbridge will vibrate at a much lower frequency and at a larger amplitude because of the reduced transformed moment of inertia.

As shown in Figure 4.1, an open web joist is typically supported at the ends by a seat on the girder flange and the bottom chord is not connected to the girders. This support detail provides much less flexural continuity than shear connected beams, reducing both the lateral stiffness of the girder panel and the participation of the mass of adjacent bays in resisting walker-induced vibration. These effects are accounted for as follows:

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1. The reduced lateral stiffness requires that the coefficient 1.8 in Equation (4.3b) be reduced to 1.6 when joist seats are present.
2. The non-participation of mass in adjacent bays means that an increase in effective joist panel weight should not be considered, that is, the 50 percent increase in panel weight, as recommended for shear-connected beam-to-girder or column connections should not be used.

Also, the separation of the girder from the concrete slab results in partial composite action and the moment of inertia of girders supporting joist seats should therefore be determined using the procedure in Section 3.6.

*Unequal Joist Spans*

For the common situation where the girder stiffnesses or effective girder panel weights in a bay are different, the following modifications to the basic design procedure are necessary.

1. The combined mode frequency should be determined using the more flexible girder, i.e. the girder with the greater value of $\Delta_g$ or lowest $f_g$.
2. The effective girder panel width should be determined using the average span length of the joists supported by the more flexible girder, i.e., the average joist span length is substituted for $L_j$ when determining $D_g$.
3. In some instances, calculations may be required for both girders to determine the critical case.

*Interior Floor Edges*

Interior floor edges, as in mezzanine areas or atria, require special consideration because of the reduced effective mass due to the free edge. Where the edge member is a joist or beam, a practical solution is to stiffen the edge by adding another joist or beam, or by choosing an edge beam with

moment of inertia 50 percent greater than for the interior beams. If the edge joist or beam is not stiffened, the estimation of natural frequency, $f_n$, and effective panel weight, $W$, should be based on the general procedure with the coefficient $C_j$ in Equation (4.3a) taken as 1.0. Where the edge member is a girder, the estimation of natural frequency, $f_n$, and effective panel weight, $W$, should be based on the general procedure, except that the girder panel width, $B_g$, should be taken as $\frac{2}{3}$ of the supported beam or joist span. See Examples 4.9 and 4.10.

Experience so far has shown that exterior floor edges of buildings do not require special consideration as do interior floor edges. Reasons for this include stiffening due to exterior cladding and walkways generally not being adjacent to exterior walls. If these conditions do not exist, the exterior floor edges should be given special consideration.

*Vibration Transmission*

Occasionally, a floor system will be judged particularly annoying because of vibration transmission transverse to the supporting joists. In these situations, when the floor is impacted at one location there is a perception that a "wave" moves from the impact location in a direction transverse to the supporting joists. The phenomenon is described in more detail in Section 7.2. The recommended criterion does not address this phenomenon, but a small change in the structural system will eliminate the problem. If one beam or joist stiffness or spacing is changed periodically, say by 50 percent in every third bay, the "wave" is interrupted at that location and floor motion is much less annoying. Fixed partitions, of course, achieve the same result.

*Summary*

Figure 4.2 is a summary of the procedure for assessing typical building floors for walking vibrations.

### 4.4 Example Calculations

The following examples are presented first in the SI system of units and then repeated in the US Customary (USC) system of units. Table 4.2 identifies the intent of each example.

*4.4.1 Footbridge Examples*

### Example 4.1—SI Units

An outdoor footbridge of span 12m with pinned supports and the cross-section shown is to be evaluated for walking vibration.

*Deck Properties*

Concrete: $m_c = 2400 \text{ kg/m}^3$
$f_c' = 30 \text{ MPa}$
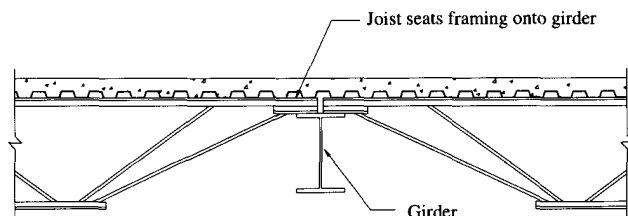$E_c = 24,000 \text{ MPa}$
Slab + deck weight = 3.6 kPa



*Fig. 4.1 Typical joist support.*

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## A. FLOOR SLAB

Determine unit weight and $d_e$ = effective depth

Calculate $D_s = d_e^3 / 12n$

## B. JOIST PANEL MODE

Calculate $I_j$ (see Sections 3.5 and 3.6 if [trusses] or open web joists)

Calculate $w_j$ and $\Delta_j = \dfrac{5w_j L_j^4}{384 E_s I_j}$

Calculate $f_j = \boxed{0.18} \ \sqrt{g / \Delta_j}$ and $D_j = I_j / S$

Calculate $B_j = C_j (D_s / D_j)^{1/4} L_j \le 2/3 \times$ floor width

$C_j = \boxed{2.0}$ for interior panels; $\boxed{1.0}$ for edge panels

Calculate $W_j = w_j B_j L_j$ [(x 1.5 if continuous)]

## C. GIRDER PANEL MODE

Base calculations on girder with [smaller frequency.]

Calculate $I_g$ (see Sections 3.5 and 3.6 if truss or joist girder or if girder supports open web joists).

Calculate $w_g$ and $\Delta_g = \dfrac{5 w_g L_g^4}{384 E_s I_g}$ with correction if only one beam.

Calculate $f_g = \boxed{0.18} \ \sqrt{g / \Delta_g}$ and $D_g = I_g / L_j$

   Use average of supported joist span lengths, if different, for $L_j$

For interior panel, calculate

$\quad B_g = \boxed{C_g (D_j / D_g)^{1/4} L_j} \le 2/3$ floor length

$\quad C_g = 1.8$ if shear connected; 1.6 if not.

For edge panel, calculate $B_g = 2/3 L_j$

Calculate $W_g = w_g B_g L_g$ ($\times 1.5$ if continuous)

## D. COMBINED PANEL MODE

If $B_j > L_g$, reduce $\Delta_g$ by $L_g / B_j \ge 0.5$

Calculate $f_n = 0.18 \sqrt{g / (\Delta_j + \Delta_g)}$

Calculate $W = \dfrac{\Delta_j}{\Delta_j + \Delta_g} W_j + \dfrac{\Delta_g}{\Delta_j + \Delta_g} W_g$

Choose $\beta$ from Table 4.1 (0.02, 0.03, 0.05)

Calculate $\dfrac{a_p}{g} = \dfrac{P_o \exp(-0.35 f_n)}{\beta W}$ where $P_o \approx 0.29$ kN (65 lb)

Compare with $a_o / g$ from Table 4.1 (0.5%, 1.5%)

## E. CHECK STIFFNESS CRITERION IF $f_n > 9$ Hz

## F. REDESIGN IF NECESSARY

*Fig. 4.2 Floor evaluation calculation procedure.*

| Table 4.2 Summary of Walking Excitation Examples | | |
|---|---|---|
| **Example** | **Units** | **Description** |
| 4.1 | SI | Outdoor Footbridge |
| 4.2 | USC | Same as Example 4.1 |
| 4.3 | SI | Typical Interior Bay of an Office Building—Hot Rolled Framing |
| 4.4 | USC | Same as Example 4.3 |
| 4.5 | SI | Typical Interior Bay of an Office Building — Open Web Joist Framing, $f_n > 9$–10 Hz. |
| 4.6 | USC | Same as Example 4.5 |
| 4.7 | SI | Mezzanine with Beam Edge Member |
| 4.8 | USC | Same as Example 4.7 |
| 4.9 | SI | Mezzanine with Girder Edge Member |
| 4.10 | USC | Same as Example 4.9 |
| Note: USC means US Customary | | |

Because the footbridge is not supported by girders, only the joist or beam panel mode needs to be investigated.

*Beam Mode Properties*

Since $0.4 L_j = 0.4 \times 12$ m = 4.8 m is greater than 1.5 m, the full width of the slab is effective. Using a dynamic modulus of elasticity of $1.35 E_c$, the transformed moment of inertia is calculated as follows:

$n$ = modular ratio = $E_s / 1.35 E_c$

$\quad = 200{,}000/(1.35 \times 24{,}000) = 6.17$

$$\bar{y} = \frac{2 \times 8{,}370(525/2) - (3{,}000/6.17)(150)(150/2)}{2 \times 8{,}370 + (3{,}000/6.17)(150)} = -12 \text{ mm}$$

$\quad = 12$ mm above the top flange of the beam

$I_j = 2[350 \times 10^6 + 8{,}370(525/2 + 12)^2]$

$\quad + (3{,}000/6.17)(150)^3 / 12$

$\quad + (3{,}000/6.17)(150)(150/2 - 12)^2$

$\quad = 2{,}388 \times 10^6$ mm

The weight per linear meter per beam is:

$w_j = 3.6 \times 3 + 2 \times 66 \times 0.00981 = 12.1$ kN/m

and the corresponding deflection is

$$\Delta_j = \frac{5 w_j L_j^4}{384 E_s I_j} = \frac{5 \times 12.1 \times 12{,}000^4}{384 \times 200{,}000 \times 2{,}388 \times 10^6} = 6.84 \text{ mm}$$

*Beam Properties*

W530×66
$A = 8{,}370 \text{ mm}^2$
$I_x = 350 \times 10^6 \text{ mm}^4$
$d = 525$ mm

*Cross Section*



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The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{9,806}{6.84}} = 6.81 \text{ Hz}$$

The effective beam panel width, $B_j$, is 3 m, since the entire footbridge will vibrate as a simple beam. The weight of the beam panel is then

$$W_j = w_j L_j = 12.1 \text{ x } 12 = 145 \text{ kN}$$

*Evaluation*

From Table 4.1, $\beta = 0.01$ for outdoor footbridges, and

$$\beta W_j = 0.01 \text{ x } 145 = 1.45 \text{ kN}$$

From Equation (4.1), with $f_n = f_j = 6.81$ Hz and $P_0 = 0.41$ kN

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{0.41 \exp(-0.35 \text{x} 6.41)}{1.45}$$

$$= 0.030 \text{ equivalent to 3 percent gravity}$$

which is less than the acceleration limit $a_o / g$ of 5 percent for outdoor footbridges (Table 4.1). The footbridge is therefore satisfactory. Also, plotting $f_n = 6.81$ Hz and $a_p = 3.0$ percent $g$ on Figure 2.1 shows that the footbridge is satisfactory. Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 1 kN per mm does not apply.

If the same footbridge were located indoors, for instance in a shopping mall, it would not be satisfactory since the acceleration limit for this situation is 1.5 percent $g$.

## Example 4.2—USC Units

An outdoor footbridge of span 40 ft. with pinned supports and the cross-section shown is to be evaluated for walking vibration.

*Deck Properties*

Concrete: $w_c = 145$ pcf
$f_c' = 4,000$ psi
Slab + deck weight = 75 psf

*Beam Properties*

W21x44
$A = 13.0$ in.$^2$
$I_x = 843$ in.$^4$
$d = 20.66$ in.
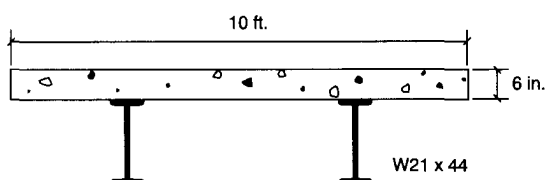
*Cross Section*



Because the footbridge is not supported by girders, only the joist or beam panel mode needs to be investigated.

*Beam Mode Properties*

Since $0.4 L_j = 0.4$ x 40 x 12 = 192 in. is greater than 5 ft. = 60 in., the full width of the slab is effective. Using a dynamic modulus of elasticity of $1.35 E_c$, the transformed moment of inertia is calculated as follows:

$$E_c = w^{1.5}\sqrt{f_c'} = 145^{1.5}\sqrt{4.0} = 3,500 \text{ ksi}$$

$$n = \text{modular ratio} = E_s / 1.35 E_c$$

$$= 29,000/(1.35 \times 3,500) = 6.15$$

$$\bar{y} = \frac{2 \times 13.0(20.66/2) - (120/6.15)(6.0)(6.0/2)}{2 \times 13.0 + (120/6.15)(6.0)} = -0.58 \text{ in.}$$

$$= 0.58 \text{ in. above the top flange of the beam}$$

$$I_j = 2[843 + 13.0(20.66/2 + 0.58)^2]$$

$$+ (120/6.15)(6.0)^3/12$$

$$+ (120/6.15)(6.0)(6.0/2 - 0.58)^2$$

$$= 5,818 \text{ in.}^4$$

The weight per linear ft per beam is:

$$w_j = 75 \times 10 + 2 \times 44 = 838 \text{ plf}$$

and the corresponding deflection is

$$\Delta_j = \frac{5 w_j L_j^4}{384 E_s I_j} = \frac{5 \times 838 \times 40^4 \times 1,728}{384 \times 29 \times 10^6 \times 5,818} = 0.286 \text{ in.}$$

The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{386}{0.286}} = 6.61 \text{ Hz}$$

The effective beam panel width, $B_j$, is 10 ft., since the entire footbridge will vibrate as a simple beam. The weight of the beam panel is then

$$W_j = w_j L_j = 838 \times 40 = 33,520 \text{ lbs} = 33.5 \text{ kips}$$

*Evaluation*

From Table 4.1, $\beta = 0.01$ for outdoor footbridges, and

$$\beta W_j = 0.01 \text{ x } 33.5 = 0.335 \text{ kips}$$

From Equation (4.1), with $f_n = f_j = 6.61$ Hz and $P_o = 92$ lbs

$$\frac{a_p}{g} = \frac{P_0 \exp(-0.35 f_n)}{\beta W} = \frac{92 \exp(-0.35 \times 6.61)}{335}$$

$$= 0.027 \text{ equivalent to 2.7 percent gravity}$$

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which is less than the acceleration limit $a_o/g$ of 5 percent for outdoor footbridges (Table 4.1). The footbridge is therefore satisfactory. Also, plotting $f_n = 6.61$ Hz and $a_p = 2.7$ percent $g$ on Figure 2.1 shows that the footbridge is satisfactory. Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 5.7 kips per in. does not apply.

If the same footbridge were located indoors, for instance in a shopping mall, it would not be satisfactory since the acceleration limit for this situation is 1.5 percent $g$.

### 4.4.2 Typical Interior Bay of an Office Building Examples

## Example 4.3—SI Units

Determine if the hot-rolled framing system for the typical interior bay shown in Figure 4.3 satisfies the criterion for walking vibration. The structural system supports office floors without full height partitions. Use 0.5 kPa for live load and 0.2 kPa for the weight of mechanical equipment and ceiling.

*Deck Properties:*

Concrete: $m_c = 1{,}850$ kg/m³
$\quad\quad\quad\quad f_c' = 30$ MPa
$\quad\quad\quad\quad E_c = 16{,}000$ MPa
Floor thickness = 80 mm + 50 mm ribs
$\quad\quad\quad\quad\quad\quad = 130$ mm
Slab + deck weight = 2 kPa

*Beam Properties*

W460×52
$A = 6{,}640$ mm²
$I_x = 212 \times 10^6$ mm⁴
$d = 450$ mm

*Girder Properties*

W530×74
$A = 9{,}490$ mm²
$I_x = 410 \times 10^6$ mm⁴
$d = 529$ mm

*Beam Mode Properties*

With an effective concrete slab width of 3 m $< 0.4L_j = 0.4$ x 10.5 = 4.2 m, considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of $1.35E_c$, the transformed moment of inertia is:

$$n = E_s / 1.35E_c = 200{,}000 / (1.35 \times 16{,}000) = 9.26$$

$$\bar{y} = \frac{6{,}640(50 + 450/2) - (3{,}000/9.26)(80)(80/2)}{6{,}640 + (3{,}000/9.26)(80)}$$

$$= 24.2 \text{ mm below top of form deck}$$

$$I_j = 212 \times 10^6 + 6{,}640\,(50 + 450/2 - 24.2)^2$$

$$+ (3{,}000/9.26)(80)^3/12$$

$$+ (3{,}000/9.26)(80)(24.2 + 80/2)^2$$

$$= 750 \times 10^6 \text{ mm}^4$$

For each beam, the uniform distributed loading is

$$w_j = 3(0.5 + 2 + 0.2 + 52 \text{ x } 0.00981/3) = 8.61 \text{ kN/m}$$

which includes 0.5 kPa live load and 0.2 kPa for mechanical/ceiling. The corresponding deflection is

$$\Delta_j = \frac{5W_jL_j^4}{384E_sI_j} = \frac{5 \times 8.61 \times (10{,}500)^4}{384 \times 200{,}000 \times 750 \times 10^6} = 9.08 \text{ mm}$$

The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{9{,}806}{9.08}} = 5.91 \text{ Hz}$$

Using an average concrete thickness of 105 mm, the transformed moment of inertia per unit width in the slab direction is

$$D_s = d_e^3/(12n) = 105^3/(12 \times 9.26) = 10{,}420 \text{ mm}^3$$

The transformed moment of inertia per unit width in the beam direction is (beam spacing is 3 m)

$$D_j = I_j / S = 750 \times 10^6 / 3{,}000 = 250{,}000 \text{ mm}^3$$

The effective beam panel width from Equation (4.3a) with $C_j = 2.0$ is
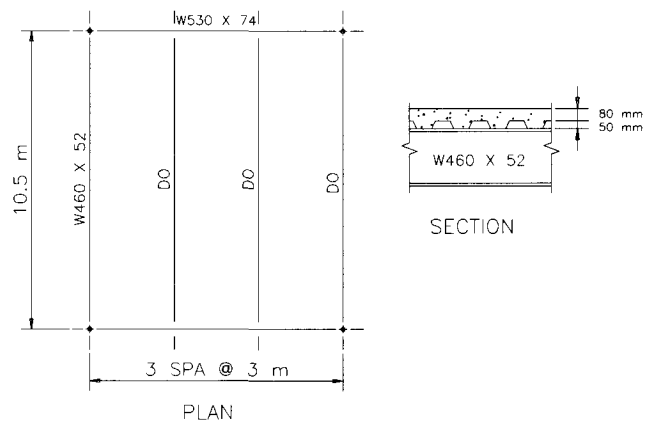
$$B_j = C_j(D_s / D_j)^{1/4}L_j$$

Fig. 4.3 Interior bay floor framing details for Example 4.3.

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$$= 2.0(10,420/250,000)^{1/4}(10.5) = 9.49 \text{ m}$$

which must be less than ⅔ times the floor width. Since this is a typical interior bay, the actual floor width is at least three times the girder span, 3 x 9 = 27 m. And, since ⅔ x 27 = 18 m > 9.49 m, the effective beam panel width is 9.49 m.

The weight of the beam panel is calculated from Equation (4.2), adjusted by a factor of 1.5 to account for continuity:

$$W_j = 1.5(w_j / S)B_j L_j = 1.5(8.61/3)(9.49 \times 10.5) = 429 \text{ kN}$$

*Girder Mode Properties*

With an effective slab width of

$$0.4L_g = 0.4 \times 9 = 3.6 \text{ m} < L_j = 10.5 \text{ m}$$

and considering the concrete in the deck ribs, the transformed moment of inertia is found as follows:

Avg. concrete depth = 80 + 50/2 = 105 mm

$$\bar{y} = \frac{9,490(25 + 529 / 2) - (3,600 / 9.26)(105)(105 / 2)}{9,490 + (3,600 / 9.26)(105)}$$

= 12 mm below effective slab

$$I_g = 410 \times 10^6 + 9,490 \,(25 + 529/2 - 12)^2$$

$$+ \,(3,600/9.26)(105)^3/12$$

$$+ \,(3,600/9.26)(105)(12+105/2)^2$$

$$= 1,348 \times 10^6 \text{ mm}^4$$

For each girder, the equivalent uniform loading is

$$w_g = L_j(w_j / S) + \text{girder weight per unit length}$$

$$= 10.5(8.61/3) + 74 \times 0.00981 = 30.9 \text{ kN/m}$$

and the corresponding deflection is

$$\Delta_g = \frac{5w_g L_g^4}{384E_s I_g} = \frac{5 \times 30.9 \times (9,000)^4}{384 \times 200,000 \times 1,348 \times 10^6} = 9.79 \text{ mm}$$

and

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} = 0.18\sqrt{\frac{9,806}{9.79}} = 5.69 \text{ Hz}$$

With $D_j = 250,000 \text{ mm}^3$ and $D_g = I_g / L_j = 1,348 \times 10^6/10,500 = 128,380 \text{ mm}$, the effective girder panel width using Equation (4.3b) with $C_g = 1.8$ is

$$B_g = C_g(D_j / D_g)^{1/4}L_g$$

$$= 1.8(250,000/128,380)^{1/4}(9) = 19.1 \text{ m},$$

which must be less than ⅔ times the floor length. Since this is a typical interior bay, the actual floor length is at least three times the beam span, 3 x 10.5 = 31.5 m. And, since ⅔ x 31.5

= 21 m > 19.1 m, the girder panel width is 19.1 m. From Equation (4.2), the girder panel weight is

$$W_g = (w_g / L_j)B_g L_g = (30.9/10.5)(19.1 \times 9) = 506 \text{ kN}$$

The girder panel weight was not increased by 50 percent as was done in the joist panel weight calculation since continuity effects generally are not realized when girders frame directly into columns.

*Combined Mode Properties*

Since the girder span (9 m) is less than the joist panel width (9.49 m), the girder deflection, $\Delta_g$, is reduced according to Equation (4.5):

$$\Delta_g' = \frac{L_g}{B_j}\Delta_g = \frac{9}{9.49} \times 9.79 = 9.28 \text{ mm}$$

From Equation (3.4), the floor fundamental frequency is

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g')} = 0.18\sqrt{9,806 / (9.08 + 9.28)}$$

$$= 4.15 \text{ Hz}$$

and from Equation (4.4), the equivalent combined mode panel weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g'}W_j + \frac{\Delta_g'}{\Delta_j + \Delta_g'}W_g$$

$$= \frac{9.08}{9.08 + 9.28}(429) + \frac{9.28}{9.08 + 9.28}(506) = 468 \text{ kN}$$

For office occupancy without full height partitions, ß = 0.03 from Table 4.1, thus

$$\beta W = 0.03 \times 468 = 14.0 \text{ kN}$$

*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 0.29$ kN,

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35f_n)}{\beta W} = \frac{0.29\exp(-0.35 \times 4.15)}{14.0}$$

$$= 0.0048 \text{ equivalent to } 0.48 \text{ percent } g,$$

which is less than the acceleration limit $a_o / g$ of 0.5 percent. The floor is therefore judged satisfactory. Also, plotting $f_n = 4.15$ Hz and $a_p = 0.48$ percent $g$ on Figure 2.1 shows that the floor is satisfactory. Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 1 kN per mm does not apply.

**Example 4.4—USC Units**

Determine if the hot-rolled framing system for the typical interior bay shown in Figure 4.4 satisfies the criterion for walking vibration. The structural system supports the office

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floors without full height partitions. Use 11 psf live load and 4 psf for the weight of mechanical equipment and ceiling.

*Deck Properties*

Concrete: $w_c = 110$ pcf.
$\quad f_c' = 4000$ psi
Floor thickness $= 3.25$ in. $+ 2$ in. ribs
$\quad\quad\quad = 5.25$ in.
Slab + deck weight $= 42$ psf

*Beam Properties*

W18×35
$A = 10.30$ in.$^2$
$I_x = 510$ in.$^4$
$d = 17.70$ in.

*Girder Properties*

W21×50
$A = 14.70$ in.$^2$
$I_x = 984$ in.$^4$
$d = 20.83$ in.

*Beam Mode Properties*

With an effective concrete slab width of 120 in. $= 10$ ft $<$ $0.4L_j = 0.4 \times 35 = 14$ ft, considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of $1.35E_c$, the transformed moment of inertia is:

$$E_c = w^{1.5}\sqrt{f_c'} = 110^{1.5}\sqrt{4.0} = 2,300 \text{ ksi}$$

$$n = \text{modular ratio} = E_s / 1.35E_c = 29,000 / (1.35 \times 2,300)$$

$$= 9.31$$
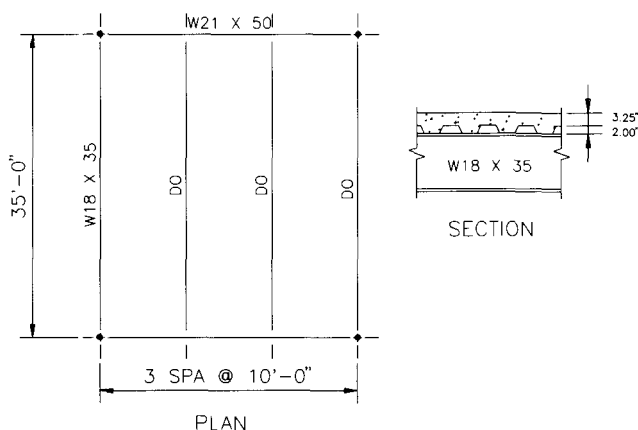


*Fig. 4.4 Interior bay floor framing details for Example 4.4.*

$$\bar{y} = \frac{10.30(2 + 17.70 / 2) - (120 / 9.31)(3.25)(3.25 / 2)}{10.30 + (120 / 9.31)(3.25)}$$

$$= 0.837 \text{ in. below top of form deck}$$

$$I_j = 510 + 10.30(2 + 17.70 / 2 - 0.837)^2$$

$$+ (120 / 9.31)(3.25)^3 / 12$$

$$+ (120/9.31)(3.25)(0.837 + 3.25/2)^2$$

$$= 1833 \text{ in.}^4$$

For each beam, the uniform distributed loading is

$$w_j = 10(11 + 42 + 4 + 35/10) = 605 \text{ plf}$$

which includes 11 psf live load and 4 psf for mechanical/ceiling, and the corresponding deflection is

$$\Delta_j = \frac{5w_jL_j^4}{384E_sI_j} = \frac{5 \times 605 \times 35^4 \times 1,728}{384 \times 29 \times 10^6 \times 1,833} = 0.384 \text{ in.}$$

The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{386}{0.384}} = 5.71 \text{ Hz}$$

Using an average concrete thickness of 4.25 in., the transformed moment of inertia per unit width in the slab direction is

$$D_s = 12d_e^3 / (12n) = 12 \times 4.25^3 / (12 \times 9.31) = 8.25 \text{ in.}^4/\text{ft}$$

The transformed moment of inertia per unit width in the beam direction is (beam spacing is 10 ft)

$$D_j = I_j / S = 1,833/10 = 183 \text{ in.}^4/\text{ft}$$

The effective beam panel width from Equation (4.3a) with $C_j = 2.0$ is

$$B_j = C_j(D_s / D_j)^{1/4}L_j = 2.0(8.25/183)^{1/4}(35) = 32.2 \text{ ft}$$

which must be less than ⅔ times the floor width. Since this is a typical interior bay, the actual floor width is at least three times the girder span, $3 \times 30 = 90$ ft. And, since ⅔ × 90 = 60 ft $> 32.2$ ft, the effective beam panel width is 32.2 ft.

The weight of the beam panel is calculated from Equation (4.2), adjusted by a factor of 1.5 to account for continuity:

$$W_j = 1.5(w_j / S)B_jL_j = 1.5(605/10)(32.2 \times 35)$$

$$= 102,275 \text{ lbs} = 102 \text{ kips}$$

*Girder Mode Properties*

With an effective slab width of

$$0.4L_g = 0.4 \times 30 \times 12 = 144 \text{ in.} < L_j = 35 \times 12 = 420 \text{ in.}$$

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and considering the concrete in the deck ribs, the transformed moment of inertia is found as follows:

Avg. concrete depth = 3.25 + 2.0/2 = 4.25 in.

$$\bar{y} = \frac{14.70(1 + 20.83/2) - (144/9.31)(4.25)(4.25/2)}{14.70 + (144/9.31)(4.25)}$$

= 0.349 in. below effective slab

$$I_g = 984 + 14.70(1 + 20.83/2 - 0.349)^2$$

$$+ (144/9.31)(4.25)^3/12$$

$$+ (144/9.31)(4.25)(0.349 + 4.25/2)^2$$

= 3,285 in.$^4$

For each girder, the equivalent uniform loading is

$$w_g = L_j(w_j/S) + \text{girder weight per unit length}$$

$$= 35(605/10) + 50 = 2,168 \text{ plf.}$$

and the corresponding deflection is

$$\Delta_g = \frac{5w_g L_g^4}{384 E_s I_g} = \frac{5 \times 2,168 \times 30^4 \times 1,728}{384 \times 29 \times 10^6 \times 3,285} = 0.415 \text{ in.}$$

From Equation (3.3), the girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} = 0.18\sqrt{\frac{386}{0.415}} = 5.49 \text{ Hz}$$

With $D_j = 183$ in.$^4$/ft and $D_g = I_g/L_j = 3,285/35 = 93.9$ in.$^4$/ft, the effective girder panel width using Equation (4.3b) with $C_g = 1.8$ is

$$B_g = C_g(D_j/D_g)^{1/4}L_g = 1.8(183/93.9)^{1/4}(30) = 63.8 \text{ ft,}$$

But, the girder panel width must be less than ⅔ times the floor length. Since this is a typical interior bay, the actual floor length is at least three times the joist span, 3 x 35 = 105 ft. And, since ⅔ x 105 = 70 ft > 63.8 ft, the girder panel width is 63.8 ft. From Equation (4.2), the girder panel weight is

$$W_g = (w_g/L_j)B_g L_g = (2,168/35)(63.8 \times 30)$$

$$= 118,556 \text{ lb} = 119 \text{ kips}$$

The girder panel weight was not increased by 50 percent as was done in the joist panel weight calculation since continuity effects generally are not realized when girders frame directly into columns.

*Combined Mode properties:*

In this case the girder span (30 ft) is less than the joist panel width (32.2 ft) and the girder deflection, $\Delta_g$, is therefore reduced according to Equation (4.5):

$$\Delta_g' = \frac{L_g}{B_j}\Delta_g = \frac{30}{32.2} \times 0.415 = 0.387 \text{ in.}$$

From Equation (3.4), the floor fundamental frequency is

$$f_n = 0.18\sqrt{g/(\Delta_j + \Delta_g')} = 0.18\sqrt{386/(0.384 + 0.387)}$$

= 4.03 Hz

and from Equation (4.4), the equivalent panel mode panel weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g'}W_j + \frac{\Delta_g'}{\Delta_j + \Delta_g'}W_g$$

$$= \frac{0.384}{0.384 + 0.387}(102) + \frac{0.387}{0.384 + 0.387}(119) = 111 \text{ kips}$$

For office occupancy without full height partitions, ß = 0.03 from Table 4.1, thus

$$\beta W = 0.03 \times 111 = 3.33 \text{ kips.}$$

*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 65$ lbs,

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{65 \exp(-0.35 \times 4.03)}{3,330}$$

= 0.0048 equivalent to 0.48 percent $g$

which is less than the acceleration limit $a_o/g$ of 0.5 percent. The floor is therefore judged marginally satisfactory. Also, plotting $f_n = 4.03$ Hz and $a_p = 0.48$ percent $g$ on Figure 2.1 shows that the floor is marginally satisfactory. Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 5.7 kips per in. does not apply.

**Example 4.5—SI Units**

The framing system shown in Figure 4.5 was designed for a heavy floor loading. The system is to be evaluated for normal office occupancy. The office space will not have full height partitions. Use 0.5 kPa for live load and 0.2 kPa for the weight of mechanical equipment and ceiling.

*Deck Properties*

Concrete: $m_c = 1,850$ kg/m$^3$
$f_c' = 30$ MPa
$E_c = 16,000$ MPa
Floor thickness = 40 mm + 25 mm ribs
= 65 mm
Slab + deck weight = 1 kPa

*Joist Properties*

30$K$8   $wt = 0.19$ kN/m
$A = 1,055$ mm$^2$

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$I_{chords} = 141 \times 10^6 \text{ mm}^4$

$D = 762 \text{ mm}$

$y_c = 340 \text{ mm}$

*Girder Properties*

W760×134
$A = 17,000 \text{ mm}^2$
$I_x = 1,500 \times 10^6 \text{ mm}^4$
$d = 750 \text{ mm}$

*Beam Mode Properties*

With an effective concrete slab width of 750 mm < 0.4 $L_j$ = 0.4 x 8,500 = 3,400 mm, considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of 1.35 $E_c$, the transformed moment of inertia is calculated using the procedure of Section 3.6:

$n$ = modular ratio = $E_s / 1.35 E_c = 200,000/(1.35 \times 16,000)$
= 9.26

The transformed moment of inertia using the actual chord areas is

$$\bar{y} = \frac{1,055(25 + 340) - (750/9.26)(40)(40/2)}{1,055 + (750/9.26)(40)}$$

= 74.6 mm below top of form deck

$I_{comp} = 141 \times 10^6 + 1,055(25 + 340 - 74.6)^2$

$+ (750/9.26)(40)^3/12 + (750/9.26)(40)(74.6 + 40/2)^2$

$= 259 \times 10^6 \text{ mm}^4$

Since $6 \leq L_j/d = 8,500/762 = 11.2 \leq 24$, Equation (3.16) is applicable:

$C_t = 0.90(1 - e^{-0.28(L_j/D)})^{2.8} = 0.90(1 - e^{-0.28(11.2)})^{2.8}$

= 0.79

Using Equation (3.19) and then (3.18)

$$\gamma = \frac{1}{C_t} - 1 = \frac{1}{0.79} - 1 = 0.266$$

and

$$I_j = I_{eff} = \frac{1}{\dfrac{\gamma}{I_{chords}} + \dfrac{1}{I_{comp}}} = \frac{1}{\dfrac{0.266}{141 \times 10^6} + \dfrac{1}{259 \times 10^6}}$$

$= 174 \times 10^6 \text{ mm}^4$

For each joist, the uniform distributed loading is

$w_j = 0.750(0.5 + 1.0 + 0.2) + 0.19 = 1.47 \text{ kN/m}$

which includes 0.5 kPa live load and 0.2 kPa for mechanical/ceiling, and the corresponding deflection is

$$\Delta_j = \frac{5w_j L_j^4}{384 E_s I_j} = \frac{5 \times 1.47 \times (8,500)^4}{384 \times 200,000 \times 174 \times 10^6} = 2.87 \text{ mm}$$

The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{9,806}{2.87}} \approx 10.5 \text{ Hz}$$
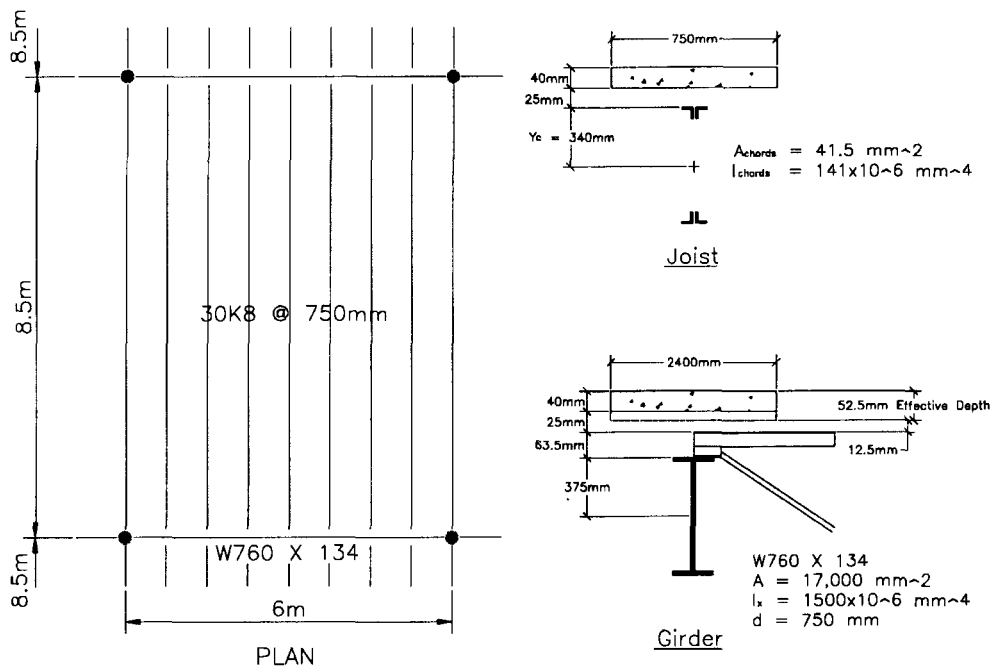


Fig. 4.5 Interior bay floor framing details for Example 4.5.

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Using an average concrete thickness, 52.5 mm, the transformed moment of inertia per unit width in the slab direction is

$$D_s = d_e^3 / (12n) = 52.5^3/(12 \times 9.26) = 1,302 \text{ mm}^3$$

The transformed moment of inertia per unit width in the joist direction is (joist spacing is 750 mm)

$$D_j = I_j / S = 174 \times 10^6/750 = 232,000 \text{ mm}^3$$

The effective beam panel width from Equation (4.3a) with $C_j = 2.0$ is

$$B_j = C_j(D_s / D_j)^{1/4}L_j = 2.0(1,302/232,000)^{1/4}(8.5) = 4.65 \text{ m}$$

which must be less than ⅔ times the floor width. Since this is a typical interior bay, the actual floor width is at least three times the girder span, 3 x 6 = 18 m. And, since ⅔ x 18 = 12 m > 4.65 m, the effective beam panel width is 4.65 m.

The weight of the beam panel is calculated from Equation (4.2), without adjustment for continuity:

$$W_j = (w_j / S)B_jL_j = (1.47/0.75)(4.65 \times 8.5) = 77.5 \text{ kN}$$

*Girder Mode Properties*

With an effective slab width of

$$0.4L_g = 0.4 \times 6 = 2.4 \text{ m} < L_j = 8.5 \text{ m}$$

and considering the concrete in the deck ribs, the transformed moment of inertia is found as follows:

Avg. concrete depth = 40 + 25/2 = 52.5 mm

$$\bar{y} = \frac{17,000(12.5 + 63.5 + 750/2) - (2,400/9.26)(52.5)(52.5/2)}{17,000 + (2,400/9.26)(52.5)}$$

= 239 mm below the effective slab

$$I_g = 1,500 \times 10^6 + 17,000(12.5 + 63.5 + 750/2 - 239)^2$$

$$+ (2,400/9.26)(52.5)^3/12$$

$$+ (2,400/9.26)(52.5)(239 + 52.5/2)^2$$

$$= 3,225 \times 10^6 \text{ mm}^4$$

To account for the reduced girder stiffness due to flexibility of the joist seats, $I_g$ is reduced according to Equation (3.14):

$$I_g = I_{nc} + (I_c - I_{nc})/4$$

$$= [1,500 + (3,225 - 1,500)/4] \times 10^6 = 1,930 \times 10^6 \text{ mm}$$

For each girder, the equivalent uniform loading is

$$w_g = L_j(w_j / S) + \text{girder weight per unit length}$$

$$= 8.5(1.47/0.750) + 134 \times 0.00981 = 18.0 \text{ kN/m}$$

and the corresponding deflection is

$$\Delta_g = \frac{5w_gL_g^4}{384E_sI_g} = \frac{5 \times 18.0 \times (6,000)^4}{384 \times 200,000 \times 1,930 \times 10^6} = 0.787 \text{ mm}$$

From Equation (3.3), the grider mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} = 0.18\sqrt{\frac{9,806}{0.787}} = 20.1 \text{ Hz}$$

With $D_j = 232,000 \text{ mm}^3$ and $D_g = I_g / L_j = 1,930 \times 10^6/8,500 = 227,000 \text{ mm}^3$, the effective girder panel width using Equation (4.3b) with $C_g = 1.6$ is

$$B_g = C_g(D_j / D_g)^{1/4}L_g = 1.6(232,000/227,000)^{1/4}(6)$$

$$= 9.65 \text{ m,}$$

which must be less than ⅔ times the floor length. Since this is a typical interior bay, the actual floor length is at least three times the joist span, 3 x 8.5 = 25.5m. And, since ⅔ x 25.5 = 17 m > 9.65 m, the girder panel width is taken as 9.65 m. From Equation (4.2), the girder panel weight is

$$W_g = (w_g / L_j)B_gL_g = (18.0/8.5)(9.65 \times 6.0) = 123 \text{ kN}$$

*Combined Mode properties:*

In this case the girder span (6 m) is greater than the effective joist panel width ($B_j = 4.65$ m) and the girder deflection, $\Delta g$, is not reduced. From Equation (3.4),

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g)} = 0.18\sqrt{9,806 / (2.87 + 0.787)}$$

$$= 9.32 \text{ Hz}$$

and from Equation (4.4), the equivalent panel mode weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g}W_j + \frac{\Delta_g}{\Delta_j + \Delta_g}W_g$$

$$= \frac{2.87}{2.87 + 0.787}(77.5) + \frac{0.787}{2.87 + 0.787}(123) = 87.3 \text{ kN}$$

For office occupancy without full height partitions, $\beta = 0.03$ from Table 4.1, thus

$$\beta W = 0.03 \times 87.3 = 2.62 \text{ kN}$$

*Walking Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 0.29$ kN,

$$\frac{a_p}{g} = \frac{P_o\exp(-.035f_n)}{\beta W} = \frac{0.29\exp(-0.35 \times 9.32)}{2.62}$$

$$= 0.0042 \text{ equivalent to } 0.42 \text{ percent } g$$

which is less than the acceleration limit $a_o / g$ of 0.5 percent $g$ from Table 4.1 or Figure 2.1.

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*Floor Stiffness Evaluation*

Since the fundamental frequency of the system is greater than 9 Hz, the minimum stiffness requirement of 1 kN per mm applies. (See *Floor Stiffness* in Section 4.2.) The static deflection of a single tee-beam due to a 1 kN concentrated load at midspan is

$$\Delta_{oj} = \frac{PL_j^3}{48E_sI_j} = \frac{1,000(8,500)^3}{48 \times 200,000 \times 174 \times 10^6} = 0.368 \text{ mm}$$

Since all the limitations for Equation (4.7) are satisfied as follows:

$$0.018 \le \frac{d_e}{S} = \frac{52.5}{750} = 0.070 \le 0.208$$

and

$$4.5 \times 10^6 \le \frac{L_j^4}{I_t} = \frac{(8,500)^4}{174 \times 10^6} = 30.0 \times 10^6 \le 257 \times 10^6$$

and

$$2 \le \frac{L_j}{S} = \frac{8,500}{750} = 11.3 \le 30$$

Then from Equation (4.7)

$$N_{eff} = 0.49 + 34.2\frac{d_e}{S} + 9.0 \times 10^{-9}\frac{L_j^4}{I_t} - 0.0059\left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2(0.070) + (9.0 \times 10^{-9})(30.0 \times 10^6)$$

$$- 0.00059(11.3)^2$$

$$= 3.08 \text{ joists}$$

The joist panel deflection is then

$$\Delta_j = \frac{\Delta_{oj}}{N_{eff}} = \frac{0.368}{3.08} = 0.119 \text{ mm}$$

With

$$\Delta_g = \frac{PL_g^3}{48E_sI_g} = \frac{1,000(6,000)^3}{48 \times 200,000 \times 1,930 \times 10^6} = 0.0117 \text{ mm}$$

the total deflection is

$$\Delta = \Delta_j + \Delta_g / 2 = 0.119 + 0.0117/2 = 0.125 \text{ mm}$$

The floor stiffness is then

$$k_{floor} = \frac{1.0}{0.125} = 8.0 \text{ kN/m} > 1 \text{ kN/m}$$

*Final Evaluation*

Since the floor system satisfies both the walking excitation and stiffness criteria, it is judged satisfactory for offices occupancy without full height partitions.

## Example 4.6—USC Units

The framing system shown in Figure 4.6 was designed for a heavy floor loading. The system is to be evaluated for normal office occupancy. The office space will not have full height partitions. Use 11 psf for live load and 4 psf for the weight of mechanical equipment and ceiling.
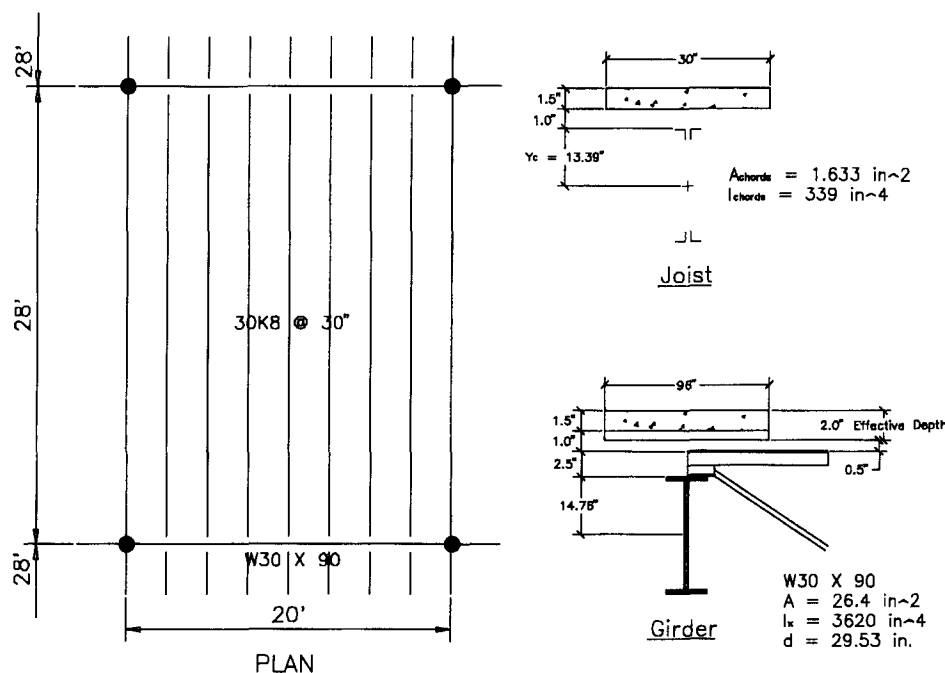


Fig. 4.6 Interior bay floor framing details for Example 4.6.

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*Deck Properties*

Concrete: $w_c$ = 110 pcf.
$f'_c$ = 3,000 psi
Floor thickness = 1.5 in. + 1 in. ribs
= 2.5 in.
Slab + deck weight = 19.3 psf

*Joist Properties*

30*K*8   *wt* = 13.2 plf
$A$ = 1.633 in.$^2$
$I_{chords}$ = 339 in.$^4$
$D$ = 30.0 in.
$y_c$ = 13.39 in.

*Girder Properties*

W30×90
$A$ = 26.40 in.$^2$
$I_x$ = 3,620 in.$^4$
$d$ = 29.53 in.

*Beam Mode Properties*

With an effective concrete slab width of 30 in. $< 0.4L_j$ = 0.4 x 28 x 12 = 134 in., considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of 1.35$E_c$, the transformed moment of inertia is calculated using the procedure of Section 3.6:

$$E_c = w^{1.5}\sqrt{f'_c} = 110^{1.5}\sqrt{3.0} = 2,000 \text{ ksi}$$

$$n = \text{modular ratio} = E_s / 1.35E_c = 29,000/(1.35 \times 2,000)$$

$$= 10.74$$

The transformed moment of inertia using the actual chord areas is

$$\bar{y} = \frac{1.633(1.0 + 13.39) - (30/10.74)(1.5)(1.5/2)}{1.633 + (30/10.74)(1.5)}$$

$$= 3.50 \text{ in. below top of form deck}$$

$$I_{comp} = 339 + 1.633(1.0 + 13.39 - 3.50)^2$$

$$+ (30/10.74)(1.5)^3/12 + (30/10.74)(1.5)(3.50 + 1.5/2)^2$$

$$= 609 \text{ in.}^4$$

Since $6 \le L_j / d = 28 \times 12/30 = 11.2 \le 24$, Equation (3.16) is applicable:

$$C_t = 0.90(1 - e^{-0.28L_j/D})^{2.8} = 0.90(1 - e^{-0.28(11.2)})^{2.8}$$

$$= 0.80$$

Using Equation (3.19) and then (3.18)

$$\gamma = \frac{1}{C_t} - 1 = \frac{1}{0.80} - 1 = 0.250$$

and

$$I_j = I_{eff} = \frac{1}{\dfrac{\gamma}{I_{chords}} + \dfrac{1}{I_{comp}}} = \frac{1}{\dfrac{0.250}{339} + \dfrac{1}{609}}$$

$$= 420 \text{ in.}^4$$

For each joist, the uniform distributed loading is

$$w_j = (30/12)(11 + 19.3 + 4) + 13.2 = 99 \text{ plf}$$

which includes 11 psf live load and 4 psf for mechanical/ceiling, and the corresponding deflection is

$$\Delta_j = \frac{5w_jL_j^4}{384E_sI_j} = \frac{5 \times 99 \times 28^4 \times 1,728}{384 \times 29 \times 10^6 \times 420} = 0.112 \text{ in.}$$

The beam mode fundamental frequency from Equation (3.3) is:

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{386}{0.112}} = 10.6 \text{ Hz}$$

Using an average concrete thickness, 2.0 in., the transformed moment of inertia per unit width in the slab direction is

$$D_s = 12d_e^3 / 12n = 12(2.0)^3/(12 \times 10.74) = 0.745 \text{ in.}^4/\text{ft}$$

The transformed moment of inertia per unit width in the joist direction is (joist spacing is 30 in.)

$$D_j = I_j / S = 420/(30/12) = 168 \text{ in.}^4/\text{ft}$$

The effective beam panel width from Equation (4.3a) with $C_j$ = 2.0 is

$$B_j = C_j(D_s / D_j)^{1/4}L_j = 2.0(0.745/168)^{1/4}(28) = 14.4 \text{ ft}$$

Since this is a typical interior bay, the actual floor width is at least three times the girder span, 3 x 20 = 60 ft. And, since 2/3 x 60 = 40 ft > 14.4 ft, the effective beam panel width is 14.7 ft.

The weight of the beam panel is calculated from Equation (4.2) without adjustment for continuity:

$$W_j = (w_j / S)B_jL_j = (99/2.5)(14.4 \times 28) = 16,000 \text{ lbs}$$

$$= 16.3 \text{ kips}$$

*Girder Mode Properties*

With an effective slab width of

$$0.4L_g = 0.4 \times 20 \times 12 = 96 \text{ in.} < L_j = 28 \times 12 = 336 \text{ in.}$$

and considering the concrete in the deck ribs, the transformed moment of inertia is found as follows:

Avg. concrete depth = 1.5 + 1.0/2 = 2.0 in.

$$\bar{y} = \frac{26.40(0.5 + 2.5 + 29.53/2) - (96/10.74)(2.0)(2.0/2)}{26.40 + (96/10.74)(2.0)}$$

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= 10.19 in. below effective slab

$$I_g = 3,620 + 26.40(0.5 + 2.5 + 29.53/2 - 10.19)^2$$

$$+ (96/10.74)(2.0)^3/12 + (96/10.74)(2.0)(10.19 + 2.0/2)^2$$

$$= 7,379 \text{ in.}^4$$

To account for the reduced girder stiffness due to flexibility of the joist seats (shoes), $I_g$ is reduced according to Equation (3.14):

$$I_g = I_{nc} + (I_c - I_{nc})/4$$

$$= 3,620 + (7,379 - 3,620)/4 = 4,560 \text{ in}^4$$

For each girder, the equivalent uniform loading is

$$w_g = L_g(w_j / S) + \text{girder weight per unit length}$$

$$= 28(99/2.5) + 90 = 1,199 \text{ plf.}$$

And the corresponding deflection is

$$\Delta_g = \frac{5w_g L_g^4}{384EI_g} = \frac{5 \times 1,199 \times 20^4 \times 1,728}{384 \times 29 \times 10^6 \times 4,560} = 0.033 \text{ in.}$$

From Equation (3.3), the girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} = 0.18\sqrt{\frac{386}{0.033}} = 19.5 \text{ Hz}$$

With

$$D_j = 168 \text{ in.}^4/\text{ft}$$
$$D_g = I_g / L_j = 4,560/28 = 163 \text{ in.}^4/\text{ft}$$

the effective girder panel width using Equation (4.3b) with $C_g = 1.6$ is

$$B_g = C_g(D_j / D_g)^{1/4}L_g = 1.6(168/163)^{1/4}(20) = 32.2 \text{ ft,}$$

which must be less than ⅔ times the floor length. Since this is a typical interior bay, the actual floor length is at least three times the joist span, 3 x 28 = 84 ft. And, since ⅔ x 84 = 56 ft > 32.2 ft, the girder panel width is taken as 31.6 ft. From Equation (4.2), the girder panel weight is

$$W_g = (w_g / L_j)B_g L_g = (1,199/28)(32.2 \times 20)$$

$$= 27,577 \text{ lb} = 27.6 \text{ kips}$$

*Combined Mode Properties*

In this case the girder span (20 ft) is greater than the effective joist panel width ($B_j = 14.7$ ft) and the girder deflection, $\Delta_g$, is not reduced. From Equation (1.5),

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g)} = 0.18\sqrt{386 / (0.112 + 0.033)}$$

$$= 9.29 \text{ Hz}$$

and from Equation (3.4), the equivalent panel mode weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g}W_j + \frac{\Delta_g}{\Delta_j + \Delta_g}W_g$$

$$= \frac{0.112}{0.112 + 0.033}(16.3) + \frac{0.033}{0.112 + 0.033}(27.6) = 18.9 \text{ kips}$$

For office occupancy without full height partitions, $\beta = 0.03$ from Table 4.1, thus

$$\beta W = 0.03 \text{ x } 18.9 = 0.564 \text{ kips} = 567 \text{ lbs}$$

*Walking Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 65$ lbs,

$$\frac{a_p}{g} = \frac{P_o\exp(-0.35f_n)}{\beta W} = \frac{65\exp(-0.35 \times 9.29)}{567}$$

$$= 0.0044 \text{ equivalent to } 0.44 \text{ percent } g$$

which is less than the acceleration limit $a_o / g$ of 0.5 percent from Table 4.1 or Figure 2.1.

*Floor Stiffness Evaluation*

Since the fundamental frequency of the system is slightly greater than 9 Hz, the minimum stiffness requirement of 5.7 kips per in. applies. (See *Floor Stiffness* in Section 4.2.) The static deflection of a single tee-beam due to a 0.224 kips concentrated load at midspan is

$$\Delta_{oj} = \frac{PL_j^3}{48E_sI_j} = \frac{0.224(28)^3(1,728)}{48 \times 29,000 \times 420} = 0.0145 \text{ in.}$$

Since all the limitations for Equation (4.7) are satisfied as follows:

$$0.018 \leq \frac{d_e}{S} = \frac{2.0}{30.0} = 0.067 \leq 0.208$$

and

$$4.5 \times 10^6 \leq \frac{L_j^4}{I_t} = \frac{(28 \times 12)^4}{420} = 30.3 \times 10^6 \leq 257 \times 10^6$$

and

$$2 \leq \frac{L_j}{S} = \frac{28 \times 12}{30} = 11.2 \leq 30$$

then from Equation (4.7)

$$N_{eff} = 0.49 + 34.2\frac{d_e}{S} + 9.0 \times 10^{-9}\frac{L_j^4}{I_t} - 0.0059\left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2(0.067) + (9.0 \times 10^{-9})(30.3 \times 10^6)$$

$$- 0.00059(11.2)^2$$

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$= 2.98$ joists

The joist panel deflection is then

$$\Delta_j = \frac{\Delta_{oj}}{N_{eff}} \approx \frac{0.0145}{2.98} = 0.0049 \text{ in.}$$

With

$$\Delta_g = \frac{PL_g^3}{48 E_s I_g} = \frac{0.224(20)^3(1,728)}{48 \times 29,000 \times 4,560} = 0.000488 \text{ in.}$$

the total deflection is

$$\Delta = \Delta_j + \Delta_g / 2 = 0.0049 + 0.000488/2$$

$$= 0.0051 \text{ in.}$$

The floor stiffness is then

$$k_{floor} = \frac{0.224}{0.0051} = 43.9 \text{ kips/in} > 5.7 \text{ kip/in}$$

*Final Evaluation*

Since the floor system satisfies both the walking excitation and stiffness criteria, it is judged satisfactory for offices occupancy without full height partitions.

### 4.4.3 Mezzanine Examples

**Example 4.7—SI Units**

Evaluate the mezzanine framing shown in Figure 4.7 for walking vibrations. The floor system supports an office occupancy without full-depth partitions. Note that framing details are the same as those for Example 4.3, except that the floor system is only one bay wide normal to the edge of the mezzanine floor. Also note that the edge member is a beam. Use 0.5 kPa live load and 0.2 kPa for the weight of mechanical equipment and ceiling.

*Beam Mode Properties*

From Example 4.3

$w_j = 8.61$ kN/m
$\Delta_j = 9.08$ mm
$f_j = 5.91$ Hz.
$D_s = 10,420$ mm$^3$      $D_j = 250,000$ mm$^3$
$B_j = 9.49/2$ m $= 4.75$ m for an unstiffened edge panel

Since the actual floor width is 9 m and $\frac{2}{3} \times 9 = 6$ m $> B_j = 4.75$ m, the effective beam panel width is 4.75 m.

The effective weight of the beam panel is calculated from Equation (4.2), adjusted by a factor of 1.5 to account for continuity in the beam direction:

$$W_j = 1.5 w_j B_j L_j = 1.5(8.61/3)(4.75 \times 10.5) = 215 \text{ kN}$$

*Girder Mode Properties*

From Example 4.3:

$\Delta_g = 9.79$ mm          $f_g = 5.69$ Hz
$B_g = 19.1$ m               $W_g = 506$ kN

*Combined Mode properties*

The girder span (9 m) is greater than the beam panel width (4.75 m), thus the girder deflection, $\Delta_g$, is not reduced as was done in Example 4.3. The fundamental frequency is then

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g)} = 0.18\sqrt{9,806 / (9.08 + 9.79)}$$

$$= 4.10 \text{ Hz}$$

and from Equation (4.4),

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g$$

$$= \frac{9.08}{9.08 + 979}(215) + \frac{9.79}{9.08 + 9.79}(506) = 366 \text{ kN}$$

For office occupancy without full height partitions, $\beta = 0.03$ from Table 4.1, thus

$$\beta W = 0.03 \times 366 \approx 11.0 \text{ kN}$$

*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 0.29$ kN,

$$\frac{a_p}{g} \approx \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{0.29 \exp(-0.35 \times 4.10)}{11.0}$$

$$= 0.0063 \text{ equivalent to } 0.63 \text{ percent } g$$

which is more than the acceleration limit $a_o / g$ of 0.5 percent from Table 4.1. The mezzanine floor framing is judged to be
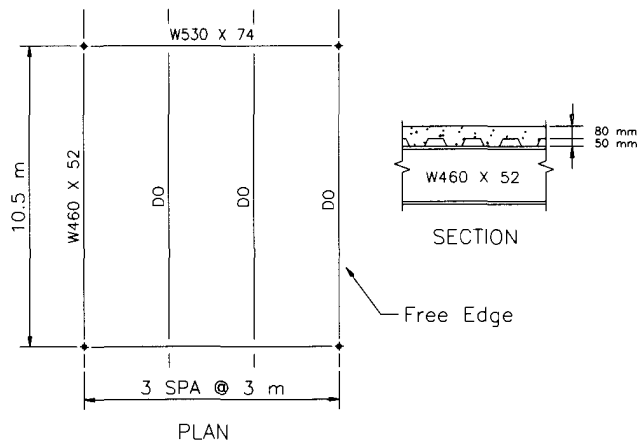


*Fig. 4.7 Mezzanine with edge beam member framing details for Example 4.7.*

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unsatisfactory for walking vibrations. Also, plotting $f_n = 4.10$ Hz and $a_p = 0.63$ percent $g$ on Figure 2.1 shows the floor to be unsatisfactory.

In this example, the edge member is a beam, and thus the beam panel width is one half of that for an interior bay. The result is that the combined panel does not have sufficient mass to satisfy the design criterion. If the mezzanine floor is only one bay wide normal to the edge beam, then both the beams and the girder need to be stiffened to satisfy the criterion. If the mezzanine floor is two or more bays wide normal to the edge beam, then, in accordance with Section 4.3, only the moment of inertia of the edge beam needs to be increased by 50 percent to satisfy the assumptions used for typical interior bays. For this example, a W460x74 ($I_x = 333 \times 10^6$ mm$^4 > 1.5 \times 212 \times 10^6 = 318 \times 10^6$ mm$^4$) is sufficient.

Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 1 kN per mm does not apply.

## Example 4.8—USC Units

Evaluate the mezzanine framing shown in Figure 4.8 for walking vibrations. The floor system supports an office occupancy without full-depth partitions. Note that framing details are the same as those for Example 4.4, except that the floor system is only one bay wide normal to the edge of the mezzanine floor. Also note that the edge member is a beam. Use 11 psf live load and 4 psf for the weight of mechanical equipment and ceiling.

*Beam Mode Properties*

From Example 4.4

$w_j = 605$ plf
$\Delta_j = 0.384$ in.
$f_j = 5.71$ Hz.
$D_s = 8.25$ in$^4$/ft.      $D_j = 183$ in$^4$/ft.
$B_j = 32.2$ ft./2 = 16.1 ft for an unstiffened edge panel

Since the actual floor width is 30 ft. and ⅔x 30 = 20 ft. > 16.1 ft., the effective beam panel width is 16.1 ft.

The effective weight of the beam panel is calculated from Equation (4.2), adjusted by a factor of 1.5 to account for continuity in the beam direction:

$W_j = 1.5 w_j B_j L_j = 1.5(605/10)(16.1 \times 35) = 51,138$ lbs

$= 51.1$ kips

*Girder Mode Properties*

From Example 4.4:

$\Delta_g = 0.415$ in
$f_g = 5.49$ Hz
$B_g = 63.8$ ft        $W_g = 119$ kips

*Combined Mode Properties*

In this case the girder span (30 ft) is greater than the joist panel width (16.1 ft), thus the girder deflection, $\Delta_g$, is not reduced as was done in Example 4.4. The fundamental frequency is then

$$f_n = 0.18\sqrt{g/(\Delta_j + \Delta_g)} = 0.18\sqrt{386/(0.384 + 0.415)}$$

$$= 3.96\,\text{Hz}$$

and from Equation (4.4),

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g}W_j + \frac{\Delta_g}{\Delta_j + \Delta_g}W_g$$

$$= \frac{0.384}{0.384 + 0.415}(51.1) + \frac{0.415}{0.384 + 0.415}(119) = 86.4\,\text{kips}$$

For office occupancy without full height partitions, $\beta = 0.03$ from Table 4.1, thus

$\beta W = 0.03$ x 86.4 = 2.59 kips = 2,590 lbs.

*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 65$ lbs,

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{65\exp(-0.35 \times 3.96)}{2,590}$$

$$= 0.0063 \text{ equivalent to } 0.63 \text{ percent } g$$

which is more than the acceleration limit $a_o / g$ of 0.5 percent for Table 4.1. The mezzanine floor framing is judged to be unsatisfactory for walking vibrations. Also, plotting $f_n =$
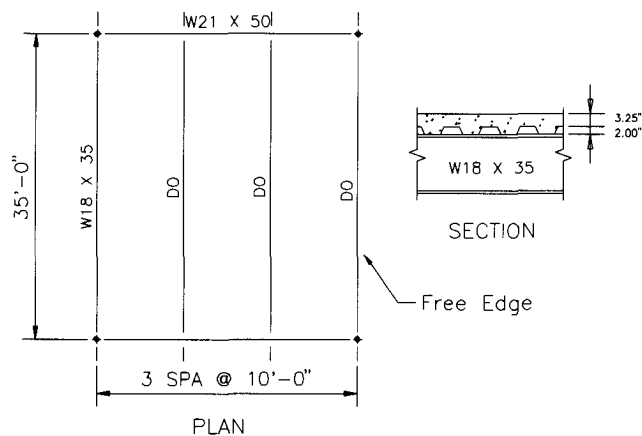


*Fig. 4.8 Mezzanine with edge beam member framing details for Example 4.8.*

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3.96 Hz and $a_p = 0.63$ percent $g$ on Figure 2.1 shows the floor to be unsatisfactory.

In this example, the edge member is a beam, and thus the beam panel width is one half of that for an interior bay. The result is that the combined panel does not have sufficient mass to satisfy the design criterion. If the mezzanine floor is only one bay wide normal to the edge beam, then both the beams and the girders need to be stiffened to satisfy the criterion. If the mezzanine floor is two or more bays wide normal to the edge beam, then, in accordance with Section 4.3, only the moment of inertia of the edge beam needs to be increased by 50 percent to satisfy the assumptions used for typical interior bays. For this example a W18x50 ($I_x = 800$ in$^4 > 1.5$ x $510 = 765$ in$^4$) is sufficient.

Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 5.7 kips per in. does not apply.

## Example 4.9—SI Units

Evaluate the mezzanine framing shown in Figure 4.9 for walking vibrations. All details are the same as in Example 4.7, except that the framing is rotated 90°. Note that the edge member is now a girder and that the basic framing is the same as that in Example 4.3. The mezzanine is assumed to be one bay wide normal to the edge girder. Use 0.5 kPa live load and 0.2 kPa for the weight of mechanical equipment and ceiling

*Beam Mode Properties*

From Example 4.3

$$w_j = 8.61 \text{ kN/m}$$
$$\Delta_j = 9.08 \text{ mm}$$
$$f_j = 5.91 \text{ Hz.}$$
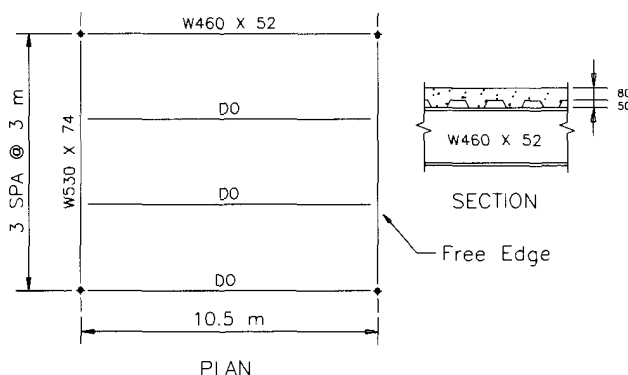


*Fig. 4.9 Mezzanine with girder edge member framing details for Example 4.9.*

$$D_s = 10,420 \text{ mm}^3 \qquad D_j = 250,000 \text{ mm}^3$$
$$B_j = 9.49 \text{ m}$$

From the framing plan, the actual floor width normal to the beams is at least 3 x 10 = 30 m and ⅔ x 30 = 20 m is greater than 9.49 m. The effective beam panel width is then 9.49 m.

The effective weight of the beam panel from Equation (4.2) is then

$$W_j = (w_j / S)B_jL_j = (8.61/3)(9.49 \times 10.5) = 286 \text{ kN}$$

*Girder Mode Properties*

For each girder, the equivalent uniform loading is

$$w_g = (L_j / 2)(w_j / S) + \text{girder weight per unit length}$$

$$= (10.5/2)(8.61/3) + 74 \times 0.00981 = 15.8 \text{ kN/m}$$

and the corresponding deflection is ($I_g = 1,187 \times 10^6$ mm$^4$)

$$\Delta_g = \frac{5w_gL_g^4}{384E_sI_g} = \frac{5 \times 15.8 \times (9,000)^4}{384 \times 200,000 \times 1,187 \times 10^6} = 5.71 \text{ mm}$$

The fundamental frequency is then

$$f_g = 0.18\sqrt{g / \Delta_g} = 0.18\sqrt{9,806 / 5.71} = 7.46 \text{ Hz}$$

As recommended in Section 4.3 under *Interior Floor Edges,* the girder panel width is limited to ⅔ of the beam span. Therefore,

$$B_g = (⅔)L_j = (⅔)(10.5) = 7.0 \text{ m}$$

From Equation (4.2), the girder panel weight is

$$W_g = w_gB_gL_g = (15.8/5.25)(7.0 \times 9) = 189.6 \text{ kN}$$

*Combined Mode Properties*

In this case the girder span (9 m) is less than the joist panel width (9.49 m), and the edge girder deflection is reduced to 5.71(9/9.49) = 5.41 mm. From Equation (3.4),

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g')} = 0.18\sqrt{9,806 / (9.08 + 5.41)}$$

$$= 4.68 \text{ Hz}$$

and from Equation (4.4),

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g'}W_j + \frac{\Delta_g'}{\Delta_j + \Delta_g'}W_g$$

$$= \frac{9.08}{9.08 + 5.41}(286) + \frac{5.41}{9.08 + 5.41}(189.6) = 250.0 \text{ kN}$$

For office occupancy with full height partitions, ß = 0.03 from Table 4.1, thus

$$\beta W = 0.03 \times 250.0 = 7.50 \text{ kN}$$

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*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o$ = 0.29 kN,

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{0.29 \exp(-0.35 \times 4.68)}{7.50}$$

$$= 0.0075 \text{ equivalent to } 0.75 \text{ percent } g$$

which is greater than the acceleration limit $a_o / g$ of 0.5 percent from Table 4.1. The floor is judged unsatisfactory as can also be seen from plotting $f_n$ = 4.68 Hz and $a_p$ = 0.75 percent $g$ on Figure 2.1. If the mezzanine floor is only one bay wide normal to the edge girder, then both the beams and the girders need to be stiffened. If the mezzanine is two or more bays wide normal to the edge girder, then only the edge girder needs to be stiffened as compared to a typical interior girder. In this case a W610x82 edge girder will be satisfactory as compared to a typical W530x74 interior girder.

Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 1 kN per mm does not apply.

## Example 4.10—USC Units

Evaluate the mezzanine framing shown in Figure 4.10 for walking vibrations. All details are the same as in Example 4.8 except that the framing is rotated 90°. Note that the edge member is now a girder and that the framing is the same as used in Example 4.4. The mezzanine is assumed to be one bay wide normal to the edge girder. Use 11 psf live load and 4 psf for the weight of mechanical and ceiling.

*Beam Mode Properties*

From Example 4.4

$w_j$ = 605 plf
$\Delta_j$ = 0.384 in.
$f_j$ = 5.71 Hz.
$D_s$ = 8.25 in.⁴/ft       $D_j$ = 183 in.⁴/ft
$B_j$ = 32.2 ft

From the framing plan, the actual floor width normal to the beams is at least 3 x 30 = 90 ft and ⅔ x 90 = 60 ft is greater than 32.2 ft. The effective beam panel width is then 32.2 ft.

The effective weight of the beam panel from Equation (4.2) is then

$$W_j = w_j B_j L_j = (605/10)(32.2 \text{ x } 35) = 68,184 \text{ lbs} = 68.2 \text{ kips}$$

*Girder Mode Properties*

For each girder, the equivalent uniform loading is

$$w_g = (L_j / 2)(w_j / S) + \text{girder weight per unit length}$$

$$= (35.0/2)(605/10) + 50 = 1,109 \text{ plf}$$

and the corresponding deflection is ($I_g$ = 2,879 in.⁴)

$$\Delta_g = \frac{5 w_g L_g^4}{384 E_s I_g} = \frac{5 \times 1,109 \times 30^4 \times 1,728}{384 \times 29 \times 10^6 \times 2,879} = 0.242 \text{ in.}$$

The fundamental frequency is then

$$f_g = 0.18 \sqrt{g / \Delta_g} = 0.18 \sqrt{386 / 0.242} = 7.19 \text{ Hz}$$

As recommended in Section 4.3 under *Interior Floor Edges,* the girder panel width is limited to ⅔ of the beam span. Therefore,

$$B_g = (2/3) L_j = (2/3)(35) = 23.3 \text{ ft}$$

From Equation (4.2), the girder panel weight is

$$W_g = w_g B_g L_g = (1,109/17.5)(23.3 \times 30) = 44,300 \text{ lbs}$$

$$= 44.3 \text{ kips}$$

*Combined Mode Properties*

In this case the girder span (30 ft) is less than the joist panel width (32.2 ft), and the edge girder deflection is reduced to 0.242(30/32.2) = 0.225 in. From Equation (3.4),

$$f_n = 0.18 \sqrt{g / (\Delta_j + \Delta_g')} = 0.18 \sqrt{386 / (0.384 + 0.225)}$$

$$= 4.53 \text{ Hz}$$

and from Equation (4.4),

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g'} W_j + \frac{\Delta_g'}{\Delta_j + \Delta_g'} W_g$$

$$= \frac{0.384}{0.384 + 0.225}(68.2) + \frac{0.225}{0.384 + 0.225}(44.3) = 59.4 \text{ kips}$$
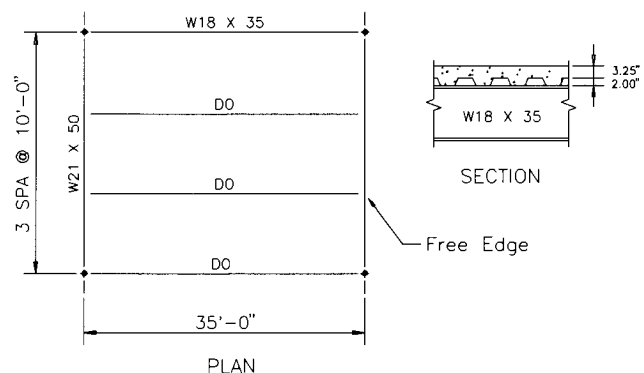


*Fig. 4.10 Mezzanine with girder edge member framing details for Example 4.10.*

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For office occupancy with full height partitions, $\beta = 0.03$ from Table 4.1, thus

$$\beta W = 0.03 \times 59.4 = 1.78 \text{ kips} = 1,780 \text{ lbs}$$

*Evaluation*

Using Equation (4.1) and from Table 4.1 for office occupancy, $P_o = 65$ lbs,

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35 f_n)}{\beta W} = \frac{65 \exp(-0.35 \times 4.53)}{1,780}$$

$$= 0.0075 \text{ equivalent to } 0.75 \text{ percent } g$$

which is greater than the acceleration limit $a_o / g$ of 0.5 percent from Table 4.1. The floor is judged unsatisfactory as can also be seen from plotting $f_n = 4.53$ Hz and $a_p = 0.75$ percent $g$ on Figure 2.1.

If the mezzanine floor is only one bay wide normal to the edge girder, then both the beams and the girder need to be stiffened to satisfy the criterion. If the mezzanine is two or more bays wide normal to the edge girder, then only the edge girder needs to be stiffened as compared to a typical interior girder. In this case a W24x55 edge girder would be satisfactory as compared to a typical W21x50 interior girder.

Since the fundamental frequency of the system is less than 9 Hz, the minimum stiffness requirement of 5.7 kips per in. does not apply.

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# Chapter 5
# DESIGN FOR RHYTHMIC EXCITATION

## 5.1 Recommended Criterion

The need for a rhythmic excitation design criterion has arisen from the increasing incidence of building vibration problems due to rhythmic activities. In a few cases, cyclic floor accelerations of as much as 50 percent gravity have resulted in structural fatigue problems. Vibrations due to rhythmic activities were first recognized in a Commentary to the 1970 National Building Code of Canada (NBC), where it was stated that resonance due to human activities can be a problem if the floor frequency is less than 5 Hz. For the 1975 NBC Commentary, this value was increased to 10 Hz "for very repetitive activities such as dancing because it is possible to get some resonance when the beat is on every second cycle of floor vibration". A design criterion for rhythmic excitation based on dynamic loading and response was first introduced in the 1985 NBC Commentary and was improved in the 1990 NBC commentary to recognize the importance of sensitive occupancies. The 1990 NBC design criterion, which uses the acceleration limits of Table 5.1, is adopted for this Design Guide. Application of this criterion will not result in fatigue problems.

The following design criterion for rhythmic excitation (see Section 2.2.2) is based on the dynamic loading function for rhythmic activities and the dynamic response of the floor structure:

$$f_n \geq (f_n)_{req'd} = f\sqrt{1 + \frac{k}{a_o/g}\frac{\alpha_i w_p}{w_t}} \tag{5.1}$$

where:

- $f_n$ = fundamental natural frequency of the structural system,
- $(f_n)_{req'd}$ = minimum natural frequency required to prevent unacceptable vibrations at each forcing frequency, $f$
- $f$ = forcing frequency = $i\,f_{step}$ (see Table 5.2)
- $i$ = number of harmonic = 1, 2, or 3 (see Table 5.2)
- $f_{step}$ = step frequency
- $k$ = a constant (1.3 for dancing, 1.7 for lively concert or sports event, 2.0 for aerobics)
- $\alpha_i$ = dynamic coefficient (see Table 5.2, which is based on Table 2.1)
- $a_o/g$ = ratio of peak acceleration limit (from Figure 2.1 in the frequency range 4-8 Hz) to the acceleration due to gravity

### Table 5.1
### Recommended Acceleration Limits for Vibrations Due to Rhythmic Activities (NBC 1990)

| Occupancies Affected by the Vibration | Acceleration Limit, % gravity |
|---|---|
| Office or residential | 0.4-0.7 |
| Dining or weightlifting | 1.5-2.5 |
| Rhythmic activity only | 4-7 |

- $w_p$ = effective weight per unit area of participants distributed over the floor panel
- $w_t$ = effective total weight per unit area distributed over the floor panel (weight of participants plus weight of floor system)

Table 5.3, based on Equation (5.1), gives minimum required natural frequencies for four typical cases. A specific evaluation of any design is obtained by application of Equation (5.1), or more accurately by application of Equations (2.4) to (2.6), with parameters for steel framed structures estimated in the following section. A computer model and the appropriate loading function described in Table 5.2 may also be used to determine vibration accelerations throughout the building. These accelerations are to be compared to the acceleration limits given in Table 5.1 for various occupancies.

## 5.2 Estimation of Parameters

The most important structural parameter that must be considered in preventing building vibration problems due to rhythmic activities is the fundamental natural frequency of vertical vibration of the structure, $f_n$. Also important is the loading function of the activity (Table 5.2) and the transmission of vibration to sensitive occupancies of the building. Of lesser importance are the equivalent weight of the floor and the damping ratio.

### Fundamental Natural Frequency, $f_n$

The floor's fundamental natural frequency is much more important in relation to rhythmic excitation than for walking excitation, and therefore more care is required for its estimation. For determining fundamental natural frequency, it is important to keep in mind that the structure extends all the way down to the foundations, and even into the ground. Equation (3.5) can be used to estimate the natural frequency of the structure, including the beams or joists, girders, and columns. Equation (3.5) is repeated here for convenience:

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| Table 5.2<br>Estimated Loading During Rhythmic Events | | | | | | |
|---|---|---|---|---|---|---|
| Activity | Forcing Frequency *f*, Hz | Weight of Participants* $w_p$ | | Dynamic Coefficient $\alpha_i$ | Dynamic Load $\alpha_i w_p$ | |
| | | kPa | psf | | kPa | psf |
| Dancing:<br>First Harmonic | 1.5–3 | 0.6 | 12.5 | 0.5 | 0.3 | 6.2 |
| Lively concert or sports event:<br>First Harmonic<br>Second Harmonic | 1.5–3<br>3–5 | 1.5<br>1.5 | 31.0<br>31.0 | 0.25<br>0.05 | 0.4<br>0.075 | 7.8<br>1.6 |
| Jumping exercises:<br>First Harmonic<br>Second Harmonic<br>Third Harmonic | 2–2.75<br>4–5.5<br>6–8.25 | 0.2<br>0.2<br>0.2 | 4.2<br>4.2<br>4.2 | 1.5<br>0.6<br>0.1 | 0.3<br>0.12<br>0.020 | 6.3<br>2.5<br>0.42 |

\* Based on maximum density of participants on the occupied area of the floor for commonly encountered conditions. For special events the density of participants can be greater.

$$f_n = 0.18\sqrt{g / (\Delta_j + \Delta_g + \Delta_c)} \qquad (5.2)$$

where

$\Delta_j$ = the elastic deflection of the floor joist or beam at mid-span due to bending and shear

$\Delta_g$ = the elastic deflection of the girder supporting the beams due to bending and shear

$\Delta_c$ = the elastic shortening of the column or wall (and the ground if it is soft) due to axial strain

and where each deflection results from the *total* weight supported by the member, including the weight of people. The flexural stiffness of floor members should be based on composite or partially composite action, as recommended in Section 3.2. Guidance for determining deflection due to shear is given in Sections 3.5 and 3.6. In the case of joists, beams, or girders continuous at supports, the deflection due to bending can be estimated using Section 3.4. The contribution of column deflection, $\Delta_c$, is generally small compared to joist and girder deflections for buildings with few (1-5) stories but becomes significant for buildings with many (> 6) stories because of the increased length of the column "spring". For a building with very many stories (> 15), the natural frequency due to the column springs alone may be in resonance with the second harmonic of the jumping frequency (Alien, 1990).

A more accurate estimate of natural frequency may be obtained by computer modeling of the total structural system.

*Acceleration Limit: $a_o / g$*

It is recommended, when applying Equation (5.1), that a limit of 0.05 (equivalent to 5 percent of the acceleration of gravity) not be exceeded, although this value is considerably less than that which participants in activities are known to accept. The 0.05 limit is intended to protect vibration sensitive occupancies of the building. A more accurate procedure is first to estimate the maximum acceleration on the activity floor by using Equations (2.5) and (2.6) and then to estimate the accelerations in sensitive occupancy locations using the fundamental mode shape. These estimated accelerations are then compared to the limits in Table 5.1. The mode shapes can be determined from computer analysis or estimated from the deflection parameters $\Delta_j$, $\Delta_g$ and $\Delta_c$ (see Example 5.3 or 5.4).

*Rhythmic Loading Parameters: $w_p$, $\alpha_i$ and $f$*

For the area used by the rhythmic activity, the distributed weight of participants, $w_p$, can be estimated from Table 5.2. In cases where participants occupy only part of the span, the value of $w_p$ is reduced on the basis of equivalent effect (moment or deflection) for a fully loaded span. Values of $\alpha_i$ and f are recommended in Table 5.2.

*Effective Weight, $w_t$*

For a simply-supported floor panel on rigid supports, the effective weight is simply equal to the distributed weight of the floor plus participants. If the floor supports an extra weight (such as a floor above), this can be taken into account by increasing the value of $w_t$. Similarly, if the columns vibrate significantly, as they do sometimes for upper floors, there is an increase in effective mass because much more mass is attached to the columns than just the floor panel supporting the rhythmic activity. The effect of an additional concentrated weight, $W_c$, can be approximated by an increase in $w_t$ of

$$2W_c\, y^2 / LB$$

where

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| Activity Acceleration Limit Construction | Forcing Frequency[1] $f$, Hz | Effective Weight of Participants $w_p$ | | Total Weight $w_t$ | | Minimum Required Fundamental Natural Frequency[3] $f_n$, Hz |
|---|---|---|---|---|---|---|
| | | kPa | psf | kPa | psf | |
| Dancing and Dining $a_o / g = 0.02$ | | | | | | |
| Heavy floor 5 kPa (100 psf) | 3 | 0.6 | 12.5 | 5.6 | 112.5 | 6.4 |
| Light floor 2.5 kPa (50 psf) | 3 | 0.6 | 12.5 | 3.1 | 62.5 | 8.1 |
| Lively Concert or Sports Event $a_o / g = 0.05$ | | | | | | |
| Heavy floor 5 kPa (100 psf) | 5 | 1.5 | 31.0 | 6.5 | 131.0 | 5.9[2] |
| Light floor 2.5 kPa (50 psf) | 5 | 1.5 | 31.0 | 4.0 | 81.0 | 6.4[2] |
| Aerobics only $a_o / g = 0.06$ | | | | | | |
| Heavy floor 5 kPa (100 psf) | 8.25 | 0.2 | 4.2 | 5.2 | 104.2 | 8.8[2] |
| Light floor 2.5 kPa (50 psf) | 8.25 | 0.2 | 4.2 | 2.7 | 54.2 | 9.2[2] |
| Jumping Exercises Shared with Weight Lifting $a_o / g = 0.02$ | | | | | | |
| Heavy floor 5 kPa (100 psf) | 8.25 | 0.12 | 2.5 | 5.12 | 102.5 | 9.2[2] |
| Light floor 2.5 kPa (50 psf) | 5.5 | 0.12 | 2.5 | 2.62 | 52.5 | 10.6[2] |

Notes to Table 5.3:
[1] Equation (5.1) is supplied to all harmonics listed in Table 5.2 and the governing forcing frequency is shown.
[2] May be reduced if, according to Equation (2.5a), damping times mass is sufficient to reduce 2nd and 3rd harmonic resonance to an acceptable level.
[3] From Equation (5.1).

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$y$ = ratio of modal displacement at the location of the weight to maximum modal displacement

$L$ = span

$B$ = effective width of the panel, which can be approximated as the width occupied by the participants

Continuity of members over supports into adjacent floor panels can also increase the effective mass, but the increase is unlikely to be greater than 50 percent. Note that only an approximate value of $w_t$ is needed for application of Equation (5.1).

*Damping Ratio,* $\beta$

This parameter does not appear in Equation (5.1) but it appears in Equation (2.5a), which applies if resonance occurs. Because participants contribute to the damping, a value of approximately 0.06 may be used, which is higher than shown in Table 4.1 for walking vibration.

## 5.3 Application of the Criterion

The designer initially should determine whether rhythmic activities are contemplated in the building, and if so, where. At an early stage in the design process it is possible to locate both rhythmic activities and sensitive occupancies so as to minimize potential vibration problems and the costs required to avoid them. It is also a good idea at this stage to consider alternative structural solutions to prevent vibration problems. Such structural solutions may include design of the structure to control the accelerations in the building and special approaches, such as isolation of the activity floor from the rest of the building or the use of mitigating devices such as tuned mass dampers.

The structural design solution involves three stages of increasing complexity. The first stage is to establish an approximate minimum natural frequency from Table 5.3 and to estimate the natural frequency of the structure using Equation (5.2). The second stage consists of hand calculations using Equation (5.1), or alternatively Equations (2.5) and (2.6), to find the minimum natural frequency more accurately, and of recalculating the structure's natural frequency using Equation (5.2), including shear deformation and continuity of beams and girders. The third stage requires computer analyses to determine natural frequencies and mode shapes, identifying the lowest critical ones, estimating vibration accelerations throughout the building in relation to the maximum acceleration on the activity floor, and finally comparing these accel-

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erations in critical locations of the building to the acceleration limits of Table 5.1.

In summary, the most important aspects of application of the rhythmic design criterion are the fundamental natural frequency of the structural system and the vibration accelerations in sensitive occupancies. Location of the activity within the building is usually the most important design decision.

## 5.4 Example Calculations

Table 5.3 shows approximate minimum required natural frequencies for typical heavy and light floor structures. Except for the fourth case (jumping exercises shared with weight lifting), the influence of sensitive occupancies affected by the vibration is not considered. A minimum natural frequency estimated from Table 5.3 and Equation (5.2) can be used to develop the initial design. Additional refinement may then be required as illustrated in the following examples which are presented first in the SI system of units and then repeated in the US Customary (USC) system of units.

### Example 5.1—Long Span Joist Supported Floor Used for Dancing—SI Units

The floor shown in Figure 5.1 is used for dining adjacent to the dancing area shown. The floor system consists of long span (14 m) joists supported on concrete block walls. The effective weight of the floor is estimated to be 3.6 kPa, including 0.6 kPa for people dancing and dining. The effective composite moment of inertia of the joists, which were selected based on strength, is $1,100 \times 10^6$ mm$^4$. (See Example 4.5 for calculation procedures.)



*Fig. 5.1  Layout of dance floor for Example 5.1.*

*First Approximation*

As a first check to determine if the floor system is satisfactory, the minimum required fundamental natural frequency is estimated from Table 5.3 by interpolation between "light" and "heavy" floors. The minimum required fundamental natural frequency is found to be 7.3 Hz.

The deflection of a composite joist due to the supported 3.6 kPa loading is

$$\Delta_j = \frac{5w_jL_j^4}{384E_sI_j} = \frac{5(3.6 \times 1.25)(14,000)^4}{384 \times 200,000 \times 1,100 \times 10^6} = 10.23 \text{ mm}$$

Since there are no girders, $\Delta_g = 0$, and since the axial deformation of the wall can be neglected, $\Delta_c = 0$. Thus, the floor's fundamental natural frequency, from Equation (5.2), is approximately

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{9,806}{10.23}} = 5.6 \text{ Hz}$$

Because $f_n = 5.6$ Hz is less than the required minimum natural frequency, 7.3 Hz, the system appears to be unsatisfactory.

*Second Approximation*

To investigate the floor design further, Equation (5.1) is used. From Table 5.1, an acceleration limit of 2 percent g is selected, that is $a_o/g = 0.02$. The floor layout is such that half the span will be used for dancing and the other half for dining. Thus, $w_p$ is reduced from 0.6 kPa (from Table 5.2) to 0.3 kPa. Using Inequality (5.1), with $f = 3$ Hz and $\alpha_1 = 0.5$ from Table 5.2 and $k = 1.3$ for dancing, the required fundamental natural frequency is

$$(f_n)_{req'd} = f\sqrt{1 + \frac{k}{a_o/g}\frac{\alpha_i w_p}{w_t}} = 3.0\sqrt{1 + \frac{1.3}{0.02} \cdot \frac{0.5 \times 0.3}{3.6}}$$

$$= 5.8 \text{ Hz.}$$

Since $f_n = 5.6$ Hz, the floor is marginally unsatisfactory and further analysis is warranted.

From Equation (2.5b), the expected maximum acceleration is

$$\frac{a_p}{g} = \frac{1.3}{(f_n/f)^2 - 1} \cdot \frac{\alpha w_p}{w_t} = \frac{1.3}{(5.6/3.0)^2 - 1} \cdot \frac{0.5 \times 0.3}{3.6}$$

$$= 0.022 \text{ equivalent to 2.2 percent } g$$

Since the recommended maximum acceleration for dancing combined with dining is 2 percent $g$ and since the floor layout might change, stiffer joists should be used.

### Example 5.2—Long Span Joist Supported Floor Used for Dining—USC Units

The floor shown in Figure 5.2 is used for dining adjacent to

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the dancing area shown. The floor system consists of long span (45 ft.) joists supported on concrete block walls. The effective weight of the floor is estimated to be 75 psf, including 12 psf for people dancing and dining. The effective composite moment of inertia of the joists, which were selected based on strength, is 2,600 in.[4] (See Example 4.6 for calculation procedures.)
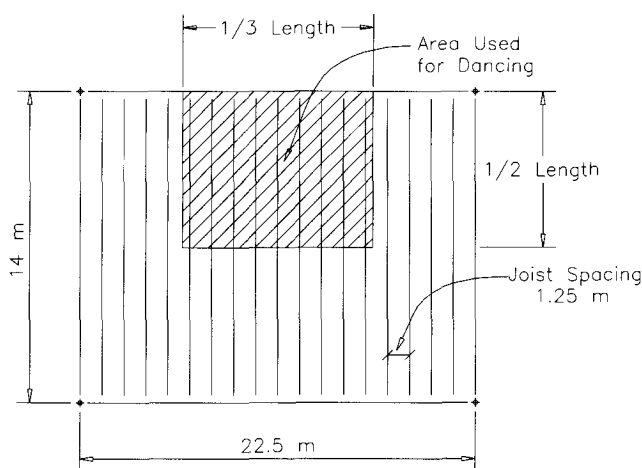
*First Approximation*

As a first check to determine if the floor system is satisfactory, the minimum required fundamental natural frequency is estimated from Table 5.3 by interpolation between "light" and "heavy" floors. The minimum required fundamental natural frequency is found to be 7.3 Hz.

The deflection of a composite joist due to the supported 75 psf loading is

$$\Delta_j = \frac{5w_j L_j^4}{384 E_s I_j} = \frac{5 \times (75 \times 48 / 12) \times 45^4 \times 1,728}{384 \times 29 \times 10^6 \times 2,600}$$

$$= 0.367 \text{ in.}$$

Since there are no girders, $\Delta_g = 0$, and since the axial deformation of the wall can be neglected, $\Delta_c = 0$. Thus, the floor's fundamental natural frequency, from Equation (5.2.), is approximately

$$f_o = 0.18 \sqrt{\frac{g}{\Delta_j}} = 0.18 \sqrt{\frac{386}{0.367}} = 5.8 \text{ Hz}$$

Because $f_n = 5.8$ Hz is less than the required minimum natural frequency, 7.3 Hz, the system appears to be unsatisfactory.

*Second Approximation*

To investigate the floor design further, Equation (5.1) is used. From Table 5.1, an acceleration limit of 2 percent $g$ is selected, that is $a_o / g = 0.02$. The floor layout is such that half the span will be used for dancing and the other half for dining. Thus, $w_p$ is reduced from 12.5 psf (from Table 5.2) to 6 psf. Using Inequality (5.1), with $f = 3$ Hz and $\alpha_1 = 0.5$ from Table 5.2 and $k = 1.3$ for dancing, the required fundamental natural frequency is

$$(f_n)_{req'd} = f \sqrt{1 + \frac{k}{a_o / g} \frac{\alpha_i w_p}{w_t}} = 3.0 \sqrt{1 + \frac{1.3}{0.02} \cdot \frac{0.5 \times 6}{75}}$$

$$= 5.7 \text{ Hz.}$$

Since $f_n = 5.8$ Hz, the floor is marginally unsatisfactory and further analysis is warranted.

From Equation (2.5b), the expected maximum acceleration is

$$\frac{a_p}{g} = \frac{1.3}{(f_n / f)^2 - 1} \cdot \frac{\alpha w_p}{w_t} = \frac{1.3}{(5.8 / 3.0)^2 - 1} \cdot \frac{0.5 \times 6}{75}$$

$$= 0.019 \text{ equivalent to } 1.9 \text{ percent } g$$

Since the recommended maximum acceleration for dancing combined with dining is 2 percent $g$ and since the floor layout might change, stiffer joists should be considered.

### Example 5.3—Second Floor of General Purpose Building Used for Aerobics—SI Units

Aerobics is to be considered for the second floor of a six story health club. The structural plan is shown in Figure 5.3.

*Fig. 5.2 Layout of dance floor for Example 5.2.*
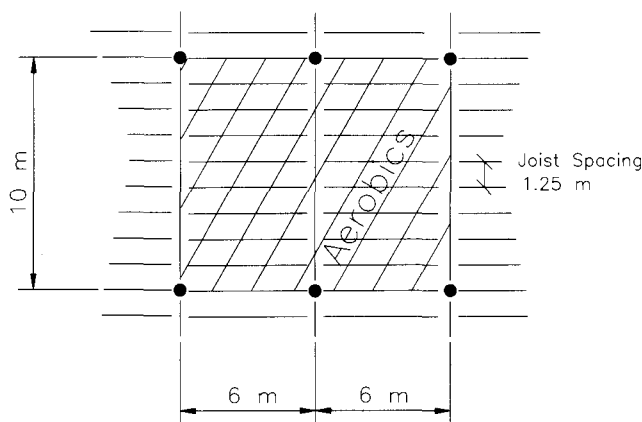


*Fig. 5.3 Aerobics floor structural layout for Example 5.3.*

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The floor construction consists of a concrete slab on open-web steel joists, supported on hot-rolled girders and steel columns. The weight of the floor is 3.1 kPa. Both the joists and the girders are simply supported and in the aerobics area the girders are composite, i.e., connected to the concrete with shear studs. The effective composite moments of inertia of the joists and girders are $108 \times 10^6 \, mm^4$ and $2,600 \times 10^6 \, mm^4$, respectively. (See Example 4.5 for calculation procedures.)

*First Approximation*

Table 5.3 indicates that the structural system should have a minimum natural frequency of approximately 9 Hz. The natural frequency of the system is estimated by use of Equation (5.2). The deflections due to the weight supported by each element (joists, girders and columns) are determined as follows:

The deflection of the joists due to the floor weight is

$$\Delta_j = \frac{5w_j L_j^4}{384 E_s I_j} = \frac{5 \times (3.1 \times 1.25) \times (6,000)^4}{384 \times 200,000 \times 108 \times 10^6} = 3.03 \text{ mm}$$

The deflection of the girders due to the floor weight is

$$\Delta_g = \frac{5w_g L_g^4}{384 E_s I_g} = \frac{5 \times (3.1 \times 6.0) \times (10,500)^4}{384 \times 200,000 \times 2,600 \times 10^6} \approx 5.66 \text{ mm}$$

The axial shortening of the columns is calculated from the axial stress due to the weight supported. Assuming an axial stress, $\sigma_a$, of 40 MPa and a column length of 5 m,

$$\Delta_c = \frac{\sigma_a L_c}{E} = \frac{40 \times 5,000}{200,000} = 1.0 \text{ mm}$$

The total deflection is

$$\Delta_T = \Delta_j + \Delta_g + \Delta_c = 3.03 + 5.66 + 1.0 = 9.69 \text{ mm}$$

and the natural frequency from Equation (5.2) is

$$f_n = 0.18\sqrt{\frac{g}{\Delta_T}} = 0.18\sqrt{9,806 / 9.69} = 5.7 \text{ Hz}$$

which is considerably less than the estimated required minimum frequency of 9.0 Hz.

*Second Approximation*

Inequality (5.1) is now used to evaluate the system further. The required frequencies for each of the jumping exercise hamonics are calculated using $k = 2.0$ for jumping, $a_o / g = 0.05$ (the accel. limit of 0.05 applies to the activity floor, not to adj. areas) and $f$, $w_p$ and $\alpha_i$ values from Table 5.2. For the first harmonic of the forcing frequency, $f = 2.75$ Hz, $\alpha_1 = 1.5$ and $w_p = 0.2$ kPa,

$$(f_n)_{req'd} = f\sqrt{1 + \frac{k}{a_o / g} \frac{\alpha_i w_p}{w_t}} = 2.75\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{1.5 \times 0.2}{3.1}}$$

$$= 6.1 \text{ Hz}$$

Similarly, for the second harmonic with $f = 5.5$ Hz, $\alpha_2 = 0.6$ and $w_p = 0.2$ kPa,

$$(f_n)_{req'd} = 5.5\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{0.6 \times 0.2}{3.1}} = 8.8 \text{ Hz}$$

And, for the third harmonic with $f = 8.75$ Hz, $\alpha_3 = 0.1$ and $w_p = 0.2$ kPa,

$$(f_n)_{req'd} = 8.25\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{0.1 \times 0.2}{3.1}} = 9.3 \text{ Hz}$$

Because the natural frequency (5.7 Hz) is less than the required frequency for all three harmonics, large, unacceptable vibrations are to be expected.

Also, because 5.7 Hz is very close to a forcing frequency for the second harmonic of the step frequency (5.5 Hz), an approximate estimate of the acceleration can be determined from the resonance response formula, Equation (2.5a):

$$\frac{a}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha_2 w_p}{w_t} = \frac{1.3}{2 \times 0.06} \cdot \frac{0.6 \times 0.2}{3.1}$$

$$= 0.42 \text{ equivalent to 42 percent } g$$

where the values of the parameters $\alpha_2$ and $w_p$ are obtained from Table 5.2 for the second harmonic of jumping exercises and 0.06 is the recommended estimate of the damping ratio of a floor-people system.

An acceleration of 42 percent of gravity implies that the vibrations will be unacceptable, not only for the aerobics floor, but also for adjacent areas on the second floor. Further, other areas of the building supported by the aerobics floor columns will be subjected to vertical accelerations of approximately 4 percent of gravity, as estimated from the mode shape, where the ratio of column deflection (1.0 mm) to total deflection at the midpoint of the activity floor (9.69 mm) is approximately 0.10. Accelerations of this magnitude are unacceptable for most occupancies.

*Conclusions*

The floor framing shown in Figure 5.4 should not be used for aerobic activities. For an acceptable structural system, the natural frequency of the structural system needs to be increased to at least 9 Hz. Significant increases in the stiffness of both the joists and the girders are required. An effective method of stiffening to achieve a natural frequency of 9 Hz is to support the aerobics floor girders at mid-span on columns directly to the foundations and to increase the stiffness of the aerobics floor joists.

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## Example 5.4—Second Floor of General Purpose Building Used for Aerobics—USC Units

Aerobics is to be considered for the second floor of a six story health club. The structural plan is shown in Figure 5.4. The floor construction consists of a concrete slab on open-web steel joists, supported on hot-rolled girders and steel columns. The weight of the floor is 65 psf. Both the joists and the girders are simply supported and in the aerobics area the girders are composite, i.e., connected to the concrete with shear studs. The effective composite moments of inertia of the joists and girders are 260 in.[4] and 6,310 in.[4], respectively. (See Example 4.6 for calculation procedures.)

*First Approximation*

Table 5.3 indicates that the structural system should have a minimum natural frequency of approximately 9 Hz. The natural frequency of the system is estimated by use of Equation (5.2). The required deflections due to the weight supported by each element (joists, girders and columns) are determined as follows:

The deflection of the joists due to the floor weight is

$$\Delta_j = \frac{5w_j L_j^4}{384 E_s I_j} = \frac{5 \times (65 \times 4) \times (20)^4 \times 1,728}{384 \times 29 \times 10^6 \times 260.0} = 0.124 \text{ in.}$$

The deflection of the girders due to the floor weight is

$$\Delta_g = \frac{5w_g L_g^4}{384 E_s I_g} = \frac{5 \times (65 \times 20) \times (36)^4 \times 1,728}{384 \times 29 \times 10^6 \times 6,310} = 0.269 \text{ in.}$$

The axial shortening of the columns is calculated from the axial stress due to the weight supported. Assuming an axial stress, $\sigma_a$, of 6 ksi and a column length of 16 ft,

$$\Delta_c = \frac{\sigma_a L_c}{E} = \frac{6.0 \times 16 \times 12}{29 \times 10^3} = 0.040 \text{ in.}$$

The total deflection is then

$$\Delta_T = \Delta_j + \Delta_g + \Delta_c = 0.124 + 0.269 + 0.040 = 0.433 \text{ in.}$$

and the natural frequency from Equation (5.2) is

$$f_n = 0.18\sqrt{\frac{g}{\Delta_T}} = 0.18\sqrt{386 / 0.433} = 5.4 \text{ Hz}$$

which is considerably less than the estimated required minimum frequency of 9.0 Hz.

*Second Approximation*

Inequality (5.1) is now used to evaluate the system further. The required frequencies for each of the jumping exercise hamonics are calculated using $k = 2.0$ for jumping, $a_o / g = 0.05$ (the accel. limit of 0.05 applies to the activity floor, not to adj. areas) and $f$, $w_p$ and $\alpha_i$ values from Table 5.2. For the first

harmonic of the forcing frequency, $f = 2.75$ Hz, $\alpha_1 = 1.5$ and $w_p = 4.2$ psf,

$$(f_n)_{req'd} = f\sqrt{1 + \frac{k}{a_o / g} \frac{\alpha_i w_p}{w_t}} = 2.75\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{1.5 \times 4.2}{65}}$$

$$= 6.1 \text{ Hz}$$

Similarly, for the second harmonic with $f = 5.5$ Hz, $\alpha_2 = 0.6$ and $w_p = 4.2$ psf,

$$(f_n)_{req'd} = 5.5\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{0.6 \times 4.2}{65}} = 8.8 \text{ Hz}$$

And, for the third harmonic with $f = 8.25$ Hz, $\alpha_{32} = 0.1$ and $w_p = 4.2$ psf,

$$(f_n)_{req'd} = 8.25\sqrt{1 + \frac{2.0}{0.05} \cdot \frac{0.1 \times 4.2}{65}} = 9.3 \text{ Hz}$$

Because the natural frequency (5.4 Hz) is less than the required frequency for all three harmonics, large, unacceptable vibrations are expected.

Also, because 5.4 Hz is very close to a forcing frequency for the second harmonic of the step frequency (5.5 Hz), an approximate estimate of the acceleration can be determined from the resonance response formula, Equation (2.5a):

$$\frac{a}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha_2 w_p}{w_t} = \frac{1.3}{2 \times 0.06} \cdot \frac{0.6 \times 4.2}{65}$$

$$= 0.42 \text{ equivalent to 42 percent } g$$
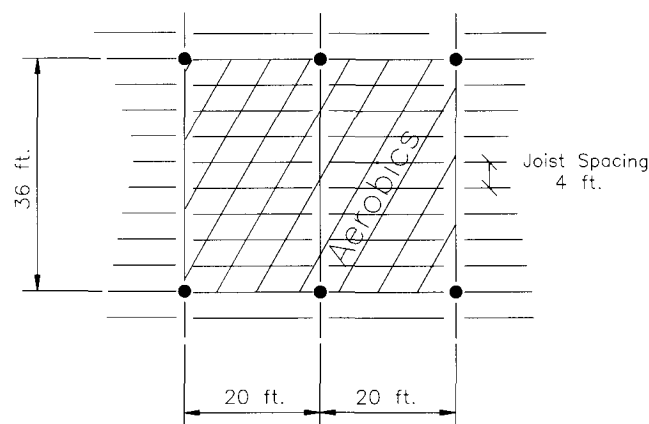


*Fig. 5.4 Aerobics floor structural layout for Example 5.4.*

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where the values of the parameters $\alpha_2$ and $w_p$ are obtained from Table 5.2 for the second harmonic of jumping exercises and 0.06 is the recommended estimate of the damping ratio of a floor-people system.

An acceleration of 42 percent of gravity implies that the vibrations will be unacceptable, not only for the aerobics floor, but also for adjacent areas on the second floor. Further, other areas of the building supported by the aerobics floor columns will be subjected to vertical accelerations of approximately 4 percent of gravity, as estimated from the mode shape, where the ratio of column deflection (0.040 in.) to total deflection at the midpoint of the activity floor (0.433 in.) is

approximately 0.10. Accelerations of this magnitude are unacceptable for most occupancies.

*Conclusions*

The floor framing shown in Figure 5.4 should not be used for aerobic activities. For an acceptable structural system, the natural frequency of the structural system needs to be increased to at least 9 Hz. Significant increases in the stiffnesses of both the joists and the girders are required. An effective method of stiffening to achieve a natural frequency of 9 Hz is to support the aerobics floor girders at mid-span on columns directly to the foundations and to increase the stiffness of the aerobics floor joists.

# Chapter 6
# DESIGN FOR SENSITIVE EQUIPMENT

## 6.1 Recommended Criterion

Floors that support sensitive equipment need to provide vibration environments that are acceptable for the equipment in question. Thus, the designer needs to determine the maximum allowed vibration to which this equipment may be subjected, so that a floor can be provided that will permit no more than this allowed vibration.

In situations where the equipment of concern is fully defined, one may generally obtain equipment vibration criteria from the equipment suppliers' installation manuals. These criteria typically specify limits on the vibrations at the equipment's supports and thus on the vibrations of the floor under the equipment. If several equipment items with different vibration sensitivities are to be supported on the same floor, the area of the floor that is expected to experience the most severe vibrations generally should be designed to accommodate the most sensitive item, unless the more sensitive items can be located in areas of lesser vibration and/or provided with added vibration isolation systems, as discussed in Section 6.4.

In cases where the equipment that is to be supported on a given floor is known only in general terms at the time the floor structure is being designed, the designer needs to rely on generic criteria. A set of such criteria that has been applied widely is given in Table 6.1, which is to be used together with Figure 6.1 (Ungar et al 1990). These criteria are expressed in terms of the greatest vibrational velocity to which various classes of equipment may be exposed. Stating these criteria in terms of velocity is most convenient in general, because the criterion for a given class of equipment corresponds to a constant value of velocity over most of the frequency range of interest. To convert a given velocity, $V$, to the corresponding acceleration, a, one may use the relation

$$a / g = 2\pi f V / g \qquad (6.1)$$

where

$f$=frequency(Hz)
$g$ = the acceleration of gravity

The values listed in Table 6.1 and the shape of the curves of Figure 6.1 were obtained from review of numerous equipment supplier's specifications and data. The shape of the curves is based on the observation that curves of constant velocity constitute conservative lower bound envelopes to many of these specifications and data (see Ungar *et al* 1990). The shape of the solid curve of Figure 6.1 also is that given

in ANSI Standard 53.29-1983 (ANSI 1983) for criteria pertaining to vibration annoyance of people in various occupancies.

As noted in Figure 6.1, for equipment without internal pneumatic isolation, the velocity values listed in Table 6.1 apply for frequencies between 8 Hz and 80 Hz, with higher values applicable below 8 Hz; for equipment with internal pneumatic isolation the tabulated values apply between 1 Hz and 80 Hz. Applicability of higher allowable velocities below 8 Hz for equipment without internal isolation results from the fact that most such equipment exhibits no internal resonances below 8 Hz, so that external disturbances at these low frequencies may be expected to result in relatively small relative motions within the equipment—and it is relative motions, rather than absolute motions, that tend to affect the operation of sensitive equipment. Equipment with internal isolation, on the other hand, is likely to exhibit resonances at frequencies below 8 Hz, so that more stringent limits need to be placed on the floor vibrations at these low frequencies.

The criterion values of Table 6.1 and Figure 6.1 apply to footfall-induced vibrations, which occur predominantly at a single frequency or at a number of frequencies that differ from each other by a factor of at least 1.4. The same criterion values may also be used to evaluate the effects of mechanical disturbances that occur at a single frequency or at a number of widely separated frequencies; for disturbances at multiple, closely spaced frequencies, however, the criterion values
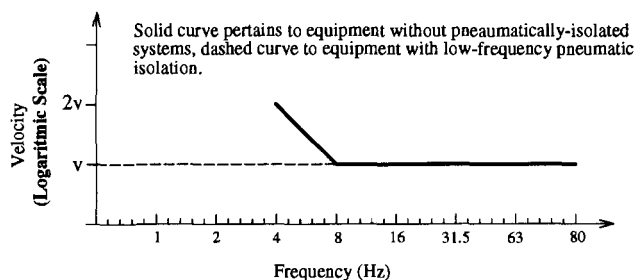


*Fig. 6.1 General criterion curve to be used with values of Table 6.1.*

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## Table 6.1
### Vibration Criteria for Sensitive Equipment

| Facility Equipment or Use | Vibrational Velocity* | |
|---|---|---|
| | (µ in./sec) | (µm/sec) |
| Computer systems; Operating Rooms**; Surgery; Bench microscopes at up to 100x magnification; | 8,000 | 200 |
| Laboratory robots | 4,000 | 100 |
| Bench microscopes at up to 400x magnification; Optical and other precision balances; Coordinate measuring machines; Metrology laboratories; Optical comparators; Microelectronics manufacturing equipment—Class A*** | 2,000 | 50 |
| Micro surgery, eye surgery, neuro surgery; Bench microscopes at magnification greater than 400x; Optical equipment on isolation tables; Microelectronics manufacturing equipment—Class B*** | 1,000 | 25 |
| Electron microscopes at up to 30,000x magnification; Microtomes; Magnetic resonance imagers; Microelectronics manufacturing equipment—Class C*** | 500 | 12 |
| Electron microscopes at greater than 30,000x magnification; Mass spectrometers; Cell implant equipment; Microelectronids manufacturing equipment—Class D*** | 250 | 6 |
| Microelectronics Manufacturing equipment—Class E***; Unisolated laser and optical research systems | 130 | 3 |

\* Value of V for Figure 6.1.
\*\* Criterion given by solid curve of Figure 6.1 corresponds to a standard mean whole-body threshold of perception (Guide 1974)
\*\*\* Class A:  Inspection, probe test, and other manufacturing support equipment.
    Class B:  Aligners, steppers, and other critical equipment for photolithography with line widths of 3 microns or more.
    Class C:  Aligners, steppers, and other critical equipment for photolithography with line widths of 1 micron.
    Class D:  Aligners, steppers, and other critical equipment for photolithography with line widths of ½ micron; includes electron-beam systems.
    Class E:  Aligners, steppers, and other critical equipment for photolithography with line widths of ¼ micron; includes electron-beam systems.

apply to disturbances observed in one-third-octave bands, rather than at single frequencies (Ungar *et al* 1990).

Table 6.1 includes some criteria for optical equipment. These are useful for preliminary design and evaluation purposes. Figure 6.2 presents more precise criteria for microscopes or other equipment used for direct visual observation of enlarged images. The criteria of Figure 6.2 are based on consideration of the capability limits of the human eye (House and Randall 1987) and consist of a maximum allowable vibrational acceleration below 3 Hz (which frequency range is generally of no concern in relation to floors of buildings), of a maximum allowable vibrational velocity between 3 Hz and 8 Hz, and of a maximum allowable vibrational displacement at frequencies above 8 Hz. The numerical values of these limits depend on the equipment's optical magnification, M, as indicated by the equations shown in Figure 6.2. The

uppermost curve of this figure pertains to 40x magnification, which is typical for surgical and workshop applications. The lowest curve pertains to 400x magnification, which is typical for laboratory bench microscopes.

It should be noted that all of the equipment criteria discussed in this section pertain to the instantaneous maximum or "peak" vibration to which the equipment is exposed; they do not consider the rate of decay of vibrations. The assumption here is that even an extremely brief exposure of equipment to vibrations above a certain limit may suffice to interfere with the equipment's operation e.g., to blur a photographic image or to misalign components. Thus, although human perception of vibrations depends on how the vibration varies with time, the dominant adverse effect of vibrations on sensitive equipment generally is independent of the time variation.

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## 6.2 Estimation of Peak Vibration of Floor due to Walking

The force pulse exerted on a floor when a person takes a step has been shown to have the idealized shape indicated in Figure 6.3. The maximum force, $F_m$, and the pulse rise time (and decay time), $t_o$, have been found to depend on the walking speed and on the person's weight, $W$, as shown in Figure 6.4 (Galbraith and Barton 1970).

The dominant footfall-induced motion of a floor typically corresponds to the floor's fundamental mode, whose response may be analyzed by considering that mode as an equivalent spring-mass system. In such a system, the maximum displacement $X_{max}$ of the spring-supported mass due to action of a force pulse like that of Figure 6.3 depends on all of the parameters of the pulse, as well as on the natural frequency $f_n$ of the spring-mass system. The same is true of the ratio $X_{max}$ to the quasi-static displacement $X_{static}$ of the mass ($A_m$ in Figure 6.5), where $X_{static}$ is the displacement of the mass due to a statically applied force of magnitude $F_m$ (Ayre 1961). However, a simple and convenient upper bound to $A_m$ which
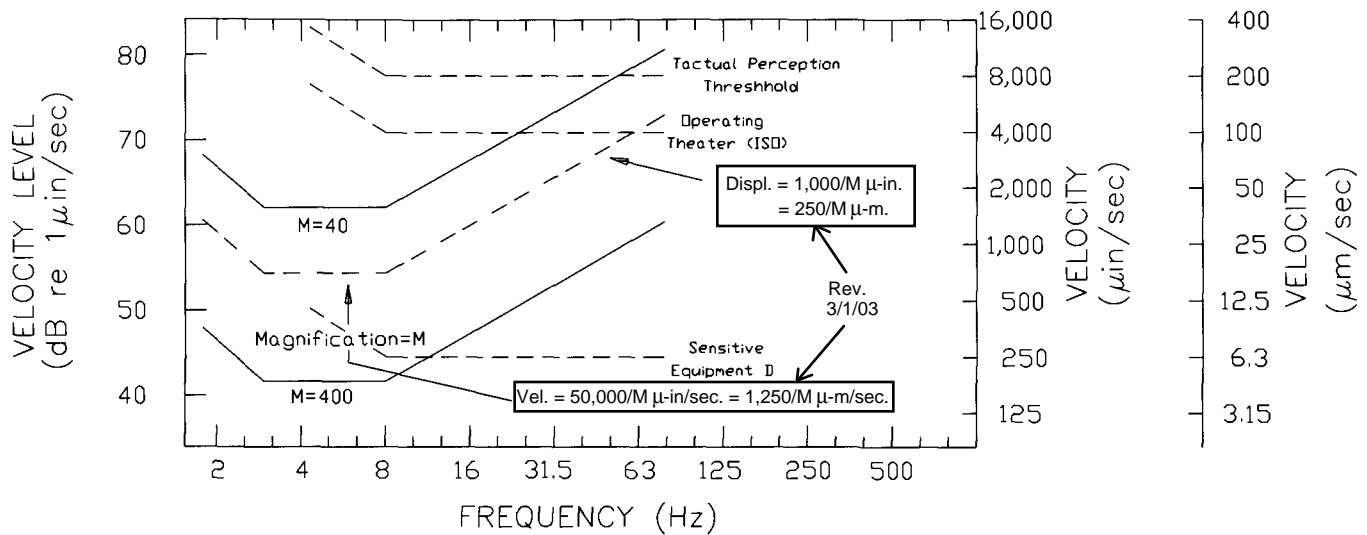


*Fig. 6.2 Suggested criteria for microscopes.*
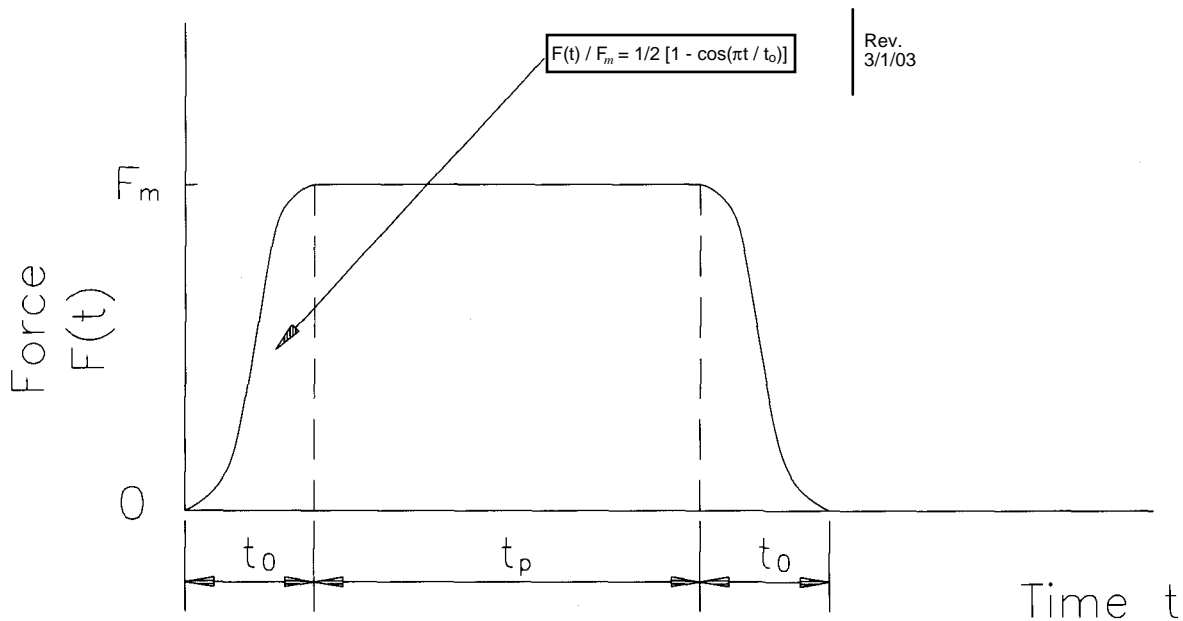


*Fig. 6.3 Idealized footstep force pulse.*

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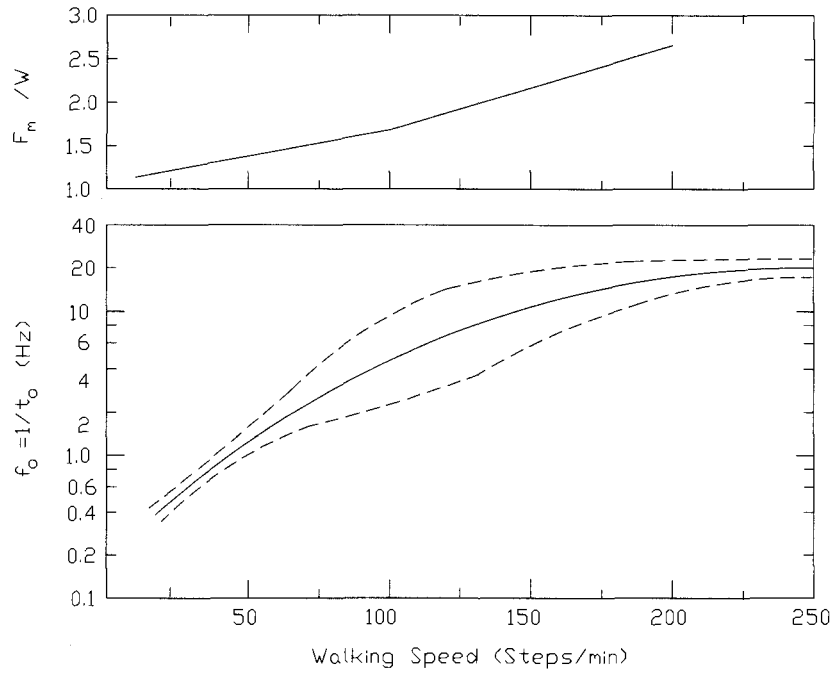
*Fig. 6.4  Dependence of maximum force, $F_m$, and
rise time, $t_o$, of footstep pulse on walking speed
(from Galbraith and Barton, 1970).*

depends only on the product $f_n t_o$ is indicated by the solid curve
of Figure 6.5. For design calculations it suffices to approxi-
mate this upper bound curve by (Ungar and White 1979)

$$A_m = X_{\max} / X_{static} = \begin{cases} 2 & \text{for } f_n t_o \le 0.5 \\[2ex] \dfrac{1}{2(f_n t_o)^2} & \text{for } f_n t_o > 0.5 \end{cases} \qquad (6.2)$$

The second part of this equation is represented by the dashed
curve of Figure 6.5, and the first part corresponds to the upper
left portion of the frame of that figure.

To determine a floor's maximum displacement due to a
footfall impulse, the floor's static displacement $X_{static} = F_m \Delta_P$
due to a point load $F_m$ at the load application point is calcu-
lated, and then Equation (6.2) is applied. Here $\Delta_P$ denotes the
floor's deflection under a unit concentrated load.

The fundamental natural frequency of the floor may be
determined as described in Chapter 3 or by means of finite-
element analysis. The flexibility $\Delta_P$ at the load application
point may be obtained by means of standard static analysis
methods, including finite-element techniques, by assuming
application of a point force at the location of concern, calcu-
lating the resulting deflection at the force application point,
and then determining the ratio of the deflection to the force.
In calculating this deflection, the local deformations of the
slab and deck should be neglected, e.g. only the deflections
of the beams and girders should be considered, taking account
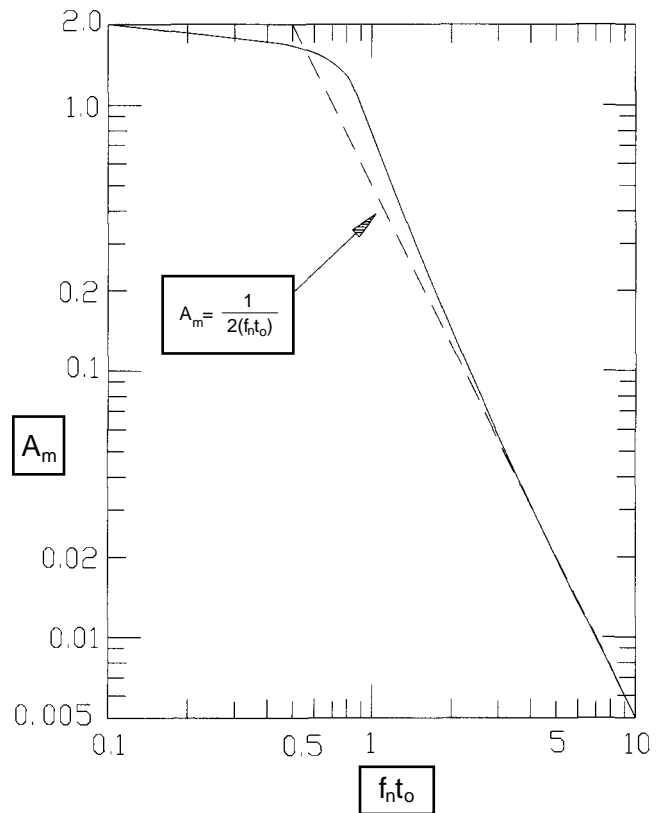


*Fig. 6.5  Maximum dynamic deflection due to footstep pulse.*

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| Table 6.2<br>Values of Footfall Impulse Parameters | | | | |
|---|---|---|---|---|
| Walking Pace<br>steps/minute | $F_m / W$<br>(from Figure 6.4) | $F_m$*<br>$\boxed{kg}$ (lb) | $f_o = 1 / t_o$, Hz<br>(from Figure 6.4) | $U_v$<br>$kN \cdot Hz^2$ (lb$\cdot$ Hz$^2$) |
| 100 (fast) | 1.7 | 1.4 (315) | 5.0 | 110 (25,000) |
| 75 (moderate) | 1.5 | 1.25 (280) | 2.5 | 25 (5,500) |
| 50 (slow) | 1.3 | 1.1 (240) | $\boxed{1.4}$ | 6.8 (1,500) |
| *For $W$= 84 kg (185 lb.) | | | | |

of composite action (see Section 3.2). Equations (4.6), (4.7), and (4.8) can be used to estimate $\Delta_P$ for a unit load at mid-bay.

## 6.3 Application of Criterion

The recommended approach for obtaining a floor that is appropriate for supporting sensitive equipment is to (1) design the floor for a static live loading somewhat greater than the design live load, (2) calculate the expected maximum velocity due to walking-induced vibrations, (3) compare the expected maximum velocity to the appropriate criteria, that is, to velocity limits indicated in Table 6.1 or Figure 6.2 or given by the manufacturer(s) of the equipment, and (4) adjust the floor framing as necessary to satisfy the criterion without over-designing the structure. For the common case where the floor fundamental natural frequency is greater than 5 Hz, the second form of Equation (6.2) applies and the maximum displacement may be expressed as

$$X_{max} = \frac{F_m \Delta_P f_o^2}{2 f_n^2} \tag{6.3}$$

where

$f_o = 1 / t_o$ (see Figure 6.3)

Since the floor vibrates at its natural frequency once it has been deflected by a footfall impulse, the maximum velocity may be determined from,

$$V = 2\pi f_n X_{max} \tag{6.4a}$$

For $f_n / f_o \gg 0.5$,

$$V = U_v \Delta_P / f_n \tag{6.4b}$$

where

$$U_v = \pi F_m f_o^2 \tag{6.5}$$

The parameter $U_v$ has been introduced to facilitate estimation and is a constant for a given walker weight and walking speed. For example, for a 84 kg (185 lb) person walking at a rapid pace of 100 steps minute (which represents a somewhat conservative design condition), from Figure 6.4, $F_m / W = 1.7$ and $F_m = 1.7 (9.81 \times 84) = 1.4$ kN (315 lb), and $f_o = 1 / t_o = 5.0$ Hz. Thus, $U_v = \pi \times 1.4 \times 5.0^2 = 110$ kN $\cdot$ Hz$^2$ (25,000 lb-Hz$^2$).

Table 6.2 shows values of $U_v$ for other 84 kg (185 lb) walker speeds. It is noted that $U_v$, and therefore the expected velocity for a particular floor, for moderate walking speed is about ¼th of that for fast walking and for slow walking is about $\frac{1}{15}$th of that for fast walking.

Rearranging Equation (6.4b) results in the following design criterion

$$\frac{\Delta_P}{f_n} \le \frac{V}{U_v} \tag{6.6}$$

That is, the ratio $\Delta_P / f_n$ should be less than the specified velocity $V$ for the equipment, divided by $U_v$. For example, for the above fast walking condition and a limiting velocity of 25 $\mu$m/sec, $\Delta_P / f_n$ should be less than $V / U_v = 25 \times 10^{-6}/110 = 0.23 \times 10^{-6}$ m/kN-Hz ($1,000 \times 10^{-6}/25,000 = 4 \times 10^{-8}$ in./lb-Hz). For slow walking, $\Delta_P / f_n$ could be permitted to be about 15 times greater, or about $3.7 \times 10^{-8}$ m/kN-Hz ($67 \times 10^{-8}$ in./lb-Hz). Locations where "fast," "moderate," and "slow" walking are expected are discussed later.

Since the natural frequency of a floor is inversely proportional to the square-root of the deflection, $\Delta_p$, due to a unit load, from Equation (6.6) the velocity $V$ is proportional to $\Delta_P^{3/2}$. This proportionality is useful for the approximate evaluation of the effects of minor design changes, because quite significant flexibility (or stiffness) changes can often be accomplished with only minor changes in the structural system. In absence of significant changes in the mass; the change in the stiffness controls the change in the natural frequency, enabling one to estimate how much the flexibility or stiffness of a given floor design needs to be changed to meet a given velocity criterion. If an initial flexibility $\Delta_{P1}$ results in a velocity $V_1$, then the flexibility $\Delta_{P2}$ that will result in a velocity $V_2$ may be found from

$$\frac{\Delta_{P2}}{\Delta_{P1}} \approx (V_2 / V_1)^{2/3} \tag{6-7}$$

For example, if a particular design of a floor is found to result in a walker-induced vibrational velocity of 50 $\mu$m/sec (2,000 $\mu$in/sec) and if the limiting velocity is 12 $\mu$m/sec (500 $\mu$in/sec), the floor flexibility needs to be changed by a factor

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of about $(12/50)^{2/3} \approx 0.4 = 1/2.5$. That is, the floor stiffness needs to be increased by a factor of 2.5.

## 6.4 Additional Considerations

As implied by the foregoing discussion, the primary structural means for reducing the footfall-induced vibrations of a floor consists of reducing its flexibility; i.e., increasing its stiffness. Comparison of the two terms on the right-hand side of Equation (4.8) permits one to determine whether the joists or beams or the girders are the prime contributors to the total flexibility, $\Delta_p$. If the first term is larger, it is primarily the joists or beams that need to be stiffened (that is, $\Delta_{jp}$ needs to be decreased); if the second term is larger, it is primarily the girder that needs to be stiffened (that is, $\Delta_{gp}$ needs to be decreased). Note that increasing the stiffness of the element that already is much stiffer than the other has only a small effect on the combined stiffness. Since the flexibility of a beam or girder varies as the cube of the element's length, a reduction in the relevant span is a very effective means for reducing the element's stiffness, provided that the necessary reduced column spacing is acceptable architecturally. Moment connections tend to have relatively little effect on the stiffness of a floor because these connections typically have relatively little initial stiffness and therefore act much like hinges for very small moments.

It should be noted that in many instances it may not be necessary to increase the stiffness of the entire floor; it often suffices to stiffen only the bay(s) in which sensitive equipment is located.

The methodology presented in the foregoing sections focuses on estimation of footfall-induced vibrations that result in the middle of a bay due to walking in the middle of that bay. Since for a given walking condition mid-bay vibrations due to walking at mid-bay are most severe, a floor that meets the vibration criterion applicable to a given situation for this mid-bay condition may be expected to meet that criterion everywhere. It thus is appropriate to design for this mid-bay condition where possible and where such design does not result in an unreasonable cost penalty.

In many situations, however, sensitive equipment may be situated at other than mid-bay locations. Also, walking—particularly rapid walking, which results in the most severe vibrations—may occur at other locations in the bay that houses the sensitive equipment or outside of that bay. It often is the case that only slow walking can occur in the relatively confined space of a laboratory, with moderate or rapid walking potentially occurring in adjacent corridors. For such situations, the various walking scenarios, as well as the distribution of vibrations over a bay or an array of bays should be considered and the floor should be designed accordingly. To employ this approach one needs to determine the vibration distributions in the floor that result from the walking scenarios of interest. The corresponding analysis is best done by use of a computer model of the floor system. However, by recog-

nizing that the vibrational deflection distribution that corresponds to the fundamental mode of a structure frequently is approximately that obtained with a static load, one may with due care often obtain a reasonable estimate of the vibration distribution by assuming this distribution to be proportional to the distribution of static deflections obtained with point forces at the walker locations.

On the other hand, some guidelines may be derived from simple qualitative considerations. One may readily visualize that footfalls that occur near a column typically will produce lesser vibrations than footfalls nearer the center of a bay. Similarly, for footfalls occurring anywhere in a bay one would expect the portions of a bay near columns to vibrate significantly less than the portions near the bay center. One also may expect vibrations to experience some attenuation as they traverse column lines, as they travel further along a floor, and as they propagate via columns and walls to adjacent floors. These considerations imply some opportunities for mitigating vibration problems by appropriate facility layout.

It is advisable to locate sensitive equipment as far as possible from heavily traveled corridors (particularly from those along which there may occur fast walking which produces comparatively severe vibration excitation). It also is advisable to place sensitive equipment as close to columns as possible. Additionally, it is advisable to locate corridors along column lines and to consider discouraging fast walking, facilitated by a long straight corridor, by dividing such a corridor into a series of shorter ones using obstructions that interfere with rapid walking.

One may also consider reducing the footfall-induced vibrations that reach sensitive equipment by providing separation joints between corridors and areas that house sensitive equipment. Such joints need to be more flexible than simple construction or expansion joints; ideally, they should involve complete structural separation, although a resilient seal may be used, if necessary.

In some situations it may be useful to provide separate structures for the sensitive equipment and for walking. For example, one might support the equipment on a structural floor, but have people walk on a corridor floor structure that is located a foot or so above the structural floor, with the corridor floor structure supported only from the columns and not making direct contact with the structural floor.

In cases where only a few sensitive items are to be located on a given floor, and particularly where the locations of these items are not known or may be changed from time to time, it often may be more cost-efficient to provide these items with special isolation devices than to design the entire floor structure to accommodate their vibration requirements. Suitable isolation devices or systems often are available from the equipment manufacturer/supplier and also may be obtained from specialty suppliers. However, any such device can provide only a limited amount of isolation, and its performance is better if it is used in conjunction with a stiffer structure;

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thus, an isolation system should not be expected to overcome vibration problems resulting from extremely flexible structures.

Unless isolation systems are used, it is important that sensitive equipment be connected rigidly to the structural floor, so that vibrations transmitted to the equipment are not amplified by the flexibility of the intervening structure. It is usually not advisable to support such equipment on a raised "computer" floor, for example, particularly where personnel also can walk on that floor. If it is necessary that this equipment have its base at the level of a raised floor, then this equipment should be provided with a pedestal that connects it rigidly to the structural floor and that it is not in direct contact with the part of the raised floor on which people can walk.

## 6.5 Example Calculations

The following examples illustrate the application of the criterion. The examples are presented first in the SI system of units and then repeated in the US Customary (USC) system of units.

### Example 6.1—SI Units

The floor framing for Example 4.5, shown in Figure 4.5, is to be investigated for supporting sensitive equipment with a velocity limitation of 200 μ$m/sec.$ The floor framing consists of 8.5 m long 30K8 joists at 750 mm on center and supported by 6 m long W760×l34 girders. The floor slab is 65 mm total depth, lightweight weight concrete, on 25 mm deep metal deck. As calculated in Example 4.5, the transformed moment of inertia of the joists is $174 \times 10^6$ mm$^4$ and that of the girders is $1,930 \times 10^6$ mm$^4$. The floor fundamental natural frequency is 9.32 Hz.

The mid-span flexibilities of the joists and girders are

$$\Delta_{oj} = \frac{L_j^3}{48E_sI_j} = \frac{8,500^3}{48 \times 200,000 \times 174 \times 10^6}$$

$$= 368 \times 10^{-6} \text{ mm/kN}$$

$$\Delta_{gP} = \frac{L_g^3}{96E_sI_g} = \frac{6,000^3}{96 \times 200,000 \times 1,930 \times 10^6}$$

$$= 5.83 \times 10^{-6} \text{ mm/kN}$$

(See Section 4.2 for explanation of the use of 1/48 and 1/96 in the above calculations.)

The mid-bay flexibility, using $N_{eff} = 3.08$ from Example 4.5, is

$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_{gP}}{2} = \frac{368 \times 10^{-6}}{3.08} + \frac{5.83 \times 10^{-6}}{2}$$

$$= 122 \times 10^{-6} \text{ mm/kN}$$

Since $f_n/f_o \gg 0.5$ for all values of $f_o$ in Table 6.2, the maximum expected velocity is given by Equation (6.4b). Using

$U_v$ values from Table 6.2, the maximum expected velocity for a 84 kg person walking at 100 steps per minute is

$$V = \frac{U_v\Delta_P}{f_n} = \frac{110 \times 122 \times 10^{-6}}{9.32} = 1,440 \times 10^{-6} \text{ m/sec}$$

$$= 1,440 \text{ μm/sec}$$

that at 75 steps per minute is

$$V = \frac{25 \times 122 \times 10^{-6}}{9.32} = 327 \times 10^{-6}/\text{sec} = 327 \text{ μm/sec}$$

and that at 50 steps per minute is

$$V = \frac{6.8 \times 122 \times 10^{-6}}{9.32} = 89 \times 10^{-6}/\text{sec} = 89 \text{ μm/sec}$$

Thus, the mid-bay location (and all other locations) of this floor is acceptable for the intended use (limiting $V = 200$ μm/sec) if only slow walking is expected. According to Table 6.1, the floor would be acceptable for operating rooms and for bench microscopes with magnifications up to l00× in the presence of only slow walking.

### Example 6.2—USC Units

The floor framing for Example 4.6, shown in Figure 4.6, is to be investigated for supporting sensitive equipment with a velocity limitation of 8,000 μin./sec. The floor framing consists of 28 ft long 30K8 joists at 30 inches on center and supported by 20 ft. long W30×90 girders. The floor slab is 2.5 in. total depth, lightweight weight concrete, on 1-in. deep metal deck. As calculated in Example 4.6, the transformed moment of inertia of the joists is 420 in$^4$. and that of the girders is 4,560 in$^4$. The floor fundamental natural frequency is 9.29 Hz.

The mid-span flexibilities of the joists and girders are

$$\Delta_{oj} = \frac{L_j^3}{48E_sI_j} = \frac{28^3 \times 1,728}{48 \times 29 \times 10^6 \times 420} = 64.9 \times 10^{-6} \text{ in./lb}$$

$$\Delta_{gP} = \frac{L_g^3}{96E_sI_g} = \frac{20^3 \times 1,728}{96 \times 29 \times 10^6 \times 4,560}$$

$$= 1.09 \times 10^{-6} \text{ in./lb}$$

(See Section 4.2 for explanation of the use of 1/48 and 1/96 in the above calculations.)

The mid-bay flexibility, using $N_{eff} = 2.98$ from Example 4.6, is

$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_{gP}}{2} = \frac{64.1 \times 10^{-6}}{2.98} + \frac{1.09 \times 10^{-6}}{2}$$

$$= 22.1 \times 10^{-6} \text{ in./lb}$$

Since $f_n/f_o \gg 0.5$ for all values of $f_o$ in Table 6.2, the maximum expected velocity is given by Equation (6.4b). Using

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$U_v$ values from Table 6.2, the maximum expected velocity for a 185 lb person walking at 100 steps per minute is

$$V = \frac{U_v \Delta_P}{f_n} = \frac{25{,}000 \times 22.1 \times 10^{-6}}{9.29}$$

$$= 0.0595 \text{ in./sec} = 59{,}500 \ \mu\text{in./sec}$$

that at 75 steps per minute is

$$V = \frac{5{,}500 \times 22.1 \times 10^{-6}}{9.29} = 0.0131 \text{ in./sec} = 13{,}100 \ \mu\text{in./sec}$$

and that at 50 steps per minute is

$$V = \frac{1{,}500 \times 22.1 \times 10^{-6}}{9.29} = 0.0036 \text{ in./sec} = 3{,}600 \ \mu\text{in./sec}$$

Thus, the mid-bay location (and all other locations) of this floor is acceptable for the intended use (limiting $V = 8{,}000$ (in./sec) if only slow walking is expected. According to Table 6.1, the floor would be acceptable for operating rooms and for bench microscopes with magnifications up to $100\times$ in the presence of only slow walking.

## Example 6.3—SI Units

The floor system of Example 4.3 is to be evaluated for sensitive equipment use. The floor framing consists of 10.5 m long W460×52 beams, spaced 3 m apart and supported on 9 m long, W530×74 girders. The floor slab is 130 mm total depth, 1,850 kg/m³ concrete on 50 mm deep metal deck. As calculated in Example 4.3, the transformed moment of inertia of the beams is $750 \times 10^6$ mm⁴ and that of the girders is $1{,}348 \times 10^6$ mm⁴. The floor fundamental frequency is 4.15 Hz.

The mid-span flexibilities of the beams and girders are

$$\Delta_{oj} = \frac{L_j^3}{96 E_s I_j} = \frac{(10{,}500)^3}{96 \times 200{,}000 \times 750 \times 10^6}$$

$$= 80.4 \times 10^{-6} \text{ mm/kN}$$

$$\Delta_{gP} = \frac{L_g^3}{96 E_s I_g} = \frac{(9{,}000)^3}{96 \times 200{,}000 \times 1{,}348 \times 10^6}$$

$$= 28.2 \times 10^{-6} \text{ mm/kN}$$

(See Section 4.2 for explanation of the use of 1/96 in the above calculations.)

From Equation (4.7) with $d_e = 80 + 50/2 = 105$ mm, the effective number of tee-beams is

$$N_{eff} = 0.49 + 34.2 \frac{d_e}{S} + (9.0 \times 10^{-9}) \frac{L_j^4}{I_t} - 0.00059 \left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2 \left(\frac{105}{3{,}000}\right) + (9.0 \times 10^{-9}) \frac{(10{,}500)^4}{750 \times 10^6}$$

$$-0.00059 \left(\frac{10{,}500}{3{,}000}\right)^2$$

$$= 1.83$$

Equation (4.7) is applicable since

$$0.018 \le \frac{d_e}{S} = \frac{105}{3{,}000} = 0.035 \le 0.208$$

$$4.5 \times 10^6 \le \frac{L_j^4}{I_j} = \frac{(10{,}500)^4}{750 \times 10^6} = 16.2 \times 10^6 \le 257 \times 10^6$$

$$2 \le \frac{L_j}{S} = \frac{10{,}500}{3{,}000} = 3.5 \le 30$$

The mid-bay flexibility then is

$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_g}{2} = \frac{80.4 \times 10^{-6}}{1.83} + \frac{28.2 \times 10^{-6}}{2}$$

$$= 58.0 \times 10^{-6} \text{ mm/kN}$$

Since $f_n/f_o$ is not $\gg 0.5$ for all values of $f_o$ in Table 6.2, Equation (6.4b) cannot be used and the more general approach is required. For a 84 kg person walking at 100 steps per minute, from Table 6.2, $F_m/W = 1.7$ and $F_m = 1.7 \times (9.81 \times 84) = 1.4$ kN. From Table 6.2, the corresponding pulse rise frequency is $f_o = 5$ Hz; then $f_n/f_o = 4.15/5 \approx 0.8$ for which $A_m = 1.1$ from the solid curve in Figure 6.5. Then, from the definition of $A_m$ in Equation (6.2),

$$X_{max} = A_m F_m \Delta_P = 1.1(1.4)(58.0 \times 10^{-6})$$

$$= 89.3 \times 10^{-6} \text{ m} = 92.1 \ \mu\text{m}$$

and from Equation (6.4)

$$V = 2\pi f_n X_{max} = 2\pi(4.15)(89.3)$$

$$= 2{,}330 \ \mu\text{m/sec}$$

Comparison of this value of the footfall-induced velocity to the criterion values in Table 6.1, indicates that the floor framing is unacceptable for any of the equipment listed in the presence of fast walking.

If slow walking, 50 steps per minute, is considered, then $f_o = \boxed{1.4 \text{ Hz}}$ and $F_m/W = 1.3$ from Table 6.2, thus $F_m = 1.3 \times (9.81 \times 84) = 1.1$ kN. Then $f_n/f_o = \boxed{4.15/1.4 = 2.96}$ and from the equation in Figure 6.5

$$A_m = \frac{1}{2(f_n/f_o)^2} = \frac{1}{\boxed{2(2.96)^2}} = \boxed{0.057}$$

Then

$$X_{max} = A_m F_m \Delta_P = \boxed{0.057}(1.1)(58.0 \times 10^{-6}) = \boxed{3.64} \times 10^{-6} \text{ m}$$

$$= \boxed{3.64} \ \mu\text{m}$$

and from Equation (6.5)

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$$V = 2\pi f_n X_{\max} = 2\pi(4.15)\boxed{(3.64)} = \boxed{94.9} \, \mu\text{m/sec}$$

According to Table 6.1, the mid-bay position of this floor is acceptable for operating rooms and bench microscopes with magnification up to 100×, if only slow walking occurs. Even with only slow walking, the floor would be expected to be unacceptable for precision balances, metrology laboratories or equipment that is more sensitive than these items.

To reduce the mid-bay velocity for fast walking to 200 urn/sec, the floor flexibility needs to be changed by the factor calculated using Equation (6.6):

$$\frac{\Delta_{P2}}{\Delta_{P1}} = (V_2 / V_1)^{2/3} = (200/2{,}330)^{2/3} = 0.195 = 1/5.1$$

That is, the floor mid-bay stiffness needs to be increased by a factor of 5.1. Such a stiffness increase is possible by use of a considerably greater amount of steel or by using shorter spans.

If the beam span is decreased to 7.5 m and the girder span to 6 m, the fundamental natural frequency, $f_n$, is increased to 8.8 Hz, and

$$\Delta_{oj} = \frac{L_j^3}{96EI_b} = \frac{(7{,}500)^3}{96 \times 200{,}000 \times 750 \times 10^6}$$

$$= 29.3 \times 10^{-6} \, \text{mm/kN}$$

$$\Delta_{gP} = \frac{L_j^3}{96EI_b} = \frac{(6{,}000)^3}{96 \times 200{,}000 \times 1{,}348 \times 10^6}$$

$$= 8.35 \times 10^{-6} \, \text{mm/kN}$$

$$N_{eff} = 0.49 + 34.2\frac{d_e}{S} + (9.0 \times 10^{-9})\frac{L_j^4}{I_t} - 0.00059\left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2\left(\frac{105}{3{,}000}\right) + (9.0 \times 10^{-9})\frac{(7{,}500)^4}{750 \times 10^6}$$

$$- 0.00059\left(\frac{7{,}500}{3{,}000}\right)^2 = 1.73$$

$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_{gP}}{2} = \frac{29.3 \times 10^{-6}}{1.73} + \frac{8.35 \times 10^{-6}}{2}$$

$$= 21.1 \times 10^{-6} \, \text{mm/kN}$$

Since $f_n / f_o$ is now much greater than 0.5 for all values of $f_o$ in Table 6.2, the maximum expected velocity is given by Equation (6.4b). Using the $U_v$ value for 100 steps per minute from Table 6.1,

$$V = \frac{U_v \Delta_P}{f_n} = \frac{100 \times 21.1 \times 10^{-6}}{8.8} = 240 \, \mu\text{m/sec}$$

Using the $U_v$ value for 50 steps per minute

$$V = \frac{6.8 \times 21.1 \times 10^{-6}}{8.8} = 16.3 \, \mu\text{m/sec}$$

Comparison of these mid-span velocities with the criterion values of Table 6.1 indicates that the mid-bay location of this floor still is not acceptable for any of the equipment listed in that table if fast walking is considered, but is acceptable for micro-surgery and the use of bench microscopes at magnifications greater than 400× if only slow walking can occur.

### Example 6.4—USC Units

The floor system of Example 4.4 is to be evaluated for sensitive equipment use. The floor framing consists of 35 ft. long W18×35 beams, spaced 10 ft. apart and supported on 30 ft long, W21×50 girders. The floor slab is 5.25 inches total depth, 110 pcf concrete on 2 in. deep metal deck. As calculated in Example 4.4, the transformed moment of inertia of the beams is 1,833 in.[4] and that of the girders is 3,285 in.[4]. The floor fundamental frequency is 4.03 Hz.

The mid-span flexibilities of the beams and girders are

$$\Delta_{oj} = \frac{L_j^3}{96EI_j} = \frac{35^3 \times 1{,}728}{96 \times 29 \times 10^6 \times 1{,}833} = 14.52 \times 10^{-6} \, \text{in./lb}$$

$$\Delta_{gP} = \frac{L_j^3}{96EI_g} = \frac{30^3 \times 1{,}728}{96 \times 29 \times 10^6 \times 3{,}285}$$

$$= 5.10 \times 10^{-6} \, \text{in./lb}$$

(See Section 4.2 for explanation of the use of 1/96 in the above calculations.)

Using Equation (4.7), with $d_e = 3.25 + 2.0/2 = 4.25$ in., the effective number of tee-beams is

$$N_{eff} = 0.49 + 34.2\frac{d_e}{S} + (9.0 \times 10^{-9})\frac{L_j^4}{I_t} - 0.00059\left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2\left(\frac{4.25}{10 \times 12}\right) + (9.0 \times 10^{-9})\frac{(35 \times 12)^4}{1{,}833}$$

$$- 0.00059\left(\frac{35 \times 12}{10 \times 12}\right)^2$$

$$= 1.85$$

Equation (4.7) is applicable since

$$0.018 \leq \frac{d_e}{S} = \frac{4.25}{10 \times 12} = 0.035 \leq 0.208$$

$$4.5 \times 10^6 \leq \frac{L_j^4}{I_j} = \frac{(35 \times 12)^4}{1{,}833} = 17.0 \times 10^6 \leq 257 \times 10^6$$

$$2 \leq \frac{L_j}{S} = \frac{35 \times 12}{10 \times 12} = 3.5 \leq 30$$

The mid-bay flexibility then is

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$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_{gP}}{2} = \frac{14.52 \times 10^{-6}}{1.85} + \frac{5.10 \times 10^{-6}}{2}$$

$$= 10.4 \times 10^{-6} \text{ in./lb}$$

Since $f_n/f_o$ is not $>> 0.5$ for all values of $f_o$ in Table 6.2, Equation (6.4b) cannot be used and the more general approach is required. For a 185 lb person walking at 100 steps per minute, from Table 6.2, $F_m/W = 1.7$ and $F_m = 1.7 \times 185 = 315$ lb. From Table 6.2, the corresponding pulse rise frequency is $f_o = 5$ Hz, then $f_n/f_o = 4.03/5 \approx 0.8$ for which $A_m = 1.1$ from the solid curve in Figure 6.5. Then, from the definition of $A_m$ in Equation (6.1),

$$X_{max} = A_m F_m \Delta_P = 1.1(315)(10.4 \times 10^{-6}) = 3,600 \times 10^{-6} \text{ in.}$$

$$= 3,600 \text{ }\mu\text{in.}$$

and from Equation (6.5)

$$V = 2\pi f_n X_{max} = 2\pi(4.03)(3,600) = 91,160 \text{ }\mu\text{in./sec}$$

Comparison of this value of the footfall-induced velocity to the criterion values in Table 6.1, indicates that the floor framing is unacceptable for any of the equipment listed in the presence of fast walking.

If slow walking, 50 steps per minute, is considered, then $f_o = \boxed{1.4 \text{ Hz}}$ and $F_m/W = 1.3$ from Table 6.2 , thus $F_m = 1.3 \times 185 = 240$ lb. Then $f_n/f_o = \boxed{4.03/1.4 = 2.88}$ and from the equation in Figure 6.5

$$A_m = \frac{1}{2(f_n/f_o)^2} = \frac{1}{\boxed{2(2.88)^2}} = \boxed{0.060}$$

Then

$$X_{max} = A_m F_m \Delta_P = \boxed{0.060}(240)(10.4 \times 10^{-6}) = \boxed{150} \times 10^{-6} \text{ in.}$$

$$= \boxed{150} \text{ }\mu\text{in.}$$

and from Equation (6.5)

$$V = 2\pi f_n X_{max} = 2\pi \boxed{(4.03)(150)} = \boxed{3,800} \text{ }\mu\text{in./sec}$$

According to Table 6.1, the mid-bay position of this floor is acceptable for operating rooms and bench microscopes with magnification up to 100×, if only slow walking occurs. Even with only slow walking, the floor would be expected to be unacceptable for precision balances, metrology laboratories or equipment that is more sensitive than these items.

To reduce the mid-bay velocity for fast walking to 8,000 $\mu$in./sec, from Equation (6.6) the floor flexibility for fast walking needs to be changed by the factor calculated using Equation (6.6):

$$\frac{\Delta_{P2}}{\Delta_{P1}} = (V_2/V_1)^{2/3} = (8,000/91,160)^{2/3} = 0.197 = 1/5.1$$

That is, the floor mid-bay stiffness needs to be increased by a factor of 5.1. Such a stiffness increase is possible by use of a considerably greater amount of steel or by using shorter spans.

If the beam span is decreased to 25 ft and girder span to 20 ft, the fundamental natural frequency, $f_n$, is increased to 8.9 Hz, and

$$\Delta_{oj} = \frac{L_j^3}{96EI_b} = \frac{25^3 \times 1,728}{96 \times 29 \times 10^6 \times 1,833} = 5.29 \times 10^{-6} \text{ in./lb}$$

$$\Delta_{gP} = \frac{L_g^3}{96EI_b} = \frac{20^3 \times 1,728}{96 \times 29 \times 10^6 \times 3,285} = 1.51 \times 10^{-6} \text{ in./lb}$$

$$N_{eff} = 0.49 + 34.2\frac{d_e}{S} + (9.0 \times 10^{-9})\frac{L_j^4}{I_t} - 0.00059\left(\frac{L_j}{S}\right)^2$$

$$= 0.49 + 34.2\left(\frac{4.25}{10 \times 12}\right) + (9.0 \times 10^{-9})\frac{(25 \times 12)^4}{1,833}$$

$$- 0.00059\left(\frac{25 \times 12}{10 \times 12}\right)^2$$

$$= 1.73$$

$$\Delta_P = \frac{\Delta_{oj}}{N_{eff}} + \frac{\Delta_{gP}}{2} = \frac{5.29 \times 10^{-6}}{1.73} + \frac{1.51 \times 10^{-6}}{2}$$

$$= 3.81 \times 10^{-6} \text{ in./lb}$$

Since $f_n/f_o$ is now much greater than 0.5 for all values of $f_o$ in Table 6.2, the maximum expected velocity is given by Equation (6.4b). Using the $U_v$ value for 100 steps per minute from Table 6.1,

$$V = \frac{U_v \Delta_P}{f_n} = \frac{25,000 \times 3.81 \times 10^{-6}}{8.9} = 0.0107 \text{ in./sec}$$

$$= 10,700 \text{ }\mu\text{in./sec}$$

Using the $U_v$ value for 50 steps per minute

$$V = \frac{1,500 \times 3.81 \times 10^{-6}}{8.9} = 642 \times 10^{-6} \text{ in./sec}$$

$$= 642 \text{ }\mu\text{in./sec}$$

Comparison of these mid-span velocities with the criterion values of Table 6.1 indicates that the mid-bay location of this floor still is not acceptable for any of the equipment listed in that table if fast walking is considered, but is acceptable for micro-surgery and the use of bench microscopes at magnifications greater than 400× if only slow walking can occur.

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# Chapter 7
# EVALUATION OF VIBRATION PROBLEMS AND REMEDIAL MEASURES

This Chapter provides guidance on vibration evaluation and on remedial measures to resolve floor vibration problems that can arise in existing buildings.

## 7.1 Evaluation

*When to Evaluate?*

Many vibration problems have been evaluated after they occurred, but the structural engineer should be aware and should advise clients that a change of use, such as the introduction of a health club or of heavy reciprocating machinery, or installation of sensitive equipment, can result in problems which may be difficult to resolve after the fact. It is always advantageous to address potential problems before they occur.

*Source Determination*

It is important, first of all, to determine the source of vibration, be it walking, rhythmic activities, equipment, or sources external to the building that transmit vibration through the ground. For example, annoying vibration in a high rise building was first thought to be caused by an earthquake or by equipment, but was found to result from aerobics on an upper floor.

*Evaluation Approaches*

Possible evaluation approaches are:

- performance tests,
- calculations, and
- vibration measurements.

A *performance test* is particularly useful prior to a change of use of an existing floor. For example, the effect of a contemplated use of a room for aerobics can be evaluated by having typical aerobics performed while people are located in sensitive occupancies to observe the resulting vibration. Two step frequencies should be used, one typically low and the other typically high. Simple walking tests with a few people placed at potential sensitive locations can be carried out for floor or roof areas contemplated for office, residential or other sensitive occupancies.

*Calculations* as described in Chapters 3 to 6 can be used to evaluate the dynamic properties of a structure and to estimate the vibration response caused by dynamic loading from human activities. Calculations, however, may be associated with significant uncertainties and therefore testing is preferable when possible.

*Measurements* can be used to evaluate the dynamic properties of a structure, as well as to quantify the vibrations associated with human activities. Dynamic properties of the structure can be determined by heel impact tests using at least two accelerometers, one in a location of maximum expected vibration, the other(s) elsewhere, including at supports such as girders and columns, as well as other sensitive occupancies of the building. Not only can the dynamic properties of the fundamental mode of vertical vibration be obtained this way, including damping ratio, natural frequency and mode shape, but also the properties of potentially troublesome higher modes. A two-channel FFT analyzer or similar instrument is generally required for these measurements. Acceleration levels during performance tests can also be obtained for comparison to the recommended limits.

Dynamic properties and acceleration levels determined by testing/calculation are needed to design retrofits and/or to make adjustments during a staged retrofit, as described later.

*Design of Retrofit*

Section 7.2 provides guidance on the choice and design of specific remedial measures for a localized vibration problem. If the vibration problem extends over a large floor area or to other floors of the building, a staged approach may be most cost efficient. An example is given later.

## 7.2 Remedial Measures

*Reduction of Effects*

In some situations it may suffice to do nothing about the structural vibration itself, but to use measures that reduce the annoyance associated with the vibration. This includes the elimination of annoying vibration cues such as noise due to rattling, and removing or altering furniture or non-structural components that vibrate in resonance with the floor motion.

*Relocation*

The vibration source (e.g., aerobics, reciprocating equipment) and/or a sensitive occupancy or sensitive equipment may be relocated. It is obviously preferable to do this before the locations are finalized. For example, a planned aerobics exercise facility might be relocated from the top floor of a building to a ground floor or to a stiff floor above an elevator

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shaft. Complaints about walking vibration can sometimes be resolved by relocating one or two sensitive people, activities, or equipment items, e.g., placing these near a column where vibrations are less severe than at mid-bay.

### Reducing Mass

Reducing the mass is usually not very effective because of the resulting reduced inertial resistance to impact or to resonant vibration. Occasionally, however, reducing the mass can increase the natural frequency sufficiently so that resonance is avoided.

### Stiffening

Vibrations due to walking or rhythmic activities can be reduced by increasing the floor natural frequency using the



(a) Additional Columns



(b) Cover Plates and Bottom Chord Reinforcement



(c) Queen Post Hanger Stiffening



(d) Stiffening of Cantilevered Beams and Joist Seats

*Fig. 7.1  Methods for stiffening floors.*

criteria in Chapters 4, 5 and 6. This is best done by increasing structural stiffness.

The structural components with the greatest dynamic flexibility (lowest fundamental frequency) are usually the ones that should be stiffened. For small dynamic loading, such as walking, an evaluation of the floor structural system considering only the girders and joists or beams usually suffices. For severe dynamic loading (e.g. rhythmic exercises, heavy equipment) the evaluation must consider the building structure as a whole, including the columns and possibly the foundations, not just the floor structure.

Some examples of stiffening are shown in Figure 7.1. New column supports down to the foundations between existing ones are most effective for flexible floor structures, Figure 7.1a, but often this approach is not acceptable to the owner. A damping element, such as a friction device or one using visco-elastic material, may absorb some vibrational energy, but recent tests of damping posts showed that their effect was limited to approximately the effective width of the joist panel (see Chapter 4).

Stiffening the supporting joists and girders by adding cover plates or rods as shown in Figure 7.1b is not particularly effective. The addition of rods to the bottom chord of joists is not very effective even if the floor system is jacked-up prior to welding of the rods. Even with jacking, the expected increase in frequency generally does not occur because only the flexural stiffness of the joist is increased, while the effect of deformation due to shear and eccentricity at joints (see Sections 3.5 and 3.6) is unchanged (Band and Murray 1996).

A technique which has been shown to be effective if there is enough ceiling space is to weld or clamp a queen post hanger to the bottom flange of a beam or joist as shown in Figure 7.1c. This arrangement substantially increases the member stiffness. The hanger can be placed around existing ducts and pipes in the ceiling space. Repairs can be carried out at nights or on week-ends by temporarily removing ceiling tiles below each member to be stiffened. The hanger should be prestressed by jacking up the floor before welding (or clamping) the last connection.

Sometimes the troublesome vibration mode involves flexure of vertical members (e.g., structural framing with cantilevers from columns or walls), in which case both horizontal and vertical stiffening will be required. In these situations, it is important to know the shape of the troublesome mode.

If the supporting member is separated from the slab, for example, in the case of overhanging beams which pass over a supporting girder or joist seats supported on the top flange of a girder, the girder can be stiffened as shown in Figure 7.1d. Generally, two to four pieces of the overhanging beam section, placed with their webs in the plane of the web of the girder and attached to both the slab and girder, provide sufficient shear connection for composite action between the slab and the girder. Similarly, composite action may be achieved for girders supporting joist seats by installing short
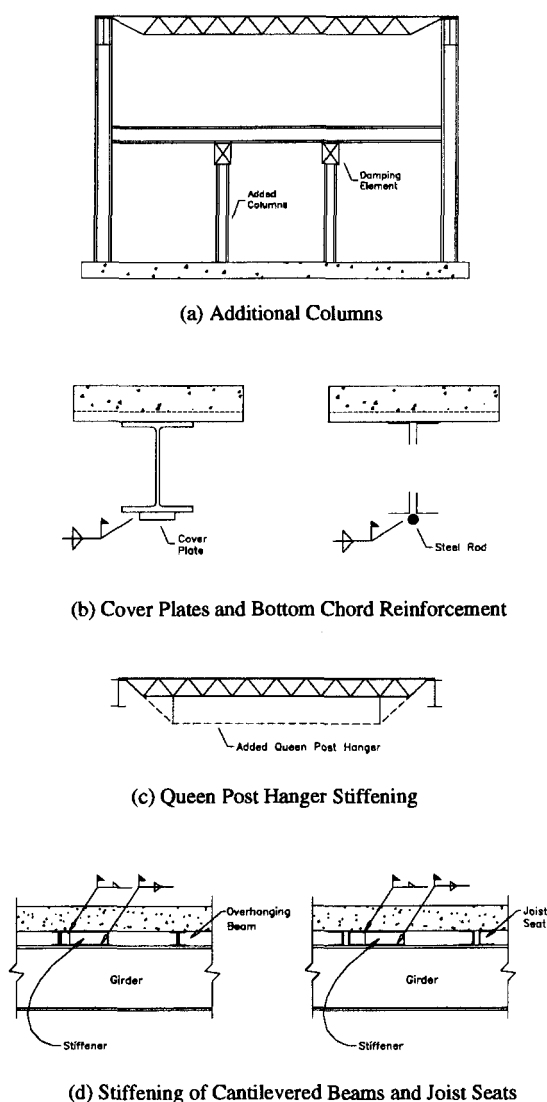
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sections of the joist seat profile as shown in Figure 7.1d. For both cases, the supporting girder should be jacked up prior to installation of the beam or joist seat shear connectors.

## Example

An example of a staged retrofit of an existing floor for walking vibration is shown in Figure 7.2. The floor construction is a concrete deck supported by open-web joists on rolled steel girders that are in turn supported on single-story columns. Unacceptable walking vibrations occurred throughout most of the floor, more so adjacent to the atrium. The problem arose due to the combined flexibility of the joists and girders ($f_n$ = 4.5Hz), the low effective mass (relatively short spans in both directions) and low damping (open floor plan). Heel impact and walking tests were carried out to determine dynamic properties and acceleration levels throughout the floor. To satisfy the design criterion in Section 4.1, both the girders and the joists required stiffening. The floor panel marked A in Figure 7.2 was first stiffened by the queen-post technique of Figure 7.1c and was found to be satisfactory ($f_n$ increased to more than 7 Hz). Then the remainder of stiffening shown in Figure 7.2 was carried out, including the addition of two stiffening posts under the atrium edge girders.

## Damping Increase

Floor vibrations can be improved by increasing the damping of the floor system. The smaller the damping is in the existing floor system, the more effective is the addition of damping. Damping in existing floors depends primarily on the presence of non-structural components, such as partitions, ceilings, mechanical service lines, furnishings and on the number of people on the floor.

The addition of non-structural components which interact with the floor structure, such as dry wall partitions in the ceiling space, provides some added damping. The addition of such partitions in the ceiling space may be beneficial for walking vibrations if the damping of the existing floor system is small. A laboratory test showed that a 1.2 m (4 ft) high double sided, drywall "false" partitions increased the damping in a two-bay test floor by approximately 20 percent. The result was a notable improvement in floor comfort but a completely acceptable floor was not achieved.

## Passive Control

Passive control of floors in the form of tuned mass dampers has been used with varying degrees of success. A tuned mass damper (TMD) is a mass attached to the floor structure through a spring and damping device. The TMD prevents build-up of resonance vibration of the floor by transfer of kinetic energy from the floor into the TMD mass and dissipating some of this kinetic energy via the damping devices. A TMD is effective, however, only if the natural frequency of the TMD nearly matches that of the troublesome mode of floor vibration. The effectiveness of a TMD tuned to the

troublesome mode of vibration can be estimated from the effective damping ratio of the floor-TMD system

$$\beta_s = 0.5 \sqrt{m / M} \qquad (7.1)$$

where

$m$ = mass of the TMD
$M$ = effective mass of the floor when vibrating in its natural mode

Thus, if the mass ratio, $m/M$, is equal to 0.01 the effective damping ratio is 0.05. This can result in a considerable reduction in resonant vibration for a lightly damped floor or footbridge, but little reduction for a floor with many partitions or many people on it, which already is relatively highly damped.

Tuned mass dampers are most effective if there is only one significant mode of vibration (Bachmann and Weber, 1995; Webster and Vaicajtis, 1992). They are much less effective if there are two or more troublesome modes of vibration whose natural frequencies are close to each other (Murray, 1996). They are ineffective for off-resonance vibrations as can occur during rhythmic activities. Finally, TMD's which are initially tuned to floor vibration modes can become out-of-tune due to changes in the floor's natural frequencies resulting from the addition or removal of materials in local areas.

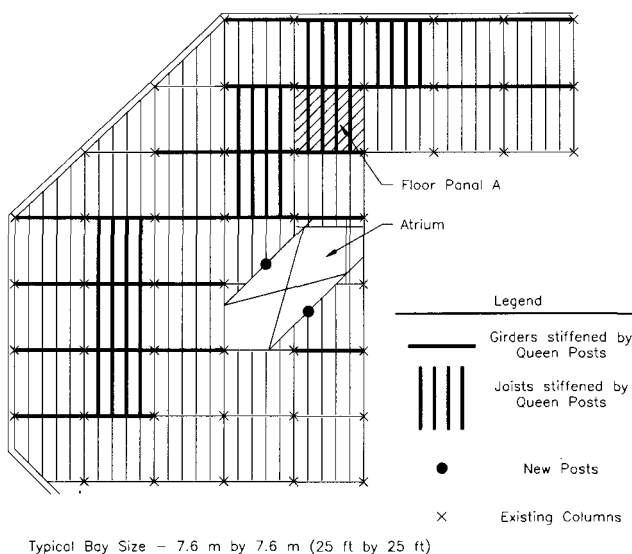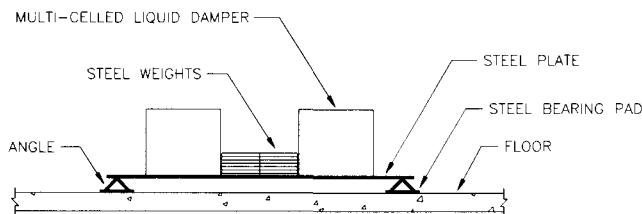To be effective for vibrations from aerobics, the mass of the TMD's must usually be much greater than for walking



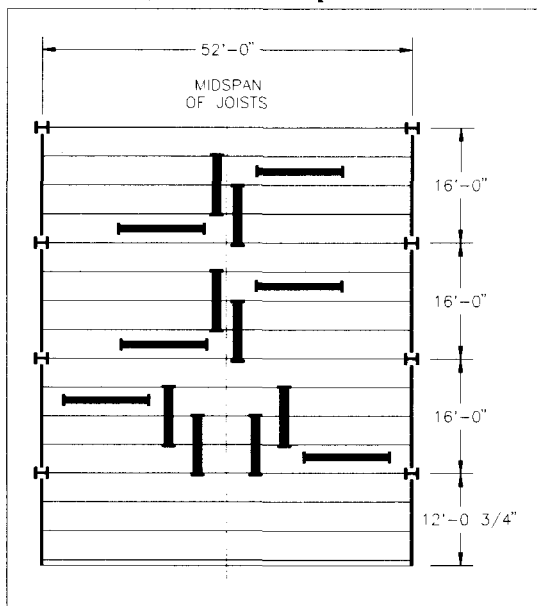*Fig. 7.2 Stiffening an existing floor for walking vibration.*

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vibration. This is because the system damping ratio, $\beta_s$, must usually be much greater to reduce aerobics vibrations in the building to acceptable levels in sensitive occupancies. The people on the floor, including the participants, already provide significant damping to the floor system. TMD's have sometimes proven successful when the effective floor mass is large relative to the number of participants and if the acceleration at resonant vibrations is less than approximately 10 percent gravity (Thornton *et al,* 1990).

### Example

Shope and Murray (1995) report the use of TMD's to improve the vibration characteristics of an existing office floor. Because of complaints of annoying floor motion on the 2nd floor of a new office building, TMD's were installed in three bays of the building (Figure 7.3). The floor system consists of 114 mm (4.5 in.) total depth normal weight concrete on 51 mm (2 in.) metal deck, open web joists and joist girders. The joists are spaced at 1.22 m (48 in.) on-center and span 15.85 m (52 ft.); the joist girders span 4.88 m (16 ft.). Heel-drop impact

tests identified two significant natural frequencies of 5.1 Hz and 6.5 Hz.

To decrease the magnitude of the floor motion, fourteen TMD's were installed. Each damper consisted of a steel plate as the spring and of two stacks of steel plates which were used to adjust the TMD frequency. Damping is provided by multi-celled liquid filled bladders confined in two rigid containers instead of conventional dashpot or damping elements connecting the additional mass to the original structure. (See Figure 7.3a.)

The dampers were located as shown in Figure 7.3b. The dampers oriented perpendicular to the joists were used to control the first mode of vibration (5.1 Hz) and those oriented parallel were used to control the second mode (6.5 Hz). The dampers were first tuned while mounted on a rigid support. After they were attached to the joists, a second tuning was done to improve the performance of the floor.

Figure 7.4 shows acceleration histories for a person walking perpendicular to the joist span before and after installation of the dampers. A significant improvement in the floor re-
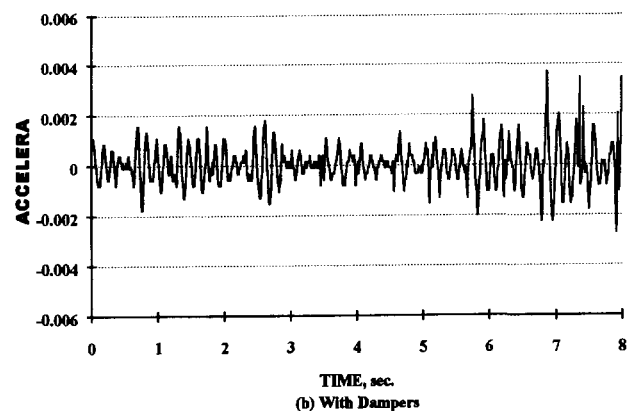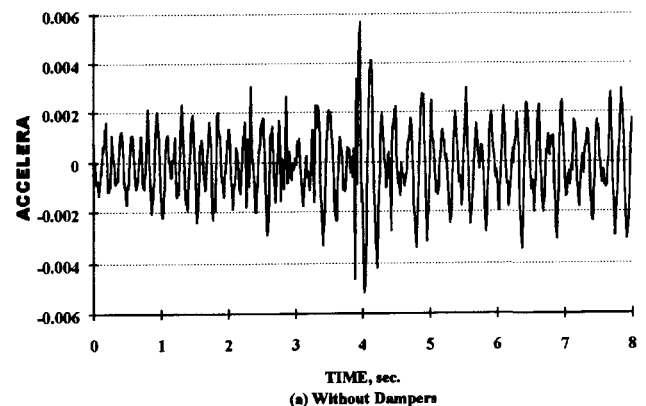


a) Tuned Mass Damper Details



b) Floor Plan

Fig. 7.3 *Office floor controlled using tuned mass dampers.*



(a) Without Dampers



(b) With Dampers

Fig. 7.4 *Office floor walking acceleration histories with and without TMDs.*

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sponse is evident. The response from occupants using the improved floor is reported to be "very positive".

### Reduction of Vibration Transmission

Extremely annoying floor vibrations sometimes occur in large open floor areas where the floor is supported by identical, equally and closely spaced joists or beams, as shown in Figure 7.5. The response of the floor due to a heel-drop type impact is shown in Figure 7.6. The response shown was measured as far as 20 m (65 ft.) from the impact location. This type of response, that is with a "beat" (periodic change in amplitude) of 1-2 seconds, is particularly annoying. The sensation is "wave-like" with waves rolling back and forth across the width of the building. Also, because of the transmission of the vibration, an occupant who is unaware of the cause of the motion is suddenly subjected to significant motion and may be particularly annoyed.

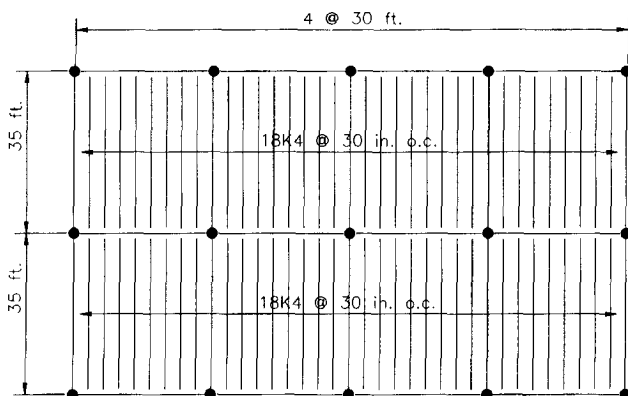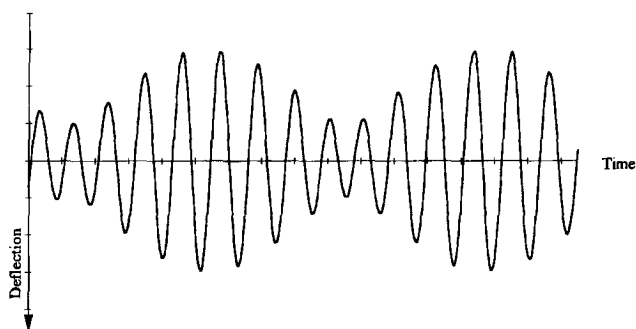Vibration transmission of the type discussed above can be reduced, if not eliminated, by periodically changing the stiffness of some of the joist members, say at the column lines, or by changing the spacing in alternate bays. In a completed structure, stiffening of joists at columns may be a practical way to reduce vibration transmission significantly.

## 7.3 Remedial Techniques in Development

### Active Control

Active control of a structure means the use of controlled energy from an external source to mitigate the motion. Although active control has been used for many years to attenuate lateral wind and earthquake induced motion in multi-story structures, permanent use for floors has not been reported. Hanagan and Murray (1994, 1995) report laboratory experiments and demonstrations using in-situ floors, but no permanent installations. They describe experiments using an electro-magnetic shaker to exert control forces on a floor system, with the shaker controlled in a feedback system via a personal computer. While adding damping to the floor system was the key objective, the collocated rate feedback control law was selected because it is robust to system changes and uncertainties (Hanagan 1994). Figure 7.7 is an illustration of the shaker.

The active control system has been tested on a laboratory floor. Figure 7.8 shows the measured velocity response to a heel-drop impact and the resulting velocity spectrum for the uncontrolled and controlled laboratory floor. Figure 7.9 shows the uncontrolled and controlled response due to walking on the laboratory floor. The active system greatly improved the floor response at the center of the floor; the maximum velocity was reduced approximately by a factor of 10.

Hanagan and Murray (1995) also report the results of tests using the active system on three different in-situ floor sys-
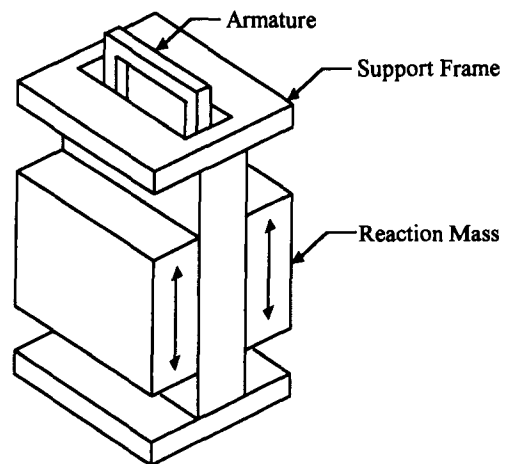


Fig. 7.5 Large open area supported by equally spaced joists.



Fig. 7.6 Floor response with "beat."



Fig. 7.7 Illustration of a reaction mass actuator: electro-magnetic shaker.

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tems. One was a college chemistry laboratory floor, the second was an office floor supported by 6.1 m (20 ft) span joists, and the third was the office floor shown in Figure 7.3. Results for the first two floors were similar to those shown in Figures 7.8 and 7.9. The fundamental frequency of these floors was above 7 Hz. The effectiveness of the active system for the third floor was not as good. It was concluded that the active system is less effective when the fundamental frequency of the floor system is below 5-6.0 Hz.

Active control of floor systems is in a developmental stage. Although the concept has been successfully demonstrated, no permanent installations are known to exist. Two reasons are offered. First is the relatively high initial cost. Hanagan (1994) reports that the cost of her control system for a typical office building bay is US$15-20,000. Second is that active control requires continuous electrical power and periodic maintenance. It is anticipated that costs will decrease rapidly in the near future as shaker development improves, but the maintenance issue is likely to remain.

### *Floating Floor for Rhythmic Activities*

An effective method for reducing building vibration due to machinery is to isolate the machinery from the building by placing the machine on soft springs. This concept can also be used for rhythmic activities by inserting a "floating floor" mass on very soft springs between the participants and the building floor supporting the activity. This idea is attractive for rhythmic activities in the upper stories of buildings, be-

cause it avoids the need to greatly stiffen the building structure, and the floating floor can be introduced when it is needed on an existing floor area and removed when it is no longer required. The increased loading due to the floating floor is offset, at least partly, by the reduced live load transmitted to the building floor. This concept has been used in several buildings in the Eastern United States and further research is underway at Virginia Polytechnic Institute and State University.

### 7.4  Protection of Sensitive Equipment

Remedial measures for reducing the exposure of sensitive equipment to vibrations induced by walking include relocation of equipment to areas where vibrations are less severe, providing vibration isolation devices for the equipment of concern, or implementing structural modifications that reduce the vibrations of floors that support the sensitive equipment. Some of the relevant issues are discussed in Section 6.4.

Equipment that is subject to excessive vibration generally may benefit from being moved to locations near columns. It is usually beneficial to move such equipment to bays in which there are no corridors and which are not directly adjacent to corridors—particularly, to heavily traveled corridors. The most favorable locations for sensitive equipment typically are at grade (that is, on the ground), but on suspended floors the best locations generally are those which are as far as possible from areas where considerable foot traffic can occur.

Vibration isolation devices are readily available for many items of sensitive equipment. These devices typically are resiliently supported platforms, tables, or cradles; the resilient supports generally consist of arrangements of steel springs,
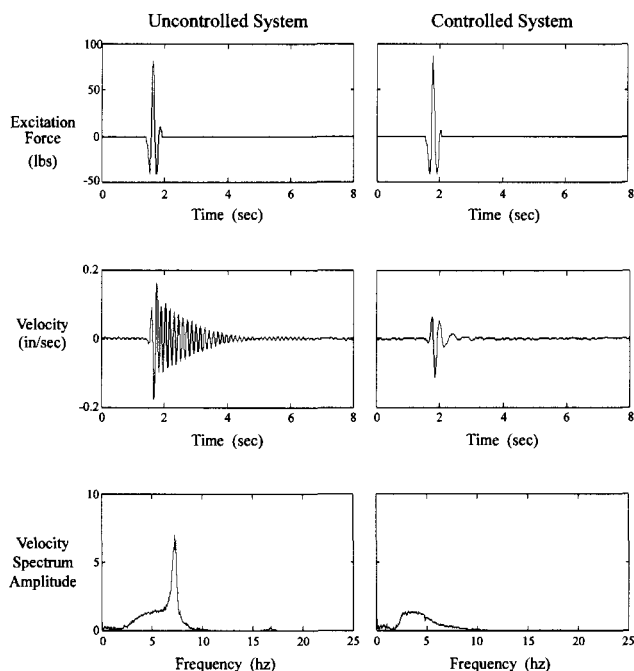


Fig. 7.8  Uncontrolled and actively controlled
floor response to a heel-drop.
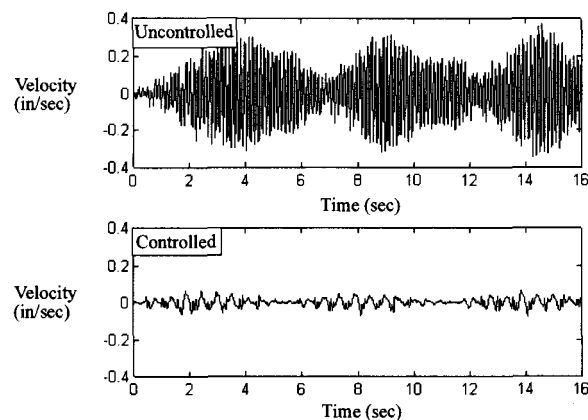


Fig. 7.9  Uncontrolled and actively controlled
floor response to walking excitation.

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of rubber elements, or of "air springs". Isolation systems often are available from the equipment manufacturers and generally can be obtained from suppliers who specialize in vibration isolation. Because selection and/or design of vibration isolation for sensitive equipment involves a number of mechanical considerations and engineering trade-offs, it usually is best left to specialists.

Structural modifications that reduce the vibrations of floors on which sensitive equipment is located include stiffening of the floors of the bays in which the equipment is situated, separating these bays from corridors in which significant walking occurs by the introduction of joints, or providing "walk-on" floors that do not communicate directly with the floors that support the sensitive equipment. Such floors might be "floated" on soft isolation systems or may be supported only at the columns, for example.

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# NOTATION

$a$ = floor acceleration

$a_p$ = peak acceleration

$a_i$ = peak acceleration for the i'th harmonic obtained from Equations (2.4) and (2.5) for use in Equation (2.6)

$a_m$ = effective maximum peak acceleration from Equation (2.6)

$a_o$ = acceleration limit, see Figure 2.1, Table 4.1 or Table 5.1

$A$ = cross sectional area of joist, beam or girder

$A_m$ = maximum dynamic amplification for footstep deflection, see Figure (6.5)

$A_O$ = initial amplitude from a heel-drop impact in Inequality (A.1)

$A_i$ = truss web member area (Section 3.5)

$B$ = effective width in Equation (4.2)

$B_j$ = effective width of joist or beam panel from Equation (4.3a)

$B_g$ = effective width of girder panel from Equation (4.3b)

$C_j$ = constant used in Equation (4.3a)

$C_g$ = constant used in Equation (4.3b)

$C_m$ = constant used in Equation (3.11)

$C_r$ = factor to determine effective moment of inertia of joist and joist girders used in Equations (3.16) and (3.17)

$d$ = depth of joist, beam or girder

$d_e$ = effective depth of slab used in Equations (4.3a) and (4.7)

$D$ = joist depth (Section 3.6); percent critical damping in Inequality A.1

$D_j$ = transformed moment of inertia of joist or beam per unit width used in Equation (4.3)

$D_g$ = transformed moment of inertia of girder per unit width used in Equation (4.3b)

$D_s$ = transformed slab moment of inertia per unit width used in Equation (4.3a)

$\exp$ = exponent of base of natural logarithm e (= 2.71828...)

$E_c$ = modulus of elasticity of concrete from ACI 318 or CSAA23.3

$E_s$ = modulus of elasticity of steel (200,000 MPa or 29 x $10^6$ psi)

$f$ = forcing frequency for rhythmic events used in Equations (2.4), (2.5) and (5.1)

$f_c'$ = compressive strength of concrete

$f_{step}$ = step frequency used in Equation (5.1)

$f_n$ = natural frequency of floor structure for the fundamental mode of vibration

$(f_n)_{req'd}$ = minimum natural frequency required to prevent unacceptable vibrations at each forcing frequency (Inequality 5.1)

$f_o$ = $1/t_o$ used in Section 6.3

$f_j$ = fundamental natural frequency of joist or beam panel

$f_g$ = fundamental natural frequency of girder panel

$F_i$ = truss member axial force due to real loads (Section 3.5)

$F_m$ = maximum footstep force, see Figure (6.3)

$g$ = acceleration due to gravity

$i$ = harmonic multiple of step frequency; member number

$I$ = moment of inertia

$I_B$ = moment of inertia of backspan to cantilever (Section 3.4)

$I_c$ = fully composite moment of inertia of girder used in Equation (3.14)

$I_C$ = moment of inertia of column (Section 3.4)

$I_{chords}$ = moment of inertia of chords of trusses or open-web joists (Sections 3.5, 3.6)

$I_{comp}$ = fully composite moment of inertia used in Equation (3.13) and (3.18)

$I_{eff}$ = effective transformed moment of inertia which accounts for shear deformation of truss or joist used in Equations (3.13) and (3.18)

$I_g$ = moment of inertia of girder; effective moment of inertia of girder in Equation (3.14)

$I_M$ = moment of inertia of main span (Section 3.4)

$I_{mod}$ = effective non-composite moment of inertia of joist or joist girder from Equation (3.15)

$I_{nc}$ = non-composite moment of inertia of girder used in Equation (3.14)

$I_S$ = moment of inertia of side span (Section 3.4)

$I_t$ = transformed moment of inertia; effective transformed moment of inertia if shear deformation are included

$I_T$ = moment of inertia of cantilever (Section 3.4)

$k$ = constant in Equations (2.7) and (5.1)

$k_b$ = $I_B/L_B$ for backspan to cantilever (Section 3.4)

$k_c$ = $I_C/L_C$ for column (Section 3.4)

$k_m$ = $I_M/L_M$ for main span member (Section 3.4)

$k_s$ = $I_S/L_S$ for side span member (Section 3.4)

$L$ = span or length of member between supports

$L_B$ = $L$ for backspan to cantilever (Section 3.4)

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$L_C$ = $L$ for column (Section 3.4)

$L_g$ = girder span

$L_i$ = truss web member length (Section 3.5)

$L_j$ = joist or beam span

$L_M$ = L for main span of horizontal member (Section 3.4)

$L_S$ = $L$ for side span of horizontal member (Section 3.4)

$L_T$ = length of cantilever (Section 3.4)

$m$ = mass of tuned mass damper in Equation (7.2)

$m_c$ = unit mass density of concrete

$M$ = effective mass of floor in Equation (7.2)

$n$ = dynamic modular ratio ($= E_s/1.35E_c$)

$n_c$ = number of connected columns (Section 3.4)

$N_{eff}$ = number of effective joists or beams from Equation (4.7)

$P$ = weight of a person in Equations (1.1) and (2.1)

$P_o$ = force constant in Equations (2.3) and (4.1)

$P_1$ = 1 kN (0.225 kips) concentrated force

$R$ = reduction factor in Equation (2.2), assumed to be 0.7 for footbridges and 0.5 for floors

$S$ = joist or beam spacing

$t$ = time

$t_o$ = pulse rise and decay time (Figure 6.3)

TMD = tuned mass damper

$U_V$ = $\pi F_m f_o^2$ (Section 6.3)

USC = United States Customary Units

$v_i$ = vertical component of web member length change (Section 3.5)

$V$ = velocity (Chapter 6)

$V_1$ = initial velocity in Equation (6.6)

$V_2$ = changed velocity in Equation (6.6)

$w$ = weight per unit area or per unit length (actual, not design)

$w_j$ = weight per unit length of joist or beam

$w_g$ = weight per unit length of girder

$w_p$ = effective weight of people per unit area (Section 5.2)

$w_t$ = effective total weight per unit area of floor (Section 5.2)

$W$ = effective weight supported by the beam or joist panel, girder panel or combined panel using Equation (4.2); weight of walker (Section 6.3)

$W_c$ = additional concentrated weight (Section 5.2)

$W_g$ = effective weight of girder panel

$W_j$ = effective weight of joist or beam panel

$X_{max}$ = maximum displacement (Section 6.2)

$X_{static}$ = static displacement due to a force $F_m$ (Section 6.2)

$y$ = ratio of modal displacement at the location of an additional concentrated weight to maximum modal displacement (Section 5.2)

$y_c$ = distance from top of top chord to center of gravity of open web joist

$\bar{y}$ = distance from bottom of effective slab to center of gravity of composite section

$\alpha_i$ = dynamic coefficient for the i'th harmonic of the step frequency, Equations (1.1), (2.4) and (2.5).

$\beta$ = modal damping ratio

$\beta_s$ = effective damping ratio in Equation (7.2)

$\gamma$ = factor in Equation (3.18)

$\gamma_i$ = angle of truss web member to vertical

$\delta_i$ = truss web member length change (Section 3.5)

$\Delta$ = total deflection due to weight supported used in Equation (3.3); deflection from a concentrated load used in Equation (4.9)

$\Delta_B$ = cantilever backspan deflection (Section 3.4)

$\Delta_c$ = deflection due to shortening of column/pile under weight supported

$\Delta_F$ = deflection of fixed cantilever due to weight supported (Section 3.4)

$\Delta_g$ = deflection of girder due to weight supported

$\Delta_g'$ = reduced deflection of girder due to weight supported, from Equation (4.5)

$\Delta_j$ = deflection of joist or beam due to weight supported

$\Delta_P$ = deflection of floor due to a concentrated force of 1 kN (225 lb.); Equation (4.8), Section 6.2

$\Delta_{P1}$ = initial flexibility in Equation (6.6)

$\Delta_{P2}$ = resulting flexibility in Equation (6.6)

$\Delta gP$ = deflection of girder due to a concentrated force of 1 kN (225 lb.) used in Section 4.2

$\Delta jP$ = deflection of joist or beam due to a concentrated force of 1 kN (225 lb.) used in Section 4.2

$\Delta_{oj}$ = deflection of single joist or beam due to a concentrated force of 1 kN (225 lb.) in Equation (4.10)

$\Delta_{ss}$ = deflection of horizontal member assumed simply-supported at column supports (Section 3.4)

$\Delta_T$ = cantilever deflection (Section 3.4)

$\gamma_i$ = angle of truss web member to vertical (Section 3.5)

$\lambda$ = $(1 + 2L_S/L_M)^2$ in Equation (3.8)

$\phi_i$ = phase angle for the i'th harmonic of the step frequency, Equation (1.1)

$\mu$ = micro ($\times 10^{-6}$)

$\sigma_a$ = axial stress in column due to weight supported

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# Appendix
# HISTORICAL DEVELOPMENT OF ACCEPTANCE CRITERIA

Attempts to quantify the response of humans to floor motion have been made for many years. Three excellent literature reviews have been conducted representing approximately 1,000 papers on the subject of human response to vibration; however, most of the cited research is concerned with ability to perform tasks in presence of steady-state or random vibrations associated with automobiles, ships or airplanes. Very little research has been completed concerning perception of motion of building structures. Nearly all of the work has involved the testing of human response using shaketables or floor motions produced by specific impacts.

Table A.1 is a chronological list of human acceptance criteria for floor vibrations. It includes two types of design criteria: criteria for human response to known or measured vibration, and design criteria related to human response that include an estimation of dynamic floor response. Three of the criteria for office/residential environments have been widely used in North America: the modified Reiher-Meister scale, the CSA Standard and the Murray criterion. Allen's criterion for rhythmic activities and Ellingwood and Tallin's criteria for shopping malls are also frequently used. The ISO Standard forms the basis of several European criteria and for criteria presented in Chapters 4 through 6 of this Design Guide. As background for understanding the evolution of acceptance criteria, a brief description of selected criteria follows. Damping ratios cited in the following are from impact decay measurements (logarithmic decrement calculations, Figure 1.2) and therefore include attenuation due to vibration transmission.

## Modified Reiher-Meister Scale.

Reiher and Meister (1931) in the early 1930's subjected a group of standing people to steady-state vibrations with frequencies of 5 to 100 Hz and amplitudes of 0.01 mm (0.0004 in.) to 10 mm (0.40 in.) and noted the subjects' reactions in ranges from "barely perceptible" to "intolerable". After studying a number of steel joist-concrete slab floor systems, Lenzen (1966) suggested that the original Reiher-Meister scale is applicable to floor systems with less than 5 percent critical damping if the amplitude scale is increased by a factor of ten. The resulting modified Reiher-Meister scale is shown in Figure A.1. Lenzen did not suggest limits on frequency or amplitude to assure acceptable floors. Murray (1975), after testing and analyzing numerous steel beam-concrete slab floors, suggested that systems with 4 to 10 percent critical damping which "plot above the upper one-half of the distinctly perceptible range will result in complaints from the

occupants, and systems in the strongly perceptible range will be unacceptable to both occupants and owners". Both Lenzen and Murray used a single impact to excite the floor systems: Lenzen used both a mechanical impactor and heel-drop impacts; Murray used only the heel-drop impact. The recommendations of Murray are based on the heel-drop impact and should not be used with any other types of impact.

McCormick (1974) presented a study of design criteria and tests for office floor vibrations, aimed at developing criteria to be used in design of two new steel-framed office towers. After reviewing some literature and performing tests on mockups for the proposed buildings, McCormick concluded that floor systems in which damping exceeds 3 percent should prove acceptable if they plot in or below the lower third of the distinctly perceptible range, although vibrations caused by normal use may be perceptible to the occupants. McCormick also suggested that a higher limit should be acceptable if damping exceeds about 10 percent.
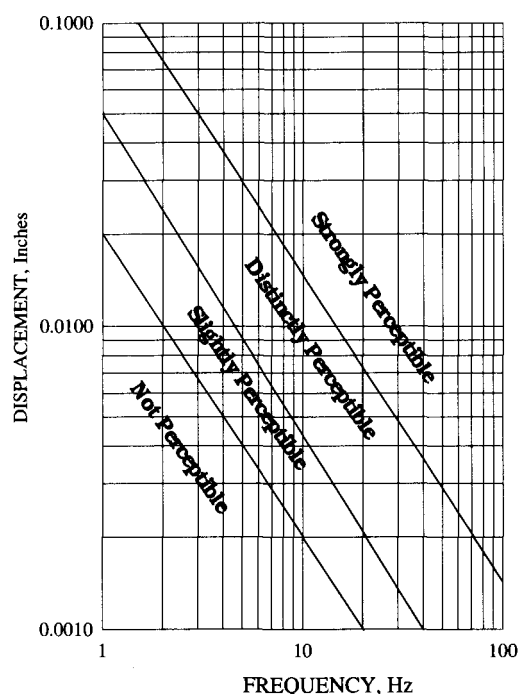


Fig. A.1  Modified Reiher-Meister Scale.

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| Table A.1 Acceptance Criteria Over Time | | | | |
|---|---|---|---|---|
| Date | Reference | Loading | Application | Comments |
| 1931 | Reiher and Meister | Steady State | General | Human response criteria |
| 1966 | Lenzen | Heel-drop | Office | Design criterion using Modified Reiher and Meister scale |
| 1970 | HUD | Heel-drop | Office | Design criterion for manufactured housing |
| 1974 | International Standards Organization | Various | Various | Human response criteria |
| 1974 | Wiss and Parmelee | Footstep | Office | Human response criteria |
| 1974 | McCormick | Heel-drop | Office | Design criterion using Modified Reiher and Meister scale |
| 1975 | Murray | Heel-drop | Office | Design criterion using Modified Reiher and Meister scale |
| 1976 | Allen and Rainer | Heel-drop | Office | Design criterion using modified ISO scale |
| 1981 | Murray | Heel-drop | Office | Design criterion based on experience |
| 1984 | Ellingwood and Tallin | Walking | Commercial | Design criterion |
| 1985 | Allen, Rainer and Pernica | Crowds | Auditorium | Design criterion related to ISO scale |
| 1986 | Ellingwood et al | Walking | Commercial | Design criterion |
| 1988 | Ohlsson | Walking | Residential/Office | Lightweight Floors |
| 1989 | International Standard ISO 2231-2 | Various | Buildings | Human response criteria |
| 1989 | Clifton | Heel-drop | Office | Design criterion |
| 1989 | Wyatt | Walking | Office/Residential | Design criterion based on ISO 2631-2 |
| 1990 | Allen | Rhythmic | Gymnasium | Design criterion for aerobics |
| 1993 | Allen and Murray | Walking | Office/Commercial | Design criterion using ISO 2631-2 |

## CSA Scale

A human response scale based on the work of Allen and Rainer (1976) is presented in Appendix G of the Canadian Standards Association Standard, CSAS16.1 (CSA 1989), to quantify the annoyance threshold for floor vibrations in residential, school, and office occupancies due to "footsteps". This scale is shown in Figure A.2. A design formula to estimate acceleration to be used with the heel-drop criteria is included in the standard. The scale was developed with data from tests on 42 long span floor systems, combined with subjective evaluation by occupants or researchers.

## Murray's Criterion

Murray (1981) recommended that floor systems designed to support office or residential environments satisfy

$$D > 35A_o f_n + 2.5 \qquad (A.1)$$

where

$D$ = percent of critical damping
$A_o$ = initial amplitude from a heel-drop impact (in.)
$f_n$ = first natural frequency (Hz)

Guidelines for estimating the three parameters are found in Murray (1991).

## ISO Scale

The International Organization for Standardization's standard ISO 2631-2:1989 (International Standard 1989) is written to cover many building vibration environments. The standard presents acceleration limits for mechanical vibrations as a function of exposure time and frequency, for both longitudinal and transverse directions of persons in standing, sitting, and lying positions.

Limits for different occupancies are given in terms of root mean square (rms) acceleration as multiples of the "baseline" curve shown in Figure A.3. For offices, ISO recommends a multiplier of 4 for continuous or intermittent vibrations and

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multipliers of 60 to 128 for transient vibrations. Intermittent vibration is defined as a string of vibration incidents such as those caused by a pile driver, whereas transient vibration is defined as rare, widely separated events, such as blasting. Walking vibration is intermittent in nature but not as frequent and repetitive as vibration caused by a pile driver.

### Ellingwood and Tallin's Criterion for Shopping Malls

Ellingwood and Tallin (1984) and Ellingwood et al. (1986) recommended a criterion for commercial floor design based on an acceleration tolerance limit of 0.005g and walking excitation. The criterion is satisfied if the maximum deflection under a 2 kN (450 lbs.) force applied anywhere on the floor system does not exceed 0.5 mm (0.02 in.), that is a stiffness of 4 kN/mm (22.5 k/in.).

### European Criteria

European acceptance criteria are generally more stringent than North American criteria, probably because of the traditional use of poured-in-place concrete floors with short spans. For instance, Bachman and Ammann (1987) recommend that concrete slab-steel framed floor systems have a first natural frequency of at least 9 Hz. Most steel framed floor systems in North American office buildings have first natural frequencies in the 5-9 Hz range, yet, the vast majority of these floors are acceptable to the occupants. Since frequency is proportional to the square root of moment of inertia, a substantial amount of material is required to satisfy the 9.0 Hz criterion. Wyatt (1983), however, has recently proposed design criteria for walking vibration which are similar to those recommended in this Design Guide for fundamental natural frequencies less than 7 Hz. His recommendations are more conservative than those in this Design Guide for higher fundamental natural frequencies. Ohlsson (1988) has proposed criteria for light-weight floor systems. He recommends that light-weight floor systems not be designed with fundamental frequencies lower than 8 Hz.

### Allen's Criteria for Rhythmic Activities

Allen (1990) presented specific guidelines for the design of floor systems supporting aerobic activities. He recommended that such floor systems be designed so that the fundamental natural frequency is greater than the forcing frequency of the highest harmonic of the step frequency that produces significant dynamic load. This criterion is explained in more detail in Section 2.2.2.
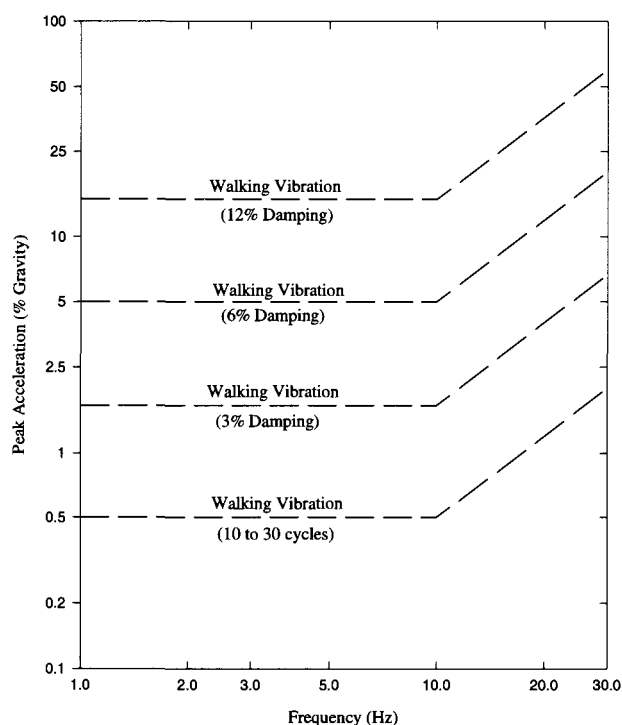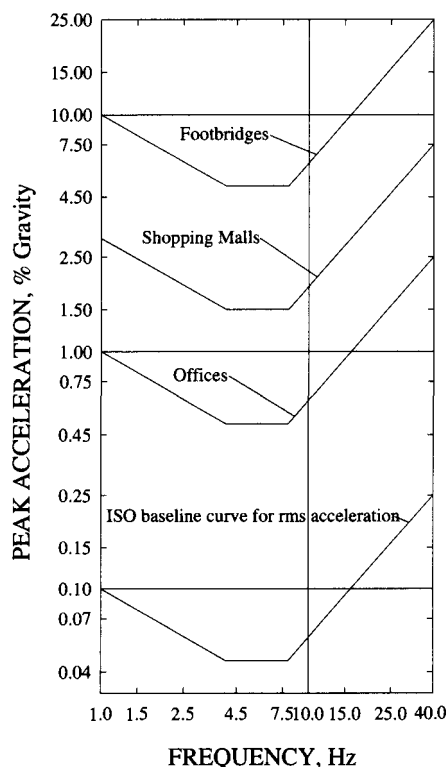


*Fig. A.2  Canadian Standards Association scale.*



*Fig. A.3  International Standards Association Scale.*

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**Revisions and Errata List**
**AISC Steel Design Guide 11, 1$^{st}$ Printing, Revision October 2003 (Digital Edition)**
**October 15, 2012**

The following list represents corrections to the revision dated October 2003 of AISC Design Guide 11, *Floor Vibrations Due to Human Activity.*

**Page(s)**        **Item**

46                In Table 6.1 the first two rows that read:

| | | |
|---|---|---|
| Computer Systems; Operating Rooms**; Surgery; Bench microscopes at up to 100x magnification; | 8,000 | 200 |
| Laboratory Robots | 4,000 | 100 |

should be replaced with:

| | | |
|---|---|---|
| Residences**; Computer Systems | 8,000 | 200 |
| Operating rooms; Surgery; Bench Microscopes up to 100x magnification; Laboratory Robots | 4,000 | 100 |