# SECTION 4: STRUCTURAL ANALYSIS AND EVALUATION

## TABLE OF CONTENTS

# STRUCTURAL ANALYSIS AND EVALUATION

## 4.1 SCOPE

This section describes methods of analysis suitable for the design and evaluation of bridges and is limited to the modeling of structures and the determination of force effects.

Other methods of analysis that are based on documented material characteristics and that satisfy equilibrium and compatibility may also be used.

In general, bridge structures are to be analyzed elastically. However, this section permits the inelastic analysis or redistribution of force effects in some continuous beam superstructures. It specifies inelastic analysis for compressive members behaving inelastically and as an alternative for extreme event limit states.

## C4.1

This section identifies and promotes the application of methods of structural analysis that are suitable for bridges. The selected method of analysis may vary from the approximate to the very sophisticated, depending on the size, complexity, and **priority** of the structure. The primary objective in the use of more sophisticated methods of analysis is to obtain a better understanding of structural behavior. Such improved understanding may often, but not always, lead to the potential for saving material.

The outlined methods of analysis, which are suitable for the determination of deformations and force effects in bridge structures, have been successfully demonstrated, and most have been used for years. Although many methods will require a computer for practical implementation, simpler methods that are amenable to hand calculation and/or to the use of existing computer programs based on line-structure analysis have also been provided. Comparison with hand calculations should always be encouraged and basic equilibrium checks should be standard practice.

With rapidly improving computing technology, the more refined and complex methods of analysis are expected to become commonplace. Hence, this section addresses the assumptions and limitations of such methods. It is important that the user understand the method employed and its associated limitations.

In general, the suggested methods of analysis are based on linear material models. This does not mean that cross-sectional resistance is limited to the linear range. This presents an obvious inconsistency in that the analysis is based on material linearity and the resistance model may be based on inelastic behavior for the strength limit states. This same inconsistency existed, however, in the load factor design method of previous editions of the AASHTO Standard Specifications, and is present in design codes of other nations using a factored design approach.

The loads and load factors, defined in Section 3, and the resistance factors specified throughout these Specifications were developed using probabilistic principles combined with analyses based on linear material models. Hence, analysis methods based on material nonlinearities to obtain force effects that are more realistic at the strength limit states and subsequent economics that may be derived are permitted only where explicitly outlined herein.

Some nonlinear behavioral effects are addressed in both the analysis and resistance sections. For example, long column behavior may be modeled via geometric nonlinear methods and may also be modeled using approximate formulae in Sections 5, 6, 7, and 8. Either method may be used, but the more refined formulations are recommended.

## 4.2 DEFINITIONS

*Accepted Method of Analysis*—A method of analysis that requires no further verification and that has become a regular part of structural engineering practice.

*Arc Span*—Distance between centers of adjacent bearings, or other points of support, measured horizontally along the centerline of a horizontally curved member.

*Aspect Ratio*—Ratio of the length to the width of a rectangle.

*Boundary Conditions*—Structural restraint characteristics regarding the support for and/or the continuity between structural models.

*Bounding*—Taking two or more extreme values of parameters to envelop the response with a view to obtaining a conservative design.

*Classical Deformation Method*—A method of analysis in which the structure is subdivided into components whose stiffness can be independently calculated. Equilibrium and compatibility among the components is restored by determining the deformations at the interfaces.

*Classical Force Method*—A method of analysis in which the structure is subdivided into statically determinate components. Compatibility among the components is restored by determining the interface forces.

*Closed-Box Section*—A cross-section composed of two vertical or inclined webs which has at least one completely enclosed cell. A closed-section member is effective in resisting applied torsion by developing shear flow in the webs and flanges.

*Closed-Form Solution*—One or more equations, including those based on convergent series, that permit calculation of force effects by the direct introduction of loads and structural parameters.

*Compatibility*—The geometrical equality of movement at the interface of joined components.

*Component*—A structural unit requiring separate design consideration; synonymous with member.

*Condensation*— Relating the variables to be eliminated from the analysis to those being kept to reduce the number of equations to be solved.

*Core Width*—The width of the superstructure of monolithic construction minus the deck overhangs.

*Cross-Section Distortion*—Change in shape of the cross-section profile due to torsional loading.

*Curved Girder*—An I-, closed-box, or tub girder that is curved in a horizontal plane.

*Damper*—A device that transfers and reduces forces between superstructure elements and/or superstructure and substructure elements, while permitting thermal movements. The device provides damping by dissipating energy under seismic, braking, or other dynamic loads.

*Deck*—A component, with or without wearing surface, directly supporting wheel loads.

*Deck System*—A superstructure in which the deck is integral with its supporting components or in which the effects or deformation of supporting components on the behavior of the deck is significant.

*Deformation*—A change in structural geometry due to force effects, including axial displacement, shear displacement, and rotations.

*Degree-of-Freedom*—One of a number of translations or rotations required to define the movement of a node. The displaced shape of components and/or the entire structure may be defined by a number of degrees-of-freedom.

*Design*—Proportioning and detailing the components and connections of a bridge to satisfy the requirements of these Specifications.

*Dynamic Degree-of-Freedom*—A degree-of-freedom with which mass or mass effects have been associated.

*Elastic*—A structural material behavior in which the ratio of stress to strain is constant, the material returns to its original unloaded state upon load removal.

*Element*—A part of a component or member consisting of one material.

*End Zone*—Region of structures where normal beam theory does not apply due to structural discontinuity and/or distribution of concentrated loads.

*Equilibrium*—A state where the sum of forces and moments about any point in space is 0.0.

*Equivalent Beam*—A single straight or curved beam resisting both flexural and torsional effects.

*Equivalent Strip*—An artificial linear element, isolated from a deck for the purpose of analysis, in which extreme force effects calculated for a line of wheel loads, transverse or longitudinal, will approximate those actually taking place in the deck.

*Finite Difference Method*—A method of analysis in which the governing differential equation is satisfied at discrete points on the structure.

*Finite Element Method*—A method of analysis in which a structure is discretized into elements connected at nodes, the shape of the element displacement field is assumed, partial or complete compatibility is maintained among the element interfaces, and nodal displacements are determined by using energy variational principles or equilibrium methods.

*Finite Strip Method*—A method of analysis in which the structure is discretized into parallel strips. The shape of the strip displacement field is assumed and partial compatibility is maintained among the element interfaces. Model displacement parameters are determined by using energy variational principles or equilibrium methods.

*First-Order Analysis*—Analysis in which equilibrium conditions are formulated on the undeformed structure; that is, the effect of deflections is not considered in writing equations of equilibrium.

*Flange Lateral Bending Stress*—The normal stress caused by flange lateral bending.

*Folded Plate Method*—A method of analysis in which the structure is subdivided into plate components, and both equilibrium and compatibility requirements are satisfied at the component interfaces.

*Footprint*—The specified contact area between wheel and roadway surface.

*Force Effect*—A deformation, stress, or stress resultant, i.e., axial force, shear force, flexural, or torsional moment, caused by applied loads, imposed deformations, or volumetric changes.

*Foundation*—A supporting element that derives its resistance by transferring its load to the soil or rock supporting the bridge.

*Frame Action*—Transverse continuity between the deck and the webs of cellular cross-section or between the deck and primary components in large bridges.

*Frame Action for Wind*—Transverse flexure of the beam web and that of framed stiffeners, if present, by which lateral wind load is partially or completely transmitted to the deck.

*Girder Radius*—The radius of the circumferential centerline of a segment of a curved girder.

*Global Analysis*—Analysis of a structure as a whole.

*Governing Position*—The location and orientation of transient load to cause extreme force effects.

*Grillage Analogy Method*—A method of analysis in which all or part of the superstructure is discretized into orthotropic components that represent the characteristics of the structure.

*Inelastic*—Any structural behavior in which the ratio of stress and strain is not constant, and part of the deformation remains after load removal.

*Lane Live Load*—The combination of tandem axle and uniformly distributed loads or the combination of the design truck and design uniformly distributed load.

*Large Deflection Theory*—Any method of analysis in which the effects of deformation upon force effects is taken into account.

*Lateral Flange Bending*—Bending of a flange about an axis perpendicular to the flange plane due to lateral loads applied to the flange and/or nonuniform torsion in the member.

*Lever Rule*—The statical summation of moments about one point to calculate the reaction at a second point.

*Linear Response*—Structural behavior in which deflections are directly proportional to loads.

*Local Analysis*—An in-depth study of strains and stresses in or among components using force effects obtained from a more global analysis.

*Member*—Same as *Component*.

*Method of Analysis*—A mathematical process by which structural deformations, forces, and stresses are determined.

*Model*—A mathematical or physical idealization of a structure or component used for analysis.

*Monolithic Construction*—Single cell steel and/or concrete box bridges, solid or cellular cast-in-place concrete deck systems, and decks consisting of precast, solid, or cellular longitudinal elements effectively tied together by transverse post-tensioning.

*M/R Method*—An approximate method for the analysis of curved box girders in which the curved girder is treated as an equivalent straight girder to calculate flexural effects and as a corresponding straight conjugate beam to calculate the concomitant St. Venant torsional moments due to curvature.

*Negative Moment*—Moment producing tension at the top of a flexural element.

*Node*—A point where finite elements or grid components meet; in conjunction with finite differences, a point where the governing differential equations are satisfied.

*Nonlinear Response*—Structural behavior in which the deflections are not directly proportional to the loads due to stresses in the inelastic range, or deflections causing significant changes in force effects, or by a combination thereof.

*Nonuniform Torsion*—An internal resisting torsion in thin-walled sections, also known as warping torsion, producing shear stress and normal stresses, and under which cross-sections do not remain plane. Members resist the externally applied torsion by warping torsion and St. Venant torsion. Each of these components of internal resisting torsion varies along the member length, although the externally applied concentrated torque may be uniform along the member between two adjacent points of torsional restraint. Warping torsion is dominant over St. Venant torsion in members having open cross-sections, whereas St. Venant torsion is dominant over warping torsion in members having closed cross-sections.

*Open Section*—A cross-section which has no enclosed cell. An open-section member resists torsion primarily by nonuniform torsion, which causes normal stresses at the flange tips.

*Orthotropic*—Perpendicular to each other, having physical properties that differ in two or more orthogonal directions.

*Panel Point*—The point where centerlines of members meet, usually in trusses, arches, cable-stayed, and suspension bridges.

*Pin Connection*—A connection among members by a notionally frictionless pin at a point.

*Pinned End*—A boundary condition permitting free rotation but not translation in the plane of action.

*Point of Contraflexure*—The point where the sense of the flexural moment changes; synonymous with point of inflection.

*Positive Moment*—Moment producing tension at the bottom of a flexural element.

*Primary Member*—A member designed to carry the loads applied to the structure as determined from an analysis.

*Rating Vehicle*—A sequence of axles used as a common basis for expressing bridge resistance.

*Refined Methods of Analysis*— Methods of structural analysis that consider the entire superstructure as an integral unit and provide the required deflections and actions.

*Restrainers*—A system of high-strength cables or rods that transfers forces between superstructure elements and/or superstructure and substructure elements under seismic or other dynamic loads after an initial slack is taken up, while permitting thermal movements.

*Rigidity*—Force effect caused by a corresponding unit deformation per unit length of a component.

*Secondary Member*—A member in which stress is not normally evaluated in the analysis.

*Second-Order Analysis*—Analysis in which equilibrium conditions are formulated on the deformed structure; that is, in which the deflected position of the structure is used in writing the equations of equilibrium.

*Series or Harmonic Method*—A method of analysis in which the load model is subdivided into suitable parts, allowing each part to correspond to one term of a convergent infinite series by which structural deformations are described.

*Shear Flow*—Shear force per unit width acting parallel to the edge of a plate element.

*Shear Lag*—Nonlinear distribution of normal stress across a component due to shear distortions.

*Shock Transmission Unit (STU)*—A device that provides a temporary rigid link between superstructure elements and/or superstructure and substructure elements under seismic, braking, or other dynamic loads, while permitting thermal movements.

*Skew Angle*—Angle between the centerline of a support and a line normal to the roadway centerline.

*Small Deflection Theory*—A basis for methods of analysis where the effects of deformation upon force effects in the structure is neglected.

*Spacing of Beams*—The center-to-center distance between lines of support.

*Spread Beams*—Beams not in physical contact, carrying a cast-in-place concrete deck.

*Stiffness*—Force effect resulting from a unit deformation.

*Strain*—Elongation per unit length.

*Stress Range*—The algebraic difference between extreme stresses.

*St. Venant Torsion*—That portion of the internal resisting torsion in a member producing only pure shear stresses on a cross-section; also referred to as pure torsion or uniform torsion.

*Submodel*—A constituent part of the global structural model.

*Superimposed Deformation*—Effect of settlement, creep, and change in temperature and/or moisture content.

*Superposition*—The situation where the force effect due to one loading can be added to the force effect due to another loading. Use of superposition is only valid when the stress-strain relationship is linearly elastic and the small deflection theory is used.

*Tandem*—Two closely spaced and mechanically interconnected axles of equal weight.

*Through-Thickness Stress*—Bending stress in a web or box flange induced by distortion of the cross-section.

*Torsional Shear Stress*—Shear stress induced by St. Venant torsion.

*Tub Section*—An open-topped section which is composed of a bottom flange, two inclined or vertical webs, and top flanges.

*Uncracked Section*—A section in which the concrete is assumed to be fully effective in tension and compression.

*V-Load Method*—An approximate method for the analysis of curved I-girder bridges in which the curved girders are represented by equivalent straight girders and the effects of curvature are represented by vertical and lateral forces applied at cross-frame locations. Lateral flange bending at brace points due to curvature is estimated.

*Warping Stress*—Normal stress induced in the cross-section by warping torsion and/or by distortion of the cross-section.

*Wheel Load*—One-half of a specified design axle load.

*Yield Line*—A plastic hinge line.

*Yield Line Method*—A method of analysis in which a number of possible yield line patterns are examined in order to determine load-carrying capacity.

## 4.3 NOTATION

$A$ = area of a stringer, beam, or component (in.$^2$) (4.6.2.2.1)
$A_b$ = cross-sectional area of barrier (in.$^2$) (C4.6.2.6.1)
$A_c$ = cross-section area—transformed for steel beams (in.$^2$) (C4.6.6)
$A_o$ = area enclosed by centerlines of elements (in.$^2$) (C4.6.2.2.1)
$A_s$ = total area of stiffeners (in.$^2$) (4.6.2.6.4)
$a$ = length of transition region for effective flange width of a concrete box beam (in.); longitudinal stiffener, spacing, or rib width in an orthotropic steel deck (in.) (4.6.2.6.2) (4.6.2.6.4)
$B$ = spacing of transverse beams (in.) (4.6.2.6.4)
$b$ = tire length (in.); width of a beam (in.); width of plate element (in.); flange width each side of the web (in.) (4.6.2.1.8) (4.6.2.2.1) (C4.6.2.2.1) (4.6.2.6.2)
$b_e$ = effective flange width corresponding to the particular position of the section of interest in the span as specified in Figure 4.6.2.6.2-1 (in.) (4.6.2.6.2)
$b_m$ = effective flange width for interior portions of a span as determined from Figure 4.6.2.6.2-2; a special case of $b_e$ (in.) (4.6.2.6.2)
$b_n$ = effective flange width for normal forces acting at anchorage zones (in.) (4.6.2.6.2)
$b_o$ = width of web projected to midplane of deck (in.) (4.6.2.6.2)
$b_s$ = effective flange width at interior support or for cantilever arm as determined from Figure 4.6.2.6.2-2; a special case of $b_e$ (in.) (4.6.2.6.2)
$C$ = continuity factor; stiffness parameter (4.6.2.1.8) (4.6.2.2.1)

| | | |
|---|---|---|
| $C_m$ | = | moment gradient coefficient (4.5.3.2.2b) |
| $C_{sm}$ | = | the dimensionless elastic seismic response coefficient (C4.7.4.3.2b) |
| $c_1$ | = | parameter for skewed supports (4.6.2.2.2e) |
| $D$ | = | web depth of a horizontally curved girder (ft.); $D_x/D_y$; width of distribution per lane (ft.) (C4.6.1.2.4b) (4.6.2.1.8) (4.6.2.2.1) |
| $D_x$ | = | flexural rigidity in direction of main bars (kip-ft.$^2$/ft.) (4.6.2.1.8) |
| $D_y$ | = | flexural rigidity perpendicular to the main bars (kip-ft.$^2$/ft.) (4.6.2.1.8) |
| $d$ | = | depth of a beam or stringer (in.); depth of member (ft.) (4.6.2.2.1) (C4.6.2.7.1) |
| $d_e$ | = | distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (ft.) (4.6.2.2.1) |
| $d_o$ | = | depth of superstructure (in.) (4.6.2.6.2) |
| $E$ | = | modulus of elasticity (ksi); equivalent width (in.); equivalent distribution width perpendicular to span (in.) (4.5.3.2.2b) (4.6.2.3) (4.6.2.10.2) |
| $E_B$ | = | modulus of elasticity of beam material (ksi) (4.6.2.2.1) |
| $E_c$ | = | modulus of elasticity of column (ksi) (C4.6.2.5) |
| $E_D$ | = | modulus of elasticity of deck material (ksi) (4.6.2.2.1) |
| $E_g$ | = | modulus of elasticity of beam or other restraining member (ksi) (C4.6.2.5) |
| $E_{MOD}$ | = | cable modulus of elasticity, modified for nonlinear effects (ksi) (4.6.3.7) |
| $E_{span}$ | = | equivalent distribution length parallel to span (in.) (4.6.2.10.2) |
| $e$ | = | correction factor for distribution; eccentricity of a lane from the center of gravity of the pattern of girders (ft.); rib spacing in orthotropic steel deck (in.) (4.6.2.2.1) (C4.6.2.2.2d) (4.6.2.6.4) |
| $e_g$ | = | distance between the centers of gravity of the beam and deck (in.) (4.6.2.2.1) |
| $f_c$ | = | factored stress, corrected to account for second-order effects (ksi) (4.5.3.2.2b) |
| $f_{2b}$ | = | stress corresponding to $M_{2b}$ (ksi) (4.5.3.2.2b) |
| $f_{2s}$ | = | stress corresponding to $M_{2s}$ (ksi) (4.5.3.2.2b) |
| $G$ | = | final force effect applied to a girder (kip or kip-ft.); shear modulus (ksi) (4.6.2.2.4) (C4.6.3.3) |
| $G_a$ | = | ratio of stiffness of column to stiffness of members resisting column bending at "$a$" end (C4.6.2.5) |
| $G_b$ | = | ratio of stiffness of column to stiffness of members resisting column bending at "$b$" end (C4.6.2.5) |
| $G_D$ | = | force effect due to design loads (kip or kip-ft.) (4.6.2.2.4) |
| $G_p$ | = | force effect due to overload truck (kip or kip-ft.) (4.6.2.2.4) |
| $g$ | = | distribution factor; acceleration of gravity (ft./sec.$^2$) (4.6.2.2.1) (C4.7.4.3.2) |
| $g_m$ | = | multiple lane live load distribution factor (4.6.2.2.4) |
| $g_1$ | = | single lane live load distribution factor (4.6.2.2.4) |
| $H$ | = | depth of fill from top of culvert to top of pavement (in.); average height of substructure supporting the seat under consideration (ft.) (4.6.2.10.2) (4.7.4.4) |
| $H$, $H_1$, $H_2$ | = | horizontal component of cable force (kip) (4.6.3.7) |
| $h$ | = | depth of deck (in.) (4.6.2.1.3) |
| $I$ | = | moment of inertia (in.$^4$) (4.5.3.2.2b) |
| $I_c$ | = | moment of inertia of column (in.$^4$); inertia of cross-section—transformed for steel beams (in.$^4$) (C4.6.2.5) (C4.6.6) |
| $I_g$ | = | moment of inertia of member acting to restrain column bending (in.$^4$) (C4.6.2.5) |
| $IM$ | = | dynamic load allowance (C4.7.2.1) |
| $I_p$ | = | polar moment of inertia (in.$^4$) (4.6.2.2.1) |
| $I_s$ | = | inertia of equivalent strip (in.$^4$) (4.6.2.1.5) |
| $J$ | = | St. Venant torsional inertia (in.$^4$) (4.6.2.2.1) |
| $K$ | = | effective length factor for columns and arch ribs; constant for different types of construction; effective length factor for columns in the plane of bending (4.5.3.2.2b) (4.6.2.2.1) (4.6.2.5) |
| $K_g$ | = | longitudinal stiffness parameter (in.$^4$) (4.6.2.2.1) |
| $k$ | = | factor used in calculation of distribution factor for multibeam bridges (4.6.2.2.1) |
| $k_s$ | = | strip stiffness factor (kip/in.) (4.6.2.1.5) |
| $L$ | = | span length of deck (ft.); span length (ft.); span length of beam (ft.) (4.6.2.1.3) (4.6.2.1.8) (4.6.2.2.1) |
| $L_{as}$ | = | effective arc span of a horizontally curved girder (ft.) (4.6.1.2.4b) |
| $L_b$ | = | spacing of brace points (ft.) (C4.6.2.7.1) |
| $L_c$ | = | unbraced length of column (in.) (C4.6.2.5) |
| $L_g$ | = | unsupported length of beam or other restraining member (in.) (C4.6.2.5) |
| $LLDF$ | = | factor for distribution of live load with depth of fill, 1.15 or 1.00, as specified in Article 3.6.1.2.6 (4.6.2.10.2) |

$L_T$ = length of tire contact area parallel to span, as specified in Article 3.6.1.2.5 (in.) (4.6.2.10.2)
$L_1$ = modified span length taken to be equal to the lesser of the actual span or 60.0 (ft.); distance between points of inflection of the transverse beam (in.) (4.6.2.3) (4.6.2.6.4)
$L_2$ = distances between points of inflection of the transverse beam (in.) (4.6.2.6.4)
$l_i$ = a notional span length (ft.) (4.6.2.6.2)
$\ell$ = unbraced length of a horizontally curved girder (ft.) (C4.6.1.2.4b)
$\ell_u$ = unsupported length of a compression member (in.); one-half of the length of the arch rib (ft.) (4.5.3.2.2b) (4.5.3.2.2c)
$M$ = major-axis bending moment in a horizontally curved girder (kip-ft.); moment due to live load in filled or partially filled grid deck (kip-in./ft.) (C4.6.1.2.4b) (4.6.2.1.8)
$M_c$ = factored moment, corrected to account for second-order effects (kip-ft.); moment required to restrain uplift caused by thermal effects (kip-in.) (4.5.3.2.2b) (C4.6.6)
$M_{lat}$ = flange lateral bending moment due to curvature (kip-ft.) (C4.6.1.2.4b)
$MM$ = multimode elastic method (4.7.4.3.1)
$M_w$ = maximum lateral moment in the flange due to the factored wind loading (kip-ft.) (C4.6.2.7.1)
$M_{1b}$ = smaller end moment on compression member due to gravity loads that result in no appreciable sidesway; positive if member is bent in single curvature, negative if bent in double curvature (kip-in.) (4.5.3.2.2b)
$M_{2b}$ = moment on compression member due to factored gravity loads that result in no appreciable sidesway calculated by conventional first-order elastic frame analysis; always positive (kip-ft.) (4.5.3.2.2b)
$M_{2s}$ = moment on compression member due to factored lateral or gravity loads that result in sidesway, Δ, greater than $\ell_u/1500$, calculated by conventional first-order elastic frame analysis; always positive (kip-ft.) (4.5.3.2.2b)
$N$ = constant for determining the lateral flange bending moment in I-girder flanges due to curvature, taken as 10 or 12 in past practice; axial force (kip); minimum support length (in.) (C4.6.1.2.4b) (C4.6.6) (4.7.4.4)
$N_b$ = number of beams, stringers, or girders (4.6.2.2.1)
$N_c$ = number of cells in a concrete box girder (4.6.2.2.1)
$N_L$ = number of design lanes (4.6.2.2.1)
$n$ = modular ratio between beam and deck (4.6.2.2.1)
$P$ = axle load (kip) (4.6.2.1.3)
$P_D$ = design horizontal wind pressure (ksf) (C4.6.2.7.1)
$P_e$ = Euler buckling load (kip) (4.5.3.2.2b)
$P_u$ = factored axial load (kip) (4.5.3.2.2b)
$P_w$ = lateral wind force applied to the brace point (kips) (C4.6.2.7.1)
$p$ = tire pressure (ksi) (4.6.2.1.8)
$p_e$ = equivalent uniform static seismic loading per unit length of bridge that is applied to represent the primary mode of vibration (kip/ft.) (C4.7.4.3.2c)
$p_e(x)$ = the intensity of the equivalent static seismic loading that is applied to represent the primary mode of vibration (kip/ft.) (C4.7.4.3.2b)
$p_o$ = a uniform load arbitrarily set equal to 1.0 (kip/ft.) (C4.7.4.3.2b)
$R$ = girder radius (ft.); load distribution to exterior beam in terms of lanes; radius of curvature (C4.6.1.2.4b) (C4.6.2.2.2d) (C4.6.6)
$r$ = reduction factor for longitudinal force effect in skewed bridges (4.6.2.3)
$S$ = spacing of supporting components (ft.); spacing of beams or webs (ft.); clear span (ft.); skew of support measured from line normal to span (°) (4.6.2.1.3) (4.6.2.2.1) (4.6.2.10.2) (4.7.4.4)
$S_b$ = spacing of grid bars (in.) (4.6.2.1.3)
$SM$ = single-mode elastic method (4.7.4.3.1)
$s$ = length of a side element (in.) (C4.6.2.2.1)
$T_G$ = temperature gradient (Δ°F) (C4.6.6)
$TH$ = time history method (4.7.4.3.1)
$T_m$ = period of bridge (sec.) (C4.7.4.3.2b)
$T_u$ = uniform specified temperature (°F) (C4.6.6)
$T_{UG}$ = temperature averaged across the cross-section (°F) (C4.6.6)
$t$ = thickness of plate-like element (in.); thickness of flange plate in orthotropic steel deck (in.) (C4.6.2.2.1) (4.6.2.6.4)
$t_g$ = depth of steel grid or corrugated steel plank including integral concrete overlay or structural concrete component, less a provision for grinding, grooving, or wear (in.) (4.6.2.2.1)
$t_o$ = depth of structural overlay (in.) (4.6.2.2.1)
$t_s$ = depth of concrete slab (in.) (4.6.2.2.1)

$V_{LD}$ = maximum vertical shear at $3d$ or $L/4$ due to wheel loads distributed laterally as specified herein (kips) (4.6.2.2.2a)
$V_{LL}$ = distributed live load vertical shear (kips) (4.6.2.2.2a)
$V_{LU}$ = maximum vertical shear at $3d$ or $L/4$ due to undistributed wheel loads (kips) (4.6.2.2.2a)
$v_s(x)$ = deformation corresponding to $p_o$ (ft.) (C4.7.4.3.2b)
$v_{s,MAX}$ = maximum value of $v_s(x)$ (ft.) (C4.7.4.3.2c)
$W$ = edge-to-edge width of bridge (ft.); factored wind force per unit length (kip/ft.); total weight of cable (kip); total weight of bridge (kip) (4.6.2.2.1) (C4.6.2.7.1) (4.6.3.7) (C4.7.4.3.2c)
$W_e$ = half the web spacing, plus the total overhang (ft.) (4.6.2.2.1)
$W_1$ = modified edge-to-edge width of bridge taken to be equal to the lesser of the actual width or 60.0 for multilane loading, or 30.0 for single-lane loading (ft.) (4.6.2.3)
$w$ = width of clear roadway (ft.); width of element in cross-section (in.) (4.6.2.2.2b) (C4.6.6)
$w(x)$ = nominal, unfactored dead load of the bridge superstructure and tributary substructure (kip/ft.) (C4.7.4.3.2) (4.7.4.3.2c)
$w_p$ = plank width (in.) (4.6.2.1.3)
$X$ = distance from load to point of support (ft.) (4.6.2.1.3)
$X_{ext}$ = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft.) (C4.6.2.2.2d)
$x$ = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.) (C4.6.2.2.2d)
$Z$ = a factor taken as 1.20 where the lever rule was not utilized, and 1.0 where the lever rule was used for a single lane live load distribution factor (4.6.2.2.4)
$z$ = vertical distance from center of gravity of cross-section (in.) (C4.6.6)
$\alpha$ = angle between cable and horizontal (°); coefficient of thermal expansion (in./in./°F); generalized flexibility (4.6.3.7) (C4.6.6) (C4.7.4.3.2b)
$\beta$ = generalized participation (C4.7.4.3.2b)
$\gamma$ = load factor; generalized mass (C4.6.2.7.1) (C4.7.4.3.2b)
$\Delta w$ = overhang width extension (in.) (C4.6.2.6.1)
$\delta_b$ = moment or stress magnifier for braced mode deflection (4.5.3.2.2b)
$\delta_s$ = moment or stress magnifier for unbraced mode deflection (4.5.3.2.2b)
$\varepsilon_u$ = uniform axial strain due to axial thermal expansion (in./in.) (C4.6.6)
$\eta_i$ = load modifier relating to ductility, redundancy, and operational importance as specified in Article 1.3.2.1 (C4.2.6.7.1)
$\theta$ = skew angle (°) (4.6.2.2.1)
$\mu$ = Poisson's ratio (4.6.2.2.1)
$\sigma_E$ = internal stress due to thermal effects (ksi) (C4.6.6)
$\phi$ = rotation per unit length (C4.6.6)
$\phi_K$ = stiffness reduction factor = 0.75 for concrete members and 1.0 for steel and aluminum members (4.5.3.2.2b)

## 4.4 ACCEPTABLE METHODS OF STRUCTURAL ANALYSIS

Any method of analysis that satisfies the requirements of equilibrium and compatibility and utilizes stress-strain relationships for the proposed materials may be used, including, but not limited to:

- Classical force and displacement methods,
- Finite difference method,
- Finite element method,
- Folded plate method,
- Finite strip method,

## C4.4

Many computer programs are available for bridge analysis. Various methods of analysis, ranging from simple formulae to detailed finite element procedures, are implemented in such programs. Many computer programs have specific engineering assumptions embedded in their code, which may or may not be applicable to each specific case.

When using a computer program, the Designer should clearly understand the basic assumptions of the program and the methodology that is implemented.

A computer program is only a tool, and the user is responsible for the generated results. Accordingly, all output should be verified to the extent possible.

Computer programs should be verified against the results of:

- Grillage analogy method,
- Series or other harmonic methods,
- Methods based on the formation of plastic hinges, and
- Yield line method.

The Designer shall be responsible for the implementation of computer programs used to facilitate structural analysis and for the interpretation and use of results.

The name, version, and release date of software used should be indicated in the contract documents.

- Universally accepted closed-form solutions,
- Other previously verified computer programs, or
- Physical testing.

The purpose of identifying software is to establish code compliance and to provide a means of locating bridges designed with software that may later be found deficient.

## 4.5 MATHEMATICAL MODELING

### 4.5.1 General

Mathematical models shall include loads, geometry, and material behavior of the structure, and, where appropriate, response characteristics of the foundation. The choice of model shall be based on the limit states investigated, the force effect being quantified, and the accuracy required.

Unless otherwise permitted, consideration of continuous composite barriers shall be limited to service and fatigue limit states and to structural evaluation.

The stiffness of structurally discontinuous railings, curbs, elevated medians, and barriers shall not be considered in structural analysis.

For the purpose of this section, an appropriate representation of the soil and/or rock that supports the bridge shall be included in the mathematical model of the foundation.

In the case of seismic design, gross soil movement and liquefaction should also be considered.

If lift-off is indicated at a bearing, the analysis shall recognize the vertical freedom of the girder at that bearing.

### C4.5.1

Service and fatigue limit states should be analyzed as fully elastic, as should strength limit states, except in case of certain continuous girders where inelastic analysis is specifically permitted, inelastic redistribution of negative bending moment and stability investigation. The extreme event limit states may require collapse investigation based entirely on inelastic modeling.

Very flexible bridges, e.g., suspension and cable-stayed bridges, should be analyzed using nonlinear elastic methods, such as the large deflection theory.

The need for sophisticated modeling of foundations is a function of the sensitivity of the structure to foundation movements.

In some cases, the foundation model may be as simple as unyielding supports. In other cases, an estimate of settlement may be acceptable. Where the structural response is particularly sensitive to the boundary conditions, such as in a fixed-end arch or in computing natural frequencies, rigorous modeling of the foundation should be made to account for the conditions present. In lieu of rigorous modeling, the boundary conditions may be varied to extreme bounds, such as fixed or free of restraint, and envelopes of force effects considered.

Where lift-off restraints are provided in the contract documents, the construction stage at which the restraints are to be installed should be clearly indicated. The analysis should recognize the vertical freedom of the girder consistent with the construction sequence shown in the contract documents.

### 4.5.2 Structural Material Behavior

#### 4.5.2.1 Elastic Versus Inelastic Behavior

For the purpose of analysis, structural materials shall be considered to behave linearly up to an elastic limit and inelastically thereafter.

Actions at the extreme event limit state may be accommodated in both the inelastic and elastic ranges.

#### 4.5.2.2 Elastic Behavior

Elastic material properties and characteristics shall be in accordance with the provisions of Sections 5, 6, 7, and 8. Changes in these values due to maturity of concrete and environmental effects should be included in the model, where appropriate.

The stiffness properties of concrete and composite members shall be based upon cracked and/or uncracked sections consistent with the anticipated behavior. Stiffness characteristics of beam-slab-type bridges may be based on full participation of concrete decks.

#### C4.5.2.2

Tests indicate that in the elastic range of structural behavior, cracking of concrete seems to have little effect on the global behavior of bridge structures. This effect can, therefore, be safely neglected by modeling the concrete as uncracked for the purposes of structural analysis *King et al., 1975; Yen et al., 1995*).

#### 4.5.2.3 Inelastic Behavior

Sections of components that may undergo inelastic deformation shall be shown to be ductile or made ductile by confinement or other means. Where inelastic analysis is used, a preferred design failure mechanism and its attendant hinge locations shall be determined. It shall be ascertained in the analysis that shear, buckling, and bond failures in the structural components do not precede the formation of a flexural inelastic mechanism. Unintended overstrength of a component in which hinging is expected should be considered. Deterioration of geometrical integrity of the structure due to large deformations shall be taken into account.

The inelastic model shall be based either upon the results of physical tests or upon a representation of load-deformation behavior that is validated by tests. Where inelastic behavior is expected to be achieved by confinement, test specimens shall include the elements that provide such confinement. Where extreme force effects are anticipated to be repetitive, the tests shall reflect their cyclic nature.

Except where noted, stresses and deformations shall be based on a linear distribution of strains in the cross-section of prismatic components. Shear deformation of deep components shall be considered. Limits on concrete strain, as specified in Section 5, shall not be exceeded.

The inelastic behavior of compressive components shall be taken into account, wherever applicable.

#### C4.5.2.3

Where technically possible, the preferred failure mechanism should be based on a response that has generally been observed to provide for large deformations as a means of warning of structural distress.

The selected mechanism should be used to estimate the extreme force effect that can be applied adjacent to a hinge.

Unintended overstrength of a component may result in an adverse formation of a plastic hinge at an undesirable location, forming a different mechanism.

### 4.5.3 Geometry

#### 4.5.3.1 Small Deflection Theory

If the deformation of the structure does not result in a significant change in force effects due to an increase in the eccentricity of compressive or tensile forces, such secondary force effects may be ignored.

#### C4.5.3.1

Small deflection theory is usually adequate for the analysis of beam-type bridges. Bridges that resist loads primarily through a couple whose tensile and compressive forces remain in essentially fixed positions relative to each other while the bridge deflects, such as in trusses and tied arches, are generally insensitive to deformations. Columns and structures in which the flexural moments are increased or decreased by deflection tend to be sensitive to deflection considerations. Such structures include suspension bridges, very flexible cable-stayed bridges, and some arches other than tied arches and frames.

In many cases, the degree of sensitivity can be assessed and evaluated by a single-step approximate method, such as the moment magnification factor method. In the remaining cases, a complete second-order analysis may be necessary.

The past traditional boundary between small- and large-deflection theory becomes less distinct as bridges and bridge components become more flexible due to advances in material technology, the change from mandatory to optional deflection limits, and the trend toward more accurate, optimized design. The Engineer needs to consider these aspects in the choice of an analysis method.

Small-deflection elastic behavior permits the use of the principle of superposition and efficient analytical solutions. These assumptions are typically used in bridge analysis for this reason. The behavior of the members assumed in these provisions is generally consistent with this type of analysis.

Superposition does not apply for the analysis of construction processes that include changes in the stiffness of the structure.

Moments from noncomposite and composite analyses may not be added for the purpose of computing stresses. The addition of stresses and deflections due to noncomposite and composite actions computed from separate analyses is appropriate.

### 4.5.3.2 Large Deflection Theory

#### *4.5.3.2.1 General*

If the deformation of the structure results in a significant change in force effects, the effects of deformation shall be considered in the equations of equilibrium.

The effect of deformation and out-of-straightness of components shall be included in stability analyses and large deflection analyses.

For slender concrete compressive components, those time- and stress-dependent material characteristics that cause significant changes in structural geometry shall be considered in the analysis.

The interaction effects of tensile and compressive axial forces in adjacent components should be considered in the analysis of frames and trusses.

#### *C4.5.3.2.1*

A properly formulated large deflection analysis is one that provides all the force effects necessary for the design. Further application of moment magnification factors is neither required nor appropriate. The presence of compressive axial forces amplifies both out-of-straightness of a component and the deformation due to nontangential loads acting thereon, thereby increasing the eccentricity of the axial force with respect to the centerline of the component. The synergistic effect of this interaction is the apparent softening of the component, i.e., a loss of stiffness. This is commonly referred to as a second-order effect. The converse is true for tension. As axial compressive stress becomes a higher percentage of the so called Euler buckling stress, this effect becomes increasingly more significant.

The second-order effect arises from the translation of applied load creating increased eccentricity. It is considered as geometric nonlinearity and is typically addressed by iteratively solving the equilibrium equations or by using geometric stiffness terms in the elastic range (*Przemieniecki, 1968*). The analyst should be aware of the characteristics of the elements employed, the assumptions upon which they are based, and the numerical procedures used in the computer code. Discussions on the subject are given by White and

Hajjar (*1991*) and Galambos (*1998*). Both references are related to metal structures, but the theory and applications are generally usable. Both contain numerous additional references that summarize the state-of-the-art in this area.

Only factored loads shall be used and no superposition of force effects shall be applied in the nonlinear range. The order of load application in nonlinear analysis shall be consistent with that on the actual bridge.

Because large deflection analysis is inherently nonlinear, the loads are not proportional to the displacements, and superposition cannot be used. Therefore, the order of load application can be important and traditional approaches, such as influence functions, are not directly applicable. The loads should be applied in the order experienced by the structure, i.e., dead load stages followed by live load stages, etc. If the structure undergoes nonlinear deformation, the loads should be applied incrementally with consideration for the changes in stiffness after each increment.

In conducting nonlinear analysis, it is prudent to perform a linear analysis for a baseline and to use the procedures employed on the problem at hand on a simple structure that can be analyzed by hand, such as a cantilever beam. This permits the analyst to observe behavior and develop insight into behavior that is not easily gained from more complex models.

### *4.5.3.2.2 Approximate Methods*

#### *4.5.3.2.2a General*

Where permitted in Sections 5, 6, and 7, the effects of deflection on force effects on beam-columns and arches which meet the provisions of these Specifications may be approximated by the single-step adjustment method known as moment magnification.

#### *C4.5.3.2.2a*

The moment magnification procedure outlined herein is one of several variations of the approximate process and was selected as a compromise between accuracy and ease of use. It is believed to be conservative. An alternative procedure thought to be more accurate than the one specified herein may be found in AISC (*1993*). This alternative procedure will require supplementary calculations not commonly made in bridge design using modern computational methods.

In some cases, the magnitude of movement implied by the moment magnification process cannot be physically attained. For example, the actual movement of a pier may be limited to the distance between the end of longitudinal beams and the backwall of the abutment. In cases where movement is limited, the moment magnification factors of elements so limited may be reduced accordingly.

#### *4.5.3.2.2b Moment Magnification—Beam Columns*

#### *C4.5.3.2.2b*

The factored moments or stresses may be increased to reflect effects of deformations as follows:

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \qquad (4.5.3.2.2b\text{-}1)$$

$$f_c = \delta_b f_{2b} + \delta_s f_{2s} \qquad (4.5.3.2.2b\text{-}2)$$

in which:

$$\delta_b = \frac{C_m}{1 - \dfrac{P_u}{\phi_K P_e}} \geq 1.0 \qquad (4.5.3.2.2b\text{-}3)$$

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_u}{\phi_K \Sigma P_e}} \qquad (4.5.3.2.2b\text{-}4)$$

where:

$M_{2b}$ = moment on compression member due to factored gravity loads that result in no appreciable sidesway calculated by conventional first-order elastic frame analysis; always positive (kip-ft.)

$M_{2s}$ = moment on compression member due to factored lateral or gravity loads that result in sidesway, $\Delta$, greater than $\ell_u/1500$, calculated by conventional first-order elastic frame analysis; always positive (kip-ft.)

$f_{2b}$ = stress corresponding to $M_{2b}$ (ksi)

$f_{2s}$ = stress corresponding to $M_{2s}$ (ksi)

$P_u$ = factored axial load (kip)

$\phi_K$ = stiffness reduction factor; 0.75 for concrete members and 1.0 for steel and aluminum members

$P_e$ = Euler buckling load (kip)

For steel/concrete composite columns, the Euler buckling load, $P_e$, shall be determined as specified in Article 6.9.5.1. For all other cases, $P_e$ shall be taken as:

$$P_e = \frac{\pi^2 EI}{(K\ell_u)^2} \qquad (4.5.3.2.2b\text{-}5)$$

where:

$E$ = modulus of elasticity (ksi)

$I$ = moment of inertia about axis under consideration (in.$^4$)

$K$ = effective length factor in the plane of bending as specified in Article 4.6.2.5. For calculation of $\delta_b$, $P_e$ shall be based on the $K$-factor for braced frames; for calculation of $\delta_s$, $P_e$ shall be based on the $K$-factor for unbraced frames

$\ell_u$ = unsupported length of a compression member (in.)

For concrete compression members, the provisions of Article 5.7.4.3 also apply.

For members braced against sidesway, $\delta_s$ shall be taken as 1.0 unless analysis indicates that a lower value may be used. For members not braced against sidesway, $\delta_b$ shall be determined as for a braced member and $\delta_s$ for an unbraced member.

For members braced against sidesway and without transverse loads between supports, $C_m$ may be taken as:

$$C_m = 0.6 + 0.4\frac{M_{1b}}{M_{2b}} \tag{4.5.3.2.2b-6}$$

The previous limit $C_m \geq 0.4$ has been shown to be unnecessary in AISC (*1994*), Chapter C, of commentary.

where:

$M_{1b}$ = smaller end moment

$M_{2b}$ = larger end moment

The ratio $M_{1b}/M_{2b}$ is considered positive if the component is bent in single curvature and negative if it is bent in double curvature.

For all other cases, $C_m$ shall be taken as 1.0.

In structures that are not braced against sidesway, the flexural members and foundation units framing into the compression member shall be designed for the sum of end moments of the compression member at the joint.

Where compression members are subject to flexure about both principal axes, the moment about each axis shall be magnified by δ, determined from the corresponding conditions of restraint about that axis.

Where a group of compression members on one level comprise a bent, or where they are connected integrally to the same superstructure, and collectively resist the sidesway of the structure, the value of $\delta_s$ shall be computed for the member group with $\Sigma P_u$ and $\Sigma P_e$ equal to the summations for all columns in the group.

#### *4.5.3.2.2c Moment Magnification—Arches*

Live load and impact moments from a small deflection analysis shall be increased by the moment magnification factor, $\delta_b$, as specified in Article 4.5.3.2.2b, with the following definitions:

$\ell_u$ = one-half of the length of the arch rib (ft.)

$K$ = effective length factor specified in Table 1

$C_m$ = 1.0

**Table 4.5.3.2.2c-1 *K* Values for Effective Length of Arch Ribs.**

| Rise to Span Ratio | 3-Hinged Arch | 2-Hinged Arch | Fixed Arch |
|---|---|---|---|
| 0.1–0.2 | 1.16 | 1.04 | 0.70 |
| 0.2–0.3 | 1.13 | 1.10 | 0.70 |
| 0.3–0.4 | 1.16 | 1.16 | 0.72 |

*4.5.3.2.3 Refined Methods*

Refined methods of analysis shall be based upon the concept of forces satisfying equilibrium in a deformed position.

*C4.5.3.2.3*

Flexural equilibrium in a deformed position may be iteratively satisfied by solving a set of simultaneous equations, or by evaluating a closed-form solution formulated using the displaced shape.

### 4.5.4 Modeling Boundary Conditions

Boundary conditions shall represent actual characteristics of support and continuity.

Foundation conditions shall be modeled in such a manner as to represent the soil properties underlying the bridge, the soil-pile interaction, and the elastic properties of piles.

### C4.5.4

If the accurate assessment of boundary conditions cannot be made, their effects may be bounded.

### 4.5.5 Equivalent Members

Nonprismatic components may be modeled by discretizing the components into a number of frame elements with stiffness properties representative of the actual structure at the location of the element.

Components or groups of components of bridges with or without variable cross-sections may be modeled as a single equivalent component provided that it represents all the stiffness properties of the components or group of components. The equivalent stiffness properties may be obtained by closed-form solutions, numerical integration, submodel analysis, and series and parallel analogies.

### C4.5.5

Standard frame elements in available analysis programs may be used. The number of elements required to model the nonprismatic variation is dependent on the type of behavior being modeled, e.g., static, dynamic, or stability analysis. Typically, eight elements per span will give sufficient accuracy for actions in a beam loaded statically with cross-sectional properties that vary smoothly. Fewer elements are required to model for deflection and frequency analyses.

Alternatively, elements may be used that are based on the assumed tapers and cross-sections. Karabalis (*1983*) provides a comprehensive examination of this issue. Explicit forms of stiffness coefficients are given for linearly tapered rectangular, flanged, and box sections. Aristizabal (*1987*) presents similar equations in a simple format that can be readily implemented into stiffness-based computer programs. Significant bibliographies are given in Karabalis (*1983*) and Aristizabal (*1987*).

## 4.6 STATIC ANALYSIS

### 4.6.1 Influence of Plan Geometry

#### 4.6.1.1 Plan Aspect Ratio

If the span length of a superstructure with torsionally stiff closed cross-sections exceeds 2.5 times its width, the superstructure may be idealized as a single-spine beam. The following dimensional definitions shall be used to apply this criterion:

- Width—the core width of a monolithic deck or the average distance between the outside faces of exterior webs.
- Length for rectangular simply supported bridges—the distance between deck joints.

  Length for continuous and/or skewed bridges—the length of the longest side of the rectangle that can be drawn within the plan view of the width of the smallest span, as defined herein.

This restriction does not apply to cast-in-place multicell box girders.

#### C4.6.1.1

Where transverse distortion of a superstructure is small in comparison with longitudinal deformation, the former does not significantly affect load distribution, hence, an equivalent beam idealization is appropriate. The relative transverse distortion is a function of the ratio between structural width and height, the latter, in turn, depending on the length. Hence, the limits of such idealization are determined in terms of the width-to-effective length ratio.

Simultaneous torsion, moment, shear, and reaction forces and the attendant stresses are to be superimposed as appropriate. The equivalent beam idealization does not alleviate the need to investigate warping effects in steel structures. In all equivalent beam idealizations, the eccentricity of loads should be taken with respect to the centerline of the equivalent beam.

#### 4.6.1.2 Structures Curved in Plan

*4.6.1.2.1 General*

The moments, shears, and other force effects required to proportion the superstructure components shall be based on a rational analysis of the entire superstructure.

The entire superstructure, including bearings, shall be considered as an integral structural unit. Boundary conditions shall represent the articulations provided by the bearings and/or integral connections used in the design. Analyses may be based on elastic small-deflection theory, unless more rigorous approaches are deemed necessary by the Engineer.

Analyses shall consider bearing orientation and restraint of bearings afforded by the substructure. These load effects shall be considered in designing bearings, cross-frames, diaphragms, bracing, and the deck.

Distortion of the cross-section need not be considered in the structural analysis.

Centrifugal force effects shall be considered in accordance with Article 3.6.3.

*C4.6.1.2.1*

Since equilibrium of horizontally curved I-girders is developed by the transfer of load between the girders, the analysis must recognize the integrated behavior of all structural components. Equilibrium of curved box girders may be less dependent on the interaction between girders. Bracing members are considered primary members in curved bridges since they transmit forces necessary to provide equilibrium.

The deck acts in flexure, vertical shear, and horizontal shear. Torsion increases the horizontal deck shear, particularly in curved box girders. The lateral restraint of the bearings may also cause horizontal shear in the deck.

Small-deflection theory is adequate for the analysis of most curved-girder bridges. However, curved I-girders are prone to deflect laterally when the girders are insufficiently braced during erection. This behavior may not be well recognized by small-deflection theory.

Classical methods of analysis usually are based on strength of materials assumptions that do not recognize cross-section deformation. Finite element analyses that model the actual cross-section shape of the I- or box girders can recognize cross-section distortion and its effect on structural behavior. Cross-section deformation of steel box girders may have a significant effect on torsional behavior, but this effect is limited by the provision of sufficient internal cross bracing.

*4.6.1.2.2 Single-Girder Torsionally Stiff Superstructures*

A horizontally curved, torsionally stiff single-girder superstructure meeting the requirements of Article 4.6.1.1 may be analyzed for global force effects as a curved spine beam.

The location of the centerline of such a beam shall be taken at the center of gravity of the cross-section, and the eccentricity of dead loads shall be established by volumetric consideration.

*C4.6.1.2.2*

In order to apply the aspect ratio provisions of Article 4.6.1.1, as specified, the plan needs to be hypothetically straightened. Force effects should be calculated on the basis of the actual curved layout.

With symmetrical cross-sections, the center of gravity of permanent loads falls outside the center of gravity. Shear center of the cross-section and the resulting eccentricity need to be investigated.

*4.6.1.2.3 Multicell Concrete Box Girders*

Horizontally curved cast-in-place multicell box girders may be designed as single-spine beams with straight segments, for central angles up to 34° within one span, unless concerns about other force effects dictate otherwise.

*C4.6.1.2.3*

A parameter study conducted by Song, Chai, and Hida (*2003*) indicated that the distribution factors from the LRFD formulae compared well with the distribution factors from grillage analyses when using straight segments on spans with central angles up to 34° in one span.

*4.6.1.2.4 Steel Multiple-Beam Superstructures*

*4.6.1.2.4a General*

Horizontally curved superstructures may be analyzed as grids or continuums in which the segments of the longitudinal beams are assumed to be straight between nodes. The actual eccentricity of the segment between the nodes shall not exceed 2.5 percent of the length of the segment.

*C4.6.1.2.4a*

An eccentricity of 2.5 percent of the length of the segment corresponds to a central angle subtended by a curved segment of about 12°.

This Article applies only to major-axis bending moment and does not apply to lateral flange bending, or torsion, which should always be examined with respect to curvature.

Bridges with even slight curvature may develop large radial forces at the abutment bearings. Therefore, thermal analysis of all curved bridges is recommended.

*4.6.1.2.4b I-Girders*

The effect of curvature on stability shall be considered for all curved I-girders.

Where I-girder bridges meet the following four conditions, the effects of curvature may be ignored in the analysis for determining the major-axis bending moments and bending shears:

- Girders are concentric;
- Bearing lines are not skewed more than 10° from radial;
- The stiffnesses of the girders are similar;
- For all spans, the arc span divided by the girder radius in feet is less than 0.06 radians where the arc span, $L_{as}$, shall be taken as follows:

*C4.6.1.2.4b*

The requirement for similar stiffness among the girders is intended to avoid large and irregular changes in stiffness which could alter transverse distribution of load. Under such conditions, a refined analysis would be appropriate. Noncomposite dead load preferably is to be distributed uniformly to the girders since the cross-frames provide restoring forces that prevent the girders from deflecting independently. Certain dead loads applied to the composite bridge may be distributed uniformly to the girders as provided in Article 4.6.2.2.1. However, heavier concentrated line loads such as parapets, sidewalks, barriers, or sound walls should not be distributed equally to the girders. Engineering judgment must be used in determining the distribution of these loads. Often the largest portion of the load on an overhang is assigned to the exterior girder, or to the exterior girder and the first interior girder. The exterior girder on the outside of the curve is often critical in curved girder bridges.

For simple spans:

$L_{as}$ = arc length of the girder (ft.)

For end spans of continuous members:

$L_{as}$ = 0.9 times the arc length of the girder (ft.)

For interior spans of continuous members:

$L_{as}$ = 0.8 times the arc length of the girder (ft.)

An I-girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Lateral flange bending effects should then be determined from an appropriate approximation and considered in the design.

Cross-frame or diaphragm members shall be designed in accordance with Articles 6.7.4 and 6.13 for forces computed by rational means.

Cross-frame spacing shall be set to limit flange lateral bending in the girders.

The effect of curvature on the torsional behavior of a girder must be considered regardless of the amount of curvature since stability and strength of curved girders is different from that of straight girders (*Hall and Yoo, 1996*).

In lieu of a refined analysis, Eq. C1 may be appropriate for determining the lateral bending moment in I-girder flanges due to curvature (*Richardson, Gordon, and Associates, 1976; United States Steel, 1984*).

$$M_{lat} = \frac{M\ell^2}{NRD} \qquad \text{(C4.6.1.2.4b-1)}$$

where:

$M_{lat}$ = flange lateral bending moment (kip-ft.)

$M$ = major-axis bending moment (kip-ft.)

$\ell$ = unbraced length (ft.)

$R$ = girder radius (ft.)

$D$ = web depth (ft.)

$N$ = a constant taken as 10 or 12 in past practice

Although the depth to be used in computing the flange lateral moment from Eq. C1 is theoretically equal to the depth, $h$, between the midthickness of the top and bottom flanges, for simplicity, the web depth, $D$, is conservatively used in Eq. C1. The Engineer may substitute the depth, $h$, for $D$ in Eq. C1, if desired. Eq. C1 assumes the presence of a cross-frame at the point under investigation, that the cross-frame spacing is relatively uniform, and that the major-axis bending moment, $M$, is constant between brace points. Therefore, at points not actually located at cross-frames, flange lateral moments from Eq. C1 may not be strictly correct. The constant, $N$, in Eq. C1 has been taken as either 10 or 12 in past practice and either value is considered acceptable depending on the level of conservatism that is desired.

Other conditions that produce torsion, such as skew, should be dealt with by other analytical means which generally involve a refined analysis.

#### *4.6.1.2.4c Closed Box and Tub Girders*

The effect of curvature on strength and stability shall be considered for all curved box girders.

Where box girder bridges meet the following three conditions, the effect of curvature may be ignored in the analysis for determination of the major-axis bending moments and bending shears:

- Girders are concentric,
- Bearings are not skewed, and

#### *C4.6.1.2.4c*

Although box-shaped girders have not been examined as carefully as I-girders with regard to approximate methods, bending moments in closed girders are less affected by curvature than are I-girders (*Tung and Fountain, 1970*). However, in a box shape, torsion is much greater than in an open shape so that web shears are affected by torsion due to curvature, skew or loads applied away from the shear center of the box. Double bearings resist significant torque compared to a box-centered single bearing.

- For all spans, the arc span divided by the girder radius is less than 0.3 radians, and the girder depth is less than the width of the box at middepth where the arc span, $L_{as}$, shall be taken as defined in Article 4.6.1.2.4b.

A box girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Lateral flange bending effects should then be found from an appropriate approximation and considered in the design.

Cross-frame or diaphragm members shall be designed in accordance with the provisions of Articles 6.7.4 and 6.13 and lateral bracing members shall be designed in accordance with Articles 6.7.5 and 6.13 for forces computed by rational means.

If the box is haunched or tapered, the shallowest girder depth should be used in conjunction with the narrowest width of the box at middepth in determining whether the effects of curvature may be ignored in calculating the major axis bending moments and bending shears.

## 4.6.2 Approximate Methods of Analysis

### 4.6.2.1 Decks

#### *4.6.2.1.1 General*

An approximate method of analysis in which the deck is subdivided into strips perpendicular to the supporting components shall be considered acceptable for decks other than:

- fully filled and partially filled grids for which the provisions of Article 4.6.2.1.8 shall apply, and
- top slabs of segmental concrete box girders for which the provisions of 4.6.2.9.4 shall apply.

Where the strip method is used, the extreme positive moment in any deck panel between girders shall be taken to apply to all positive moment regions. Similarly, the extreme negative moment over any beam or girder shall be taken to apply to all negative moment regions.

#### *C4.6.2.1.1*

This model is analogous to past AASHTO Specifications.

In determining the strip widths, the effects of flexure in the secondary direction and of torsion on the distribution of internal force effects are accounted for to obtain flexural force effects approximating those that would be provided by a more refined method of analysis.

Depending on the type of deck, modeling and design in the secondary direction may utilize one of the following approximations:

- Secondary strip designed in a manner like the primary strip, with all the limit states applicable;
- Resistance requirements in the secondary direction determined as a percentage of that in the primary one as specified in Article 9.7.3.2 (i.e., the traditional approach for reinforced concrete slab in the previous editions of the AASHTO Standard Specifications); or
- Minimum structural and/or geometry requirements specified for the secondary direction independent of actual force effects, as is the case for most wood decks.

The approximate strip model for decks is based on rectangular layouts. Currently about two-thirds of all bridges nationwide are skewed. While skew generally tends to decrease extreme force effects, it produces negative moments at corners, torsional moments in the end zones, substantial redistribution of reaction forces, and a number of other structural phenomena that should be considered in the design.

*4.6.2.1.2 Applicability*

The use of design aids for decks containing prefabricated elements may be permitted in lieu of analysis if the performance of the deck is documented and supported by sufficient technical evidence. The Engineer shall be responsible for the accuracy and implementation of any design aids used.

For slab bridges and concrete slabs spanning more than 15.0 ft. and which span primarily in the direction parallel to traffic, the provisions of Article 4.6.2.3 shall apply.

*4.6.2.1.3 Width of Equivalent Interior Strips*

The width of the equivalent strip of a deck may be taken as specified in Table 1. Where decks span primarily in the direction parallel to traffic, strips supporting an axle load shall not be taken to be greater than 40.0 in. for open grids and not greater than 144 in. for all other decks where multilane loading is being investigated. For deck overhangs, where applicable, the provisions of Article 3.6.1.3.4 may be used in lieu of the strip width specified in Table 1 for deck overhangs. The equivalent strips for decks that span primarily in the transverse direction shall not be subject to width limits. The following notation shall apply to Table 1:

$S$ = spacing of supporting components (ft.)

$h$ = depth of deck (in.)

$L$ = span length of deck (ft.)

$P$ = axle load (kip)

$S_b$ = spacing of grid bars (in.)

$+M$ = positive moment

$-M$ = negative moment

$X$ = distance from load to point of support (ft.)

*C4.6.2.1.3*

Values provided for equivalent strip widths and strength requirements in the secondary direction are based on past experience. Practical experience and future research work may lead to refinement.

To get the load per unit width of the equivalent strip, divide the total load on one design traffic lane by the calculated strip width.

**Table 4.6.2.1.3-1 Equivalent Strips.**

| Type of Deck | Direction of Primary Strip Relative to Traffic | Width of Primary Strip (in.) |
|---|---|---|
| Concrete: | | |
| • Cast-in-place | Overhang | $45.0 + 10.0X$ |
| | Either Parallel or Perpendicular | $+M$: $26.0 + 6.6S$<br>$-M$: $48.0 + 3.0S$ |
| • Cast-in-place with stay-in-place concrete formwork | Either Parallel or Perpendicular | $+M$: $26.0 + 6.6S$<br>$-M$: $48.0 + 3.0S$ |
| • Precast, post-tensioned | Either Parallel or Perpendicular | $+M$: $26.0 + 6.6S$<br>$-M$: $48.0 + 3.0S$ |
| Steel: | | |
| • Open grid | Main Bars | $1.25P + 4.0S_b$ |
| • Filled or partially filled grid | Main Bars | Article 4.6.2.1.8 applies |
| • Unfilled, composite grids | Main Bars | Article 4.6.2.1.8 applies |
| Wood: | | |
| • Prefabricated glulam<br>○ Noninterconnected | Parallel<br>Perpendicular | $2.0h + 30.0$<br>$2.0h + 40.0$ |
| ○ Interconnected | Parallel<br>Perpendicular | $90.0 + 0.84L$<br>$4.0h + 30.0$ |
| • Stress-laminated | Parallel<br>Perpendicular | $0.8S + 108.0$<br>$10.0S + 24.0$ |
| • Spike-laminated<br>○ Continuous decks or interconnected panels | Parallel<br>Perpendicular | $2.0h + 30.0$<br>$4.0h + 40.0$ |
| ○ Noninterconnected panels | Parallel<br>Perpendicular | $2.0h + 30.0$<br>$2.0h + 40.0$ |

Wood plank decks shall be designed for the wheel load of the design truck distributed over the tire contact area. For transverse planks, i.e., planks perpendicular to traffic direction:

- If $w_p \geq 10.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 10.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of $w_p$ and 10.0 in.

Only the wheel load is specified for plank decks. Addition of lane load will cause a negligible increase in force effects, however, it may be added for uniformity of the Code.

For longitudinal planks:

- If $w_p \geq 20.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 20.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of $w_p$ and 20.0 in.

where:

$w_p$ = plank width (in.)

### *4.6.2.1.4 Width of Equivalent Strips at Edges of Slabs*

#### *4.6.2.1.4a General*

For the purpose of design, the notional edge beam shall be taken as a reduced deck strip width specified herein. Any additional integral local thickening or similar protrusion acting as a stiffener to the deck that is located within the reduced deck strip width can be assumed to act with the reduced deck strip width as the notional edge beam.

#### *4.6.2.1.4b Longitudinal Edges*

Edge beams shall be assumed to support one line of wheels and, where appropriate, a tributary portion of the design lane load.

Where decks span primarily in the direction of traffic, the effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the edge of the deck and the inside face of the barrier, plus 12.0 in., plus one-quarter of the strip width, specified in either Article 4.6.2.1.3, Article 4.6.2.3, or Article 4.6.2.10, as appropriate, but not exceeding either one-half the full strip width or 72.0 in.

#### *4.6.2.1.4c Transverse Edges*

Transverse edge beams shall be assumed to support one axle of the design truck in one or more design lanes, positioned to produce maximum load effects. Multiple presence factors and the dynamic load allowance shall apply.

The effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the transverse edge of the deck and the centerline of the first line of support for the deck, usually taken as a girder web, plus one-half of the width of strip as specified in Article 4.6.2.1.3. The effective width shall not exceed the full strip width specified in Article 4.6.2.1.3.

*C4.6.2.1.4c*

For decks covered by Table A4-1, the total moment acting on the edge beam, including the multiple presence factor and the dynamic load allowance, may be calculated by multiplying the moment per unit width, taken from Table A4-1, by the corresponding full strip width specified in Article 4.6.2.1.3.

*4.6.2.1.5 Distribution of Wheel Loads*

If the spacing of supporting components in the secondary direction exceeds 1.5 times the spacing in the primary direction, all of the wheel loads shall be considered to be applied to the primary strip, and the provisions of Article 9.7.3.2 may be applied to the secondary direction.

If the spacing of supporting components in the secondary direction is less than 1.5 times the spacing in the primary direction, the deck shall be modeled as a system of intersecting strips.

The width of the equivalent strips in both directions may be taken as specified in Table 4.6.2.1.3-1. Each wheel load shall be distributed between two intersecting strips. The distribution shall be determined as the ratio between the stiffness of the strip and the sum of stiffnesses of the intersecting strips. In the absence of more precise calculations, the strip stiffness, $k_s$, may be estimated as:

$$k_s = \frac{EI_s}{S^3} \qquad (4.6.2.1.5\text{-}1)$$

where:

$I_s$ = moment of inertia of the equivalent strip (in.$^4$)

$S$ = spacing of supporting components (in.)

*C4.6.2.1.5*

This Article attempts to clarify the application of the traditional AASHTO approach with respect to continuous decks.

*4.6.2.1.6 Calculation of Force Effects*

The strips shall be treated as continuous beams or simply supported beams, as appropriate. Span length shall be taken as the center-to-center distance between the supporting components. For the purpose of determining force effects in the strip, the supporting components shall be assumed to be infinitely rigid.

The wheel loads may be modeled as concentrated loads or as patch loads whose length along the span shall be the length of the tire contact area, as specified in Article 3.6.1.2.5, plus the depth of the deck. The strips should be analyzed by classical beam theory.

The design section for negative moments and shear forces, where investigated, may be taken as follows:

- For monolithic construction, closed steel boxes, closed concrete boxes, open concrete boxes without top flanges, and stemmed precast beams, i.e., Cross-sections (b), (c), (d), (e), (f), (g), (h), (i), and (j) from Table 4.6.2.2.1-1, at the face of the supporting component,

- For steel I-beams and steel tub girders, i.e., Cross-sections (a) and (c) from Table 4.6.2.2.1-1, one-quarter the flange width from the centerline of support,

*C4.6.2.1.6*

This is a deviation from the traditional approach based on a continuity correction applied to results obtained for analysis of simply supported spans. In lieu of more precise calculations, the unfactored design live load moments for many practical concrete deck slabs can be found in Table A4-1.

For short-spans, the force effects calculated using the footprint could be significantly lower, and more realistic, than force effects calculated using concentrated loads.

Reduction in negative moment and shear replaces the effect of reduced span length in the current code. The design sections indicated may be applied to deck overhangs and to portions of decks between stringers or similar lines of support.

Past practice has been to not check shear in typical decks. A design section for shear is provided for use in nontraditional situations. It is not the intent to investigate shear in every deck.

- For precast I-shaped concrete beams and open concrete boxes with top flanges, i.e., Cross-sections (c) and (k) from Table 4.6.2.2.1-1, one-third the flange width, but not exceeding 15.0 in., from the centerline of support,

- For wood beams, i.e., Cross-section (l) from Table 4.6.2.2.1-1, one-fourth the top beam width from centerline of beam.

For open box beams, each web shall be considered as a separate supporting component for the deck. The distance from the centerline of each web and the adjacent design sections for negative moment shall be determined based on the type of construction of the box and the shape of the top of the web using the requirements outlined above.

*4.6.2.1.7 Cross-Sectional Frame Action*

Where decks are an integral part of box or cellular cross-sections, flexural and/or torsional stiffnesses of supporting components of the cross-section, i.e., the webs and bottom flange, are likely to cause significant force effects in the deck. Those components shall be included in the analysis of the deck.

If the length of a frame segment is modeled as the width of an equivalent strip, provisions of Articles 4.6.2.1.3, 4.6.2.1.5, and 4.6.2.1.6 may be used.

*C4.6.2.1.7*

The model used is essentially a transverse segmental strip, in which flexural continuity provided by the webs and bottom flange is included. Such modeling is restricted to closed cross-sections only. In open-framed structures, a degree of transverse frame action also exists, but it can be determined only by complex, refined analysis.

In normal beam-slab superstructures, cross-sectional frame action may safely be neglected. If the slab is supported by box beams or is integrated into a cellular cross-section, the effects of frame action could be considerable. Such action usually decreases positive moments, but may increase negative moments resulting in cracking of the deck. For larger structures, a three-dimensional analysis may be appropriate. For smaller structures, the analysis could be restricted to a segment of the bridge whose length is the width of an equivalent strip.

Extreme force effects may be calculated by combining the:

- Longitudinal response of the superstructure approximated by classical beam theory, and

- Transverse flexural response modeled as a cross-sectional frame.

*4.6.2.1.8 Live Load Force Effects for Fully and Partially Filled Grids and for Unfilled Grid Decks Composite with Reinforced Concrete Slabs*

Moments in kip-in./in. of deck due to live load may be determined as:

- Main bars perpendicular to traffic:

  For $L \leq 120$ in.

$$M_{transverse} = 1.28D^{0.197}L^{0.459}C \quad (4.6.2.1.8\text{-}1)$$

  For $L > 120$ in.

$$M_{transverse} = \frac{D^{0.188}\left(3.7L^{1.35} - 956.3\right)}{L}(C) \quad (4.6.2.1.8\text{-}2)$$

- Main bars parallel to traffic:

  For $L \leq 120$ in.

$$M_{parallel} = 0.73D^{0.123}L^{0.64}C \quad (4.6.2.1.8\text{-}3)$$

  For $L > 120$ in.

$$M_{parallel} = \frac{D^{0.138}\left(3.1L^{1.429} - 1088.5\right)}{L}(C) \quad (4.6.2.1.8\text{-}4)$$

where:

$L$ = span length from center-to-center of supports (in.)

$C$ = continuity factor; 1.0 for simply supported and 0.8 for continuous spans

$D$ = $D_x/D_y$

$D_x$ = flexural rigidity of deck in main bar direction (kip-in.$^2$/in.)

$D_y$ = flexural rigidity of deck perpendicular to main bar direction (kip-in.$^2$/in.)

For grid decks, $D_x$ and $D_y$ should be calculated as $EI_x$ and $EI_y$ where $E$ is the modulus of elasticity and $I_x$ and $I_y$ are the moment of inertia per unit width of deck, considering the section as cracked and using the transformed area method for the main bar direction and perpendicular to main bar direction, respectively.

Moments for fatigue assessment may be estimated for all span lengths by reducing Eq. 1 for main bars perpendicular to traffic or Eq. 3 for main bars parallel to traffic by a factor of 3.

*C4.6.2.1.8*

The moment equations are based on orthotropic plate theory considering vehicular live loads specified in Article 3.6. The equations take into account relevant factored load combinations including truck and tandem loads. The moment equations also account for dynamic load allowance, multiple presence factors, and load positioning on the deck surface to produce the largest possible moment.

Negative moment can be determined as maximum simple span positive moment times the continuity factor, $C$.

The reduction factor of 3.0 in the last sentence of Article 4.6.2.1.8 accounts for smaller dynamic load allowance (15 percent vs. 33 percent), smaller load factor (0.75 vs. 1.75) and no multiple presence (1.0 vs. 1.2) when considering fatigue. Use of Eqs. 1 and 3 for all spans is appropriate as Eqs. 1 and 3 reflect an individual design truck on short-span lengths while Eqs. 2 and 4 reflect the influence of multiple design tandems that control moment envelope on longer span lengths. The approximation produces reasonable estimates of fatigue moments, however, improved estimates can be determined using fatigue truck patch loads in the infinite series formula provided by Higgins (*2003*).

Actual $D_x$ and $D_y$ values can vary considerably depending on the specific deck design, and using assumed values based only on the general type of deck can lead to unconservative design moments. Flexural rigidity in each direction should be calculated analytically as $EI$ considering the section as cracked and using the transformed area method.

Deflection in units of in. due to vehicular live load may be determined as:

- Main bars perpendicular to traffic:

$$\Delta_{transverse} = \frac{0.0052D^{0.19}L^3}{D_x} \quad (4.6.2.1.8\text{-}5)$$

- Main bars parallel to traffic:

$$\Delta_{parallel} = \frac{0.0072D^{0.11}L^3}{D_x} \quad (4.6.2.1.8\text{-}6)$$

The deflection equations permit calculation of the midspan displacement for a deck under service load. The equations are based on orthotropic plate theory and consider both truck and tandem loads on a simply supported deck.

Deflection may be reduced for decks continuous over three or more supports. A reduction factor of 0.8 is conservative.

*4.6.2.1.9 Inelastic Analysis*

The inelastic finite element analysis or yield line analysis may be permitted by the Owner.

**4.6.2.2 Beam-Slab Bridges**

*4.6.2.2.1 Application*

The provisions of this Article may be applied to straight girder bridges and horizontally curved concrete bridges, as well as horizontally curved steel girder bridges complying with the provisions of Article 4.6.1.2.4. The provisions of this Article may also be used to determine a starting point for some methods of analysis to determine force effects in curved girders of any degree of curvature in plan.

Except as specified in Article 4.6.2.2.5, the provisions of this Article shall be taken to apply to bridges being analyzed for:

- A single lane of loading, or
- Multiple lanes of live load yielding approximately the same force effect per lane.

If one lane is loaded with a special vehicle or evaluation permit vehicle, the design force effect per girder resulting from the mixed traffic may be determined as specified in Article 4.6.2.2.5.

For beam spacing exceeding the range of applicability as specified in tables in Articles 4.6.2.2.2 and 4.6.2.2.3, the live load on each beam shall be the reaction of the loaded lanes based on the lever rule unless specified otherwise herein.

The provisions of Article 3.6.1.1.2 specify that multiple presence factors shall not be used with the approximate load assignment methods other than statical moment or lever arm methods because these factors are already incorporated in the distribution factors.

*C4.6.2.2.1*

The V-load method is one example of a method of curved bridge analysis which starts with straight girder distribution factors (*United States Steel, 1984*).

The lever rule involves summing moments about one support to find the reaction at another support by assuming that the supported component is hinged at interior supports.

When using the lever rule on a three-girder bridge, the notional model should be taken as shown in Figure C1. Moments should be taken about the assumed, or notional, hinge in the deck over the middle girder to find the reaction on the exterior girder.

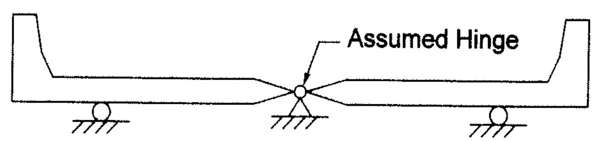



**Figure C4.6.2.2.1-1 Notional Model for Applying Lever Rule to Three-Girder Bridges.**

Provisions in Articles 4.6.2.2.2 and 4.6.2.2.3 that do not appear in earlier editions of the Standard Specifications come primarily from Zokaie et al. (*1991*). Correction factors for continuity have been deleted for two reasons:

Bridges not meeting the requirements of this Article shall be analyzed as specified in Article 4.6.3.

The distribution of live load, specified in Articles 4.6.2.2.2 and 4.6.2.2.3, may be used for girders, beams, and stringers, other than multiple steel box beams with concrete decks that meet the following conditions and any other conditions identified in tables of distribution factors as specified herein:

- Width of deck is constant;
- Unless otherwise specified, the number of beams is not less than four;
- Beams are parallel and have approximately the same stiffness;
- Unless otherwise specified, the roadway part of the overhang, $d_e$, does not exceed 3.0 ft.;
- Curvature in plan is less than the limit specified in Article 4.6.1.2.4, or where distribution factors are required in order to implement an acceptable approximate or refined analysis method satisfying the requirements of Article 4.4 for bridges of any degree of curvature in plan; and
- Cross-section is consistent with one of the cross-sections shown in Table 1.

Where moderate deviations from a constant deck width or parallel beams exist, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value for beam spacing.

- Correction factors dealing with 5 percent adjustments were thought to imply misleading levels of accuracy in an approximate method, and
- Analyses of many continuous beam-slab-type bridges indicate that the distribution coefficients for negative moments exceed those obtained for positive moments by approximately 10 percent. On the other hand, it has been observed that stresses at or near an internal bearing are reduced due to the fanning of the reaction force. This reduction is about the same magnitude as the increase in distribution factors, hence the two tend to cancel each other out, and thus are omitted from these Specifications.

In Strength Load Combination II, applying a distribution factor procedure to a loading involving a heavy permit load can be overly conservative unless lane-by-lane distribution factors are available. Use of a refined method of analysis will circumvent this situation.

A rational approach may be used to extend the provisions of this Article to bridges with splayed girders. The distribution factor for live load at any point along the span may be calculated by setting the girder spacing in the equations of this Article equal to half the sum of the center-to-center distance between the girder under consideration and the two girders to either side. This will result in a variable distribution factor along the length of the girder. While the variable distribution factor is theoretically correct, it is not compatible with existing line girder computer programs that only allow constant distribution factor. Further simplifications may be used to allow the use of such computer programs. One such simplification involves running the computer program a number of times equal to the number of spans in the bridge. For each run, the girder spacing is set equal to the maximum girder spacing in one span and the results from this run are applied to this span. This approach is guaranteed to result in conservative design. In the past, some jurisdictions applied the latter approach, but used the girder spacing at the 2/3 or 3/4 points of the span; which will also be an acceptable approximation.

Most of the equations for distribution factors were derived for constant deck width and parallel beams. Past designs with moderate exceptions to these two assumptions have performed well when the $S/D$ distribution factors were used. While the distribution factors specified herein are more representative of actual bridge behavior, common sense indicates that some exceptions are still possible, especially if the parameter $S$ is chosen with prudent judgment, or if the factors are appropriately varied at selected locations along the span.

Cast-in-place multicell concrete box girder bridge types may be designed as whole-width structures. Such cross-sections shall be designed for the live load distribution factors in Articles 4.6.2.2.2 and 4.6.2.2.3 for interior girders, multiplied by the number of girders, i.e., webs.

Additional requirements for multiple steel box girders with concrete decks shall be as specified in Article 4.6.2.2.2b.

Where bridges meet the conditions specified herein, permanent loads of and on the deck may be distributed uniformly among the beams and/or stringers.

Live load distribution factors, specified herein, may be used for permit and rating vehicles whose overall width is comparable to the width of the design truck.

The following notation shall apply to tables in Articles 4.6.2.2.2 and 4.6.2.2.3:

$A$ = area of stringer, beam or girder (in.$^2$)
$b$ = width of beam (in.)
$C$ = stiffness parameter
$D$ = width of distribution per lane (ft.)
$d$ = depth of beam or stringer (in.)
$d_e$ = distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (ft.)
$e$ = correction factor
$g$ = distribution factor
$I_p$ = polar moment of inertia (in.$^4$)
$J$ = St. Venant's torsional inertia (in.$^4$)
$K$ = constant for different types of construction
$K_g$ = longitudinal stiffness parameter (in.$^4$)
$L$ = span of beam (ft.)
$N_b$ = number of beams, stringers or girders
$N_c$ = number of cells in a concrete box girder
$N_L$ = number of design lanes as specified in Article 3.6.1.1.1
$S$ = spacing of beams or webs (ft.)
$t_g$ = depth of steel grid or corrugated steel plank including integral concrete overlay or structural concrete component, less a provision for grinding, grooving, or wear (in.)
$t_o$ = depth of structural overlay (in.)
$t_s$ = depth of concrete slab (in.)
$W$ = edge-to-edge width of bridge (ft.)
$W_e$ = half the web spacing, plus the total overhang (ft.)
$\theta$ = skew angle (°)
$\mu$ = Poisson's ratio

Unless otherwise stated, the stiffness parameters for area, moments of inertia and torsional stiffness used herein and in Articles 4.6.2.2.2 and 4.6.2.2.3 shall be taken as those of the cross-section to which traffic will be applied, i.e., usually the composite section.

Whole-width design is appropriate for torsionally-stiff cross-sections where load-sharing between girders is extremely high and torsional loads are hard to estimate. Prestressing force should be evenly distributed between girders. Cell width-to-height ratios should be approximately 2:1.

In lieu of more refined information, the St. Venant torsional inertia, $J$, may be determined as:

- For thin-walled open beam:

$$J = \frac{1}{3}\Sigma bt^3 \qquad \text{(C4.6.2.2.1-1)}$$

- For stocky open sections, e.g., prestressed I-beams, T-beams, etc., and solid sections:

$$J = \frac{A^4}{40.0 I_p} \qquad \text{(C4.6.2.2.1-2)}$$

- For closed thin-walled shapes:

$$J = \frac{4A_o^2}{\Sigma \frac{s}{t}} \qquad \text{(C4.6.2.2.1-3)}$$

where:

$b$ = width of plate element (in.)
$t$ = thickness of plate-like element (in.)
$A$ = area of cross-section (in.$^2$)
$I_p$ = polar moment of inertia (in.$^4$)
$A_o$ = area enclosed by centerlines of elements (in.$^2$)
$s$ = length of a side element (in.)

Eq. C2 has been shown to substantially underestimate the torsional stiffness of some concrete I-beams and a more accurate, but more complex, approximation can be found in Eby et al. (*1973*).

The transverse post-tensioning shown for some cross-sections herein is intended to make the units act together. A minimum 0.25 ksi prestress is recommended.

For beams with variable moment of inertia, $K_g$ may be based on average properties.

For bridge types "f," "g," "h," "i," and "j," longitudinal joints between precast units of the cross-section are shown in Table 1. This type of construction

The longitudinal stiffness parameter, $K_g$, shall be taken as:

$$K_g = n\left(I + Ae_g^{\,2}\right) \quad (4.6.2.2.1\text{-}1)$$

in which:

$$n = \frac{E_B}{E_D} \quad (4.6.2.2.1\text{-}2)$$

where:

$E_B$ = modulus of elasticity of beam material (ksi)

$E_D$ = modulus of elasticity of deck material (ksi)

$I$ = moment of inertia of beam (in.$^4$)

$e_g$ = distance between the centers of gravity of the basic beam and deck (in.)

The parameters $A$ and $I$ in Eq. 1 shall be taken as those of the noncomposite beam.

The bridge types indicated in tables in Articles 4.6.2.2.2 and 4.6.2.2.3, with reference to Table 1, may be taken as representative of the type of bridge to which each approximate equation applies.

acts as a monolithic unit if sufficiently interconnected. In Article 5.14.4.3.3f, a fully interconnected joint is identified as a flexural shear joint. This type of interconnection is enhanced by either transverse post-tensioning of an intensity specified above or by a reinforced structural overlay, which is also specified in Article 5.14.4.3.3f, or both. The use of transverse mild steel rods secured by nuts or similar unstressed dowels should not be considered sufficient to achieve full transverse flexural continuity unless demonstrated by testing or experience. Generally, post-tensioning is thought to be more effective than a structural overlay if the intensity specified above is achieved.

In some cases, the lower limit of deck slab thickness, $t_s$, shown in the range of applicability column in tables in Articles 4.6.2.2.2 and 4.6.2.2.3 is less than 7.0 in. The research used to develop the equations in those tables reflects the range of slab thickness shown. Article 9.7.1.1 indicates that concrete decks less than 7.0 in. in thickness should not be used unless approved by the Owner. Lesser values shown in tables in Articles 4.6.2.2.2 and 4.6.2.2.3 are not intended to override Article 9.7.1.1.

The load distribution factor equations for bridge type "d", cast-in-place multicell concrete box girders, were derived by first positioning the vehicle longitudinally, and then transversely, using an I-section of the box. While it would be more appropriate to develop an algorithm to find the peak of an influence surface, using the present factor for the interior girders multiplied by the number of girders is conservative in most cases.

Table C1 describes how the term $L$ (length) may be determined for use in the live load distribution factor equations given in Articles 4.6.2.2.2 and 4.6.2.2.3.

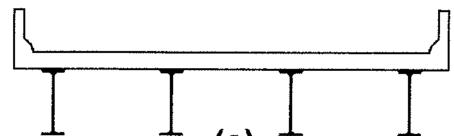
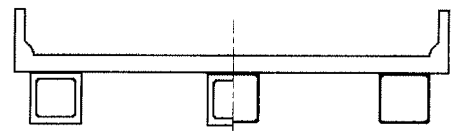
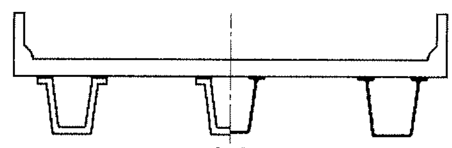
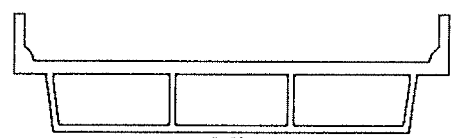
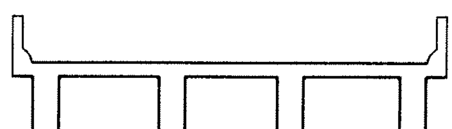
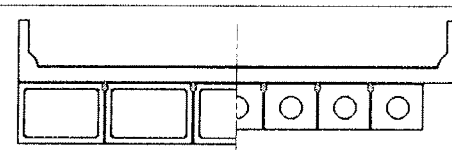
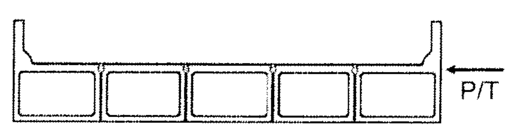
**Table C4.6.2.2.1-1 *L* for Use in Live Load Distribution Factor Equations.**

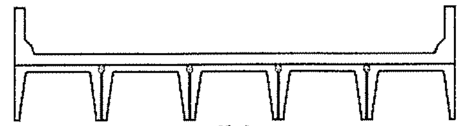
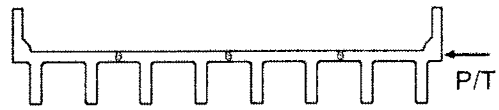
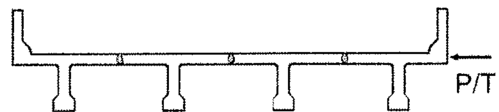
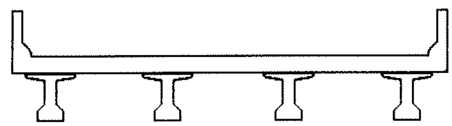
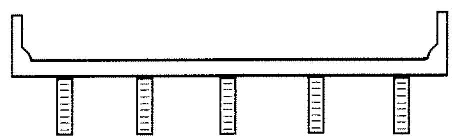
| Force Effect | $L$ (ft.) |
|---|---|
| Positive Moment | The length of the span for which moment is being calculated |
| Negative Moment—Near interior supports of continuous spans from point of contraflexure to point of contraflexure under a uniform load on all spans | The average length of the two adjacent spans |
| Negative Moment—Other than near interior supports of continuous spans | The length of the span for which moment is being calculated |
| Shear | The length of the span for which shear is being calculated |
| Exterior Reaction | The length of the exterior span |
| Interior Reaction of Continuous Span | The average length of the two adjacent spans |

Except as permitted by Article 2.5.2.7.1, regardless of the method of analysis used, i.e., approximate or refined, exterior girders of multibeam bridges shall not have less resistance than an interior beam.

In the rare occasion when the continuous span arrangement is such that an interior span does not have any positive uniform load moment (i.e., no uniform load points of contraflexure), the region of negative moment near the interior supports would be increased to the centerline of the span, and the *L* used in determining the live load distribution factors would be the average of the two adjacent spans.

**Table 4.6.2.2.1-1 Common Deck Superstructures Covered in Articles 4.6.2.2.2 and 4.6.2.2.3.**

| Supporting Components | Type Of Deck | Typical Cross-Section |
|---|---|---|
| Steel Beam | Cast-in-place concrete slab, precast concrete slab, steel grid, glued/spiked panels, stressed wood |  (a) |
| Closed Steel or Precast Concrete Boxes | Cast-in-place concrete slab |  (b) |
| Open Steel or Precast Concrete Boxes | Cast-in-place concrete slab, precast concrete deck slab |  (c) |
| Cast-in-Place Concrete Multicell Box | Monolithic concrete |  (d) |
| Cast-in-Place Concrete Tee Beam | Monolithic concrete |  (e) |
| Precast Solid, Voided or Cellular Concrete Boxes with Shear Keys | Cast-in-place concrete overlay |  (f) |
| Precast Solid, Voided, or Cellular Concrete Box with Shear Keys and with or without Transverse Post-Tensioning | Integral concrete |   (g) |

| Supporting Components | Type Of Deck | Typical Cross-Section |
|---|---|---|
| Precast Concrete Channel Sections with Shear Keys | Cast-in-place concrete overlay |  (h) |
| Precast Concrete Double Tee Section with Shear Keys and with or without Transverse Post-Tensioning | Integral concrete |  (i) |
| Precast Concrete Tee Section with Shear Keys and with or without Transverse Post-Tensioning | Integral concrete |  (j) |
| Precast Concrete I or Bulb-Tee Sections | Cast-in-place concrete, precast concrete |  (k) |
| Wood Beams | Cast-in-place concrete or plank, glued/spiked panels or stressed wood |  (l) |

For cast-in-place concrete multicell box shown as cross-section Type "d" in Table 1, the distribution factors in Article 4.6.2.2.2 and 4.6.2.2.3 shall be taken to apply to a notional shape consisting of a web, overhangs of an exterior web, and the associated half flanges between a web under consideration and the next adjacent web or webs.

*4.6.2.2.2 Distribution Factor Method for Moment and Shear*

*4.6.2.2.2a Interior Beams with Wood Decks*

The live load flexural moment and shear for interior beams with transverse wood decks may be determined by applying the lane fraction specified in Table 1 and Eq. 1.

When investigation of shear parallel to the grain in wood components is required, the distributed live load shear shall be determined by the following expression:

$$V_{LL} = 0.50\left[(0.60V_{LU}) + V_{LD}\right] \quad (4.6.2.2.2a\text{-}1)$$

where:

$V_{LL}$ = distributed live load vertical shear (kips)

$V_{LU}$ = maximum vertical shear at $3d$ or $L/4$ due to undistributed wheel loads (kips)

$V_{LD}$ = maximum vertical shear at $3d$ or $L/4$ due to wheel loads distributed laterally as specified herein (kips)

For undistributed wheel loads, one line of wheels is assumed to be carried by one bending member.

**Table 4.6.2.2.2a-1 Distribution of Live Load Per Lane for Moment and Shear in Interior Beams with Wood Decks.**

| Type of Deck | Applicable Cross-Section from Table 4.6.2.2.1-1 | One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
|---|---|---|---|---|
| Plank | a, l | $S/6.7$ | $S/7.5$ | $S \leq 5.0$ |
| Stressed Laminated | a, l | $S/9.2$ | $S/9.0$ | $S \leq 6.0$ |
| Spike Laminated | a, l | $S/8.3$ | $S/8.5$ | $S \leq 6.0$ |
| Glued Laminated Panels on Glued Laminated Stringers | a, l | $S/10.0$ | $S/10.0$ | $S \leq 6.0$ |
| Glue Laminated Panels on Steel Stringers | a, l | $S/8.8$ | $S/9.0$ | $S \leq 6.0$ |

*4.6.2.2.2b Interior Beams with Concrete Decks*

The live load flexural moment for interior beams with concrete decks may be determined by applying the lane fraction specified in Table 1.

For preliminary design, the terms $K_g/(12.0Lt_s^3)$ and $I/J$ may be taken as 1.0.

For the concrete beams, other than box beams, used in multibeam decks with shear keys:

- Deep, rigid end diaphragms shall be provided to ensure proper load distribution; and
- If the stem spacing of stemmed beams is less than 4.0 ft. or more than 10.0 ft., a refined analysis complying with Article 4.6.3 shall be used.

For multiple steel box girders with a concrete deck in bridges satisfying the requirements of Article 6.11.2.3, the live load flexural moment may be determined using the appropriate distribution factor specified in Table 1.

*C4.6.2.2.2b*

Where the spacing of the box girders varies along the length of the bridge, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value of $N_L$. In either case, the value of $N_L$ shall be determined as specified in Article 3.6.1.1.1, using the width, $w$, taken at the section under consideration.

The results of analytical and model studies of simple span multiple box section bridges, reported in Johnston and Mattock (*1967*), showed that folded plate theory could be used to analyze the behavior of bridges of this type. The folded plate theory was used to obtain the maximum load per girder, produced by various critical combinations of loading on 31 bridges having various spans, numbers of box girders, and numbers of traffic lanes.

Multiple presence factors, specified in Table 3.6.1.1.2-1, are not applied because the multiple factors in past editions of the Standard Specifications were considered in the development of the equation in Table 1 for multiple steel box girders.

The lateral load distribution obtained for simple spans is also considered applicable to continuous structures.

The bridges considered in the development of the equations had interior end diaphragms only, i.e., no interior diaphragms within the spans, and no exterior diaphragms anywhere between boxes. If interior or exterior diaphragms are provided within the span, the transverse load distribution characteristics of the bridge will be improved to some degree. This improvement can be evaluated, if desired, using the analysis methods identified in Article 4.4.

**Table 4.6.2.2.2b-1 Distribution of Live Loads Per Lane for Moment in Interior Beams.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | Distribution Factors | Range of Applicability |
|---|---|---|---|
| Wood Deck on Wood or Steel Beams | a, l | See Table 4.6.2.2.2a-1 | |
| Concrete Deck on Wood Beams | l | One Design Lane Loaded: $S/12.0$<br>Two or More Design Lanes Loaded: $S/10.0$ | $S \le 6.0$ |
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections | a, e, k and also i, j if sufficiently connected to act as a unit | One Design Lane Loaded:<br>$0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$<br>Two or More Design Lanes Loaded:<br>$0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0\,Lt_s^3}\right)^{0.1}$ | $3.5 \le S \le 16.0$<br>$4.5 \le t_s \le 12.0$<br>$20 \le L \le 240$<br>$N_b \ge 4$<br>$10{,}000 \le K_g \le$ 7,000,000 |
| | | use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule | $N_b = 3$ |
| Cast-in-Place Concrete Multicell Box | d | One Design Lane Loaded:<br>$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$<br>Two or More Design Lanes Loaded:<br>$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$ | $7.0 \le S \le 13.0$<br>$60 \le L \le 240$<br>$N_c \ge 3$<br><br>If $N_c > 8$ use $N_c = 8$ |
| Concrete Deck on Concrete Spread Box Beams | b, c | One Design Lane Loaded:<br>$\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25}$<br>Two or More Design Lanes Loaded:<br>$\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$ | $6.0 \le S \le 18.0$<br>$20 \le L \le 140$<br>$18 \le d \le 65$<br>$N_b \ge 3$ |
| | | Use Lever Rule | $S > 18.0$ |
| Concrete Beams used in Multibeam Decks | f<br><br>g if sufficiently connected to act as a unit | One Design Lane Loaded:<br>$k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$<br>where: $k = 2.5(N_b)^{-0.2} \ge 1.5$<br>Two or More Design Lanes Loaded:<br>$k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.06}$ | $35 \le b \le 60$<br>$20 \le L \le 120$<br>$5 \le N_b \le 20$ |

<table>
<tr><th>Type of Superstructure</th><th>Applicable Cross-Section from Table 4.6.2.2.1-1</th><th>Distribution Factors</th><th>Range of Applicability</th></tr>
<tr><td rowspan="2"></td><td>h</td><td rowspan="2">Regardless of Number of Loaded Lanes:<br>$S/D$<br>where:<br>$C = K(W/L) \leq K$<br>$D = 11.5 - N_L + 1.4N_L(1-0.2C)^2$<br>when $C \leq 5$<br>$D = 11.5 - N_L$ when $C > 5$<br>$K = \sqrt{\frac{(1+\mu)I}{J}}$<br>for preliminary design, the following values of $K$ may be used:<br><br>Beam Type — $K$<br>Nonvoided rectangular beams — 0.7<br>Rectangular beams with circular voids: — 0.8<br>Box section beams — 1.0<br>Channel beams — 2.2<br>T-beam — 2.0<br>Double T-beam — 2.0</td><td rowspan="2">Skew $\leq 45°$<br><br>$N_L \leq 6$</td></tr>
<tr><td>g, i, j if connected only enough to prevent relative vertical displacement at the interface</td></tr>
<tr><td>Open Steel Grid Deck on Steel Beams</td><td>a</td><td>One Design Lane Loaded:<br>$S/7.5$ If $t_g < 4.0$<br>$S/10.0$ If $t_g \geq 4.0$<br><br>Two or More Design Lanes Loaded:<br>$S/8.0$ If $t_g < 4.0$<br>$S/10.0$ If $t_g \geq 4.0$</td><td>$S \leq 6.0$<br><br><br>$S \leq 10.5$</td></tr>
<tr><td>Concrete Deck on Multiple Steel Box Girders</td><td>b, c</td><td>Regardless of Number of Loaded Lanes:<br>$0.05 + 0.85\frac{N_L}{N_b} + \frac{0.425}{N_L}$</td><td>$0.5 \leq \frac{N_L}{N_b} \leq 1.5$</td></tr>
</table>

*4.6.2.2.2c Interior Beams with Corrugated Steel Decks*

The live load flexural moment for interior beams with corrugated steel plank deck may be determined by applying the lane fraction, $g$, specified in Table 1.

**Table 4.6.2.2.2c-1 Distribution of Live Load Per Lane for Moment in Interior Beams with Corrugated Steel Plank Decks.**

| One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
|---|---|---|
| $S/9.2$ | $S/9.0$ | $S \leq 5.5$<br>$t_g \geq 2.0$ |

*4.6.2.2.2d Exterior Beams*

The live load flexural moment for exterior beams may be determined by applying the lane fraction, $g$, specified in Table 1.

The distance, $d_e$, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier.

In beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

*C4.6.2.2.2d*

This additional investigation is required because the distribution factor for girders in a multigirder cross-section, Types "a," "e," and "k" in Table 4.6.2.2.1-1, was determined without consideration of diaphragm or cross-frames. The recommended procedure is an interim provision until research provides a better solution.

The procedure outlined in this section is the same as the conventional approximation for loads on piles.

$$R = \frac{N_L}{N_b} + \frac{X_{ext}\sum^{N_L} e}{\sum^{N_b} x^2} \qquad \text{(C4.6.2.2.2d-1)}$$

where:

$R$ = reaction on exterior beam in terms of lanes

$N_L$ = number of loaded lanes under consideration

$e$ = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft.)

$x$ = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.)

$X_{ext}$ = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft.)

$N_b$ = number of beams or girders

**Table 4.6.2.2.2d-1 Distribution of Live Loads Per Lane for Moment in Exterior Longitudinal Beams.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
|---|---|---|---|---|
| Wood Deck on Wood or Steel Beams | a, l | Lever Rule | Lever Rule | N/A |
| Concrete Deck on Wood Beams | l | Lever Rule | Lever Rule | N/A |
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections | a, e, k and also i, j if sufficiently connected to act as a unit | Lever Rule | $g = e\, g_{interior}$<br>$e = 0.77 + \frac{d_e}{9.1}$ | $-1.0 \le d_e \le 5.5$ |
| | | | use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule | $N_b = 3$ |
| Cast-in-Place Concrete Multicell Box | d | $g = \frac{W_e}{14}$ | $g = \frac{W_e}{14}$ | $W_e \le S$ |
| | | or the provisions for a whole-width design specified in Article 4.6.2.2.1 | | |
| Concrete Deck on Concrete Spread Box Beams | b, c | Lever Rule | $g = e\, g_{interior}$<br>$e = 0.97 + \frac{d_e}{28.5}$ | $0 \le d_e \le 4.5$<br>$6.0 < S \le 18.0$ |
| | | | Use Lever Rule | $S > 18.0$ |
| Concrete Box Beams Used in Multibeam Decks | f, g | $g = e\, g_{interior}$<br>$e = 1.125 + \frac{d_e}{30} \ge 1.0$ | $g = e\, g_{interior}$<br>$e = 1.04 + \frac{d_e}{25} \ge 1.0$ | $d_e \le 2.0$ |
| Concrete Beams Other than Box Beams Used in Multibeam Decks | h | Lever Rule | Lever Rule | N/A |
| | i, j if connected only enough to prevent relative vertical displacement at the interface | | | |
| Open Steel Grid Deck on Steel Beams | a | Lever Rule | Lever Rule | N/A |
| Concrete Deck on Multiple Steel Box Girders | b, c | As specified in Table 4.6.2.2.2b-1 | | |

*4.6.2.2.2e Skewed Bridges*

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10°, the bending moment in the beams may be reduced in accordance with Table 1.

*C4.6.2.2.2e*

Accepted reduction factors are not currently available for cases not covered in Table 1.

**Table 4.6.2.2.2e-1 Reduction of Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | Any Number of Design Lanes Loaded | Range of Applicability |
|---|---|---|---|
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections | a, e, k and also i, j if sufficiently connected to act as a unit | $1-c_1(\tan\theta)^{1.5}$<br>$c_1 = 0.25\left(\dfrac{K_g}{12.0Lt_s^3}\right)^{0.25}\left(\dfrac{S}{L}\right)^{0.5}$<br>If $\theta < 30^\circ$ then $c_1 = 0.0$<br>If $\theta > 60^\circ$ use $\theta = 60^\circ$ | $30^\circ \le \theta \le 60^\circ$<br>$3.5 \le S \le 16.0$<br>$20 \le L \le 240$<br>$N_b \ge 4$ |
| Concrete Deck on Concrete Spread Box Beams, Cast-in-Place Multicell Box Concrete Box Beams and Double T-Sections used in Multibeam Decks | b, c, d, f, g | $1.05 - 0.25\tan\theta \le 1.0$<br>If $\theta > 60^\circ$ use $\theta = 60^\circ$ | $0^\circ \le \theta \le 60^\circ$ |

*4.6.2.2.2f Flexural Moments and Shear in Transverse Floorbeams*

If the deck is supported directly by transverse floorbeams, the floorbeams may be designed for loads determined in accordance with Table 1.

The fractions provided in Table 1 shall be used in conjunction with the 32.0-kip design axle load alone. For spacings of floorbeams outside the given ranges of applicability, all of the design live loads shall be considered, and the lever rule may be used.

**Table 4.6.2.2.2f-1 Distribution of Live Load per Lane for Transverse Beams for Moment and Shear.**

| Type of Deck | Fraction of Wheel Load to Each Floorbeam | Range of Applicability |
|---|---|---|
| Plank | $\dfrac{S}{4}$ | N/A |
| Laminated Wood Deck | $\dfrac{S}{5}$ | $S \le 5.0$ |
| Concrete | $\dfrac{S}{6}$ | $S \le 6.0$ |
| Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab | $\dfrac{S}{4.5}$ | $t_g \le 4.0$<br>$S \le 5.0$ |
| Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab | $\dfrac{S}{6}$ | $t_g > 4.0$<br>$S \le 6.0$ |
| Steel Bridge Corrugated Plank | $\dfrac{S}{5.5}$ | $t_g \ge 2.0$ |

### *4.6.2.2.3 Distribution Factor Method for Shear*

#### *4.6.2.2.3a Interior Beams*

The live load shear for interior beams may be determined by applying the lane fractions specified in Table 1. For interior beam types not listed in Table 1, lateral distribution of the wheel or axle adjacent to the end of span shall be that produced by use of the lever rule.

For preliminary design, the term *I/J* may be taken as 1.0.

For concrete box beams used in multibeam decks, if the values of *I* or *J* do not comply with the limitations in Table 1, the distribution factor for shear may be taken as that for moment.

**Table 4.6.2.2.3a-1 Distribution of Live Load per Lane for Shear in Interior Beams.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
|---|---|---|---|---|
| Wood Deck on Wood or Steel Beams | a, l | See Table 4.6.2.2.2a-1 | | |
| Concrete Deck on Wood Beams | l | Lever Rule | Lever Rule | N/A |
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T-and Double T-Sections | a, e, k and also i, j if sufficiently connected to act as a unit | $0.36+\frac{S}{25.0}$ | $0.2+\frac{S}{12}-\left(\frac{S}{35}\right)^{2.0}$ | $3.5 \le S \le 16.0$<br>$20 \le L \le 240$<br>$4.5 \le t_s \le 12.0$<br>$N_b \ge 4$ |
| | | Lever Rule | Lever Rule | $N_b = 3$ |
| Cast-in-Place Concrete Multicell Box | d | $\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$ | $\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1}$ | $6.0 \le S \le 13.0$<br>$20 \le L \le 240$<br>$35 \le d \le 110$<br>$N_c \ge 3$ |
| Concrete Deck on Concrete Spread Box Beams | b, c | $\left(\frac{S}{10}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$ | $\left(\frac{S}{7.4}\right)^{0.8}\left(\frac{d}{12.0L}\right)^{0.1}$ | $6.0 \le S \le 18.0$<br>$20 \le L \le 140$<br>$18 \le d \le 65$<br>$N_b \ge 3$ |
| | | Lever Rule | Lever Rule | $S > 18.0$ |
| Concrete Box Beams Used in Multibeam Decks | f, g | $\left(\frac{b}{130L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05}$ | $\left(\frac{b}{156}\right)^{0.4}\left(\frac{b}{12.0L}\right)^{0.1}\left(\frac{I}{J}\right)^{0.05}\left(\frac{b}{48}\right)$<br>$\frac{b}{48} \ge 1.0$ | $35 \le b \le 60$<br>$20 \le L \le 120$<br>$5 \le N_b \le 20$<br>$25{,}000 \le J \le 610{,}000$<br>$40{,}000 \le I \le 610{,}000$ |
| Concrete Beams Other Than Box Beams Used in Multibeam Decks | h<br>i, j if connected only enough to prevent relative vertical displacement at the interface | Lever Rule | Lever Rule | N/A |
| Open Steel Grid Deck on Steel Beams | a | Lever Rule | Lever Rule | N/A |
| Concrete Deck on Multiple Steel Box Beams | b, c | As specified in Table 4.6.2.2.2b-1 | | |

*4.6.2.2.3b Exterior Beams*

The live load shear for exterior beams shall be determined by applying the lane fractions specified in Table 1. For cases not addressed in Table 4.6.2.2.3a-1 and Table 1, the live load distribution to exterior beams shall be determined by using the lever rule.

The parameter $d_e$ shall be taken as positive if the exterior web is inboard of the curb or traffic barrier and negative if it is outboard.

The additional provisions for exterior beams in beam-slab bridges with cross-frames or diaphragms, specified in Articles 4.6.2.2.2d, shall apply.

**Table 4.6.2.2.3b-1 Distribution of Live Load per Lane for Shear in Exterior Beams.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
|---|---|---|---|---|
| Wood Deck on Wood or Steel Beams | a, l | Lever Rule | Lever Rule | N/A |
| Concrete Deck on Wood Beams | l | Lever Rule | Lever Rule | N/A |
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Beams | a, e, k and also i, j if sufficiently connected to act as a unit | Lever Rule | $g = e\, g_{interior}$<br>$e = 0.6 + \frac{d_e}{10}$ | $-1.0 \le d_e \le 5.5$ |
| | | | Lever Rule | $N_b = 3$ |
| Cast-in-Place Concrete Multicell Box | d | Lever Rule | $g = e\, g_{interior}$<br>$e = 0.64 + \frac{d_e}{12.5}$ | $-2.0 \le d_e \le 5.0$ |
| | | or the provisions for a whole-width design specified in Article 4.6.2.2.1 | | |
| Concrete Deck on Concrete Spread Box Beams | b, c | Lever Rule | $g = e\, g_{interior}$<br>$e = 0.8 + \frac{d_e}{10}$ | $0 \le d_e \le 4.5$ |
| | | | Lever Rule | $S > 18.0$ |
| Concrete Box Beams Used in Multibeam Decks | f, g | $g = e\, g_{interior}$<br>$e = 1.25 + \frac{d_e}{20} \ge 1.0$ | $g = e\, g_{interior}\left(\frac{48}{b}\right)$<br>$\frac{48}{b} \le 1.0$<br>$e = 1 + \left(\frac{d_e + \frac{b}{12} - 2.0}{40}\right)^{0.5} \ge 1.0$ | $d_e \le 2.0$<br>$35 \le b \le 60$ |
| Concrete Beams Other Than Box Beams Used in Multibeam Decks | h<br>i, j if connected only enough to prevent relative vertical displacement at the interface | Lever Rule | Lever Rule | N/A |
| Open Steel Grid Deck on Steel Beams | a | Lever Rule | Lever Rule | N/A |
| Concrete Deck on Multiple Steel Box Beams | b, c | As specified in Table 4.6.2.2.2b-1 | | |

### 4.6.2.2.3c *Skewed Bridges*

Shear in the exterior beam at the obtuse corner of the bridge shall be adjusted when the line of support is skewed. The value of the correction factor shall be obtained from Table 1. It is applied to the lane fraction specified in Table 4.6.2.2.3a-1 for interior beams and in Table 4.6.2.2.3b-1 for exterior beams.

In determining the end shear in multibeam bridges, the skew correction at the obtuse corner shall be applied to all the beams.

### *C4.6.2.2.3c*

Verifiable correction factors are not available for cases not covered in Table 1.

The equal treatment of all beams in a multibeam bridge is conservative regarding positive reaction and shear. However, it is not necessarily conservative regarding uplift in the case of large skew and short exterior spans of continuous beams. A supplementary investigation of uplift should be considered using the correction factor from Table 1, i.e., the terms other than 1.0, taken as negative for the exterior beam at the acute corner.

**Table 4.6.2.2.3c-1 Correction Factors for Load Distribution Factors for Support Shear of the Obtuse Corner.**

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | Correction Factor | Range of Applicability |
|---|---|---|---|
| Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Section | a, e, k and also i, j if sufficiently connected to act as a unit | $1.0+0.20\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3}\tan\theta$ | $0° \leq \theta \leq 60°$<br>$3.5 \leq S \leq 16.0$<br>$20 \leq L \leq 240$<br>$N_b \geq 4$ |
| Cast-in-Place Concrete Multicell Box | d | $1.0+\left(0.25+\frac{12.0L}{70d}\right)\tan\theta$ | $0° < \theta \leq 60°$<br>$6.0 < S \leq 13.0$<br>$20 \leq L \leq 240$<br>$35 \leq d \leq 110$<br>$N_c \geq 3$ |
| Concrete Deck on Spread Concrete Box Beams | b, c | $1.0+\frac{\sqrt{\frac{Ld}{12.0}}}{6S}\tan\theta$ | $0° < \theta \leq 60°$<br>$6.0 \leq S \leq 11.5$<br>$20 \leq L \leq 140$<br>$18 \leq d \leq 65$<br>$N_b \geq 3$ |
| Concrete Box Beams Used in Multibeam Decks | f, g | $1.0+\frac{12.0L}{90d}\sqrt{\tan\theta}$ | $0° < \theta \leq 60°$<br>$20 \leq L \leq 120$<br>$17 \leq d \leq 60$<br>$35 \leq b \leq 60$<br>$5 \leq N_b \leq 20$ |

#### *4.6.2.2.4 Curved Steel Bridges*

Approximate analysis methods may be used for analysis of curved steel bridges. The Engineer shall ascertain that the approximate analysis method used is appropriate by confirming that the method satisfies the requirements stated in Article 4.4.

In curved systems, consideration should be given to placing parapets, sidewalks, barriers and other heavy line loads at their actual location on the bridge. Wearing surface and other distributed loads may be assumed uniformly distributed to each girder in the cross-section.

#### *C4.6.2.2.4*

The V-load method (*United States Steel, 1984*) has been a widely used approximate method for analyzing horizontally curved steel I-girder bridges. The method assumes that the internal torsional load on the bridge—resulting solely from the curvature—is resisted by self-equilibrating sets of shears between adjacent girders. The V-load method does not directly account for sources of torque other than curvature and the method does not account for the horizontal shear stiffness of the concrete deck. The method is only valid for loads such as normal highway loadings. For exceptional loadings, a more refined analysis is required. The method assumes a linear distribution of girder shears across the bridge section; thus, the girders at a given cross-section should have approximately the same vertical stiffness. The V-load method is also not directly applicable to structures with reverse curvature or to a closed-framed system with horizontal lateral bracing near, or in the plane of one or both flanges. The V-load method does not directly account for girder twist; thus, lateral deflections, which become important on bridges with large spans and/or sharp skews and vertical deflections, may be significantly underestimated. In certain situations, the V-load method may not detect uplift at end bearings. The method is best suited for preliminary design, but may also be suitable for final design of structures with radial supports or supports skewed less than approximately 10°.

The M/R method provides a means to account for the effect of curvature in curved box girder bridges. The method and suggested limitations on its use are discussed by Tung and Fountain (*1970*).

Vertical reactions at interior supports on the concave side of continuous-span bridges may be significantly underestimated by both the V-load and M/R methods.

Live load distribution factors for use with the V-load and M/R methods may be determined using the appropriate provisions of Article 4.6.2.2.

Strict rules and limitations on the applicability of both of these approximate methods do not exist. The Engineer must determine when approximate methods of analysis are appropriate.

*4.6.2.2.5 Special Loads with Other Traffic*

Except as specified herein, the provisions of this Article may be applied where the approximate methods of analysis for the analysis of beam-slab bridges specified in Article 4.6.2.2 and slab-type bridges specified in Article 4.6.2.3 are used. The provisions of this Article shall not be applied where either:

- the lever rule has been specified for both single lane and multiple lane loadings, or
- the special requirement for exterior girders of beam-slab bridge cross-sections with diaphragms specified in Article 4.6.2.2.2d has been utilized for simplified analysis.

Force effects resulting from heavy vehicles in one lane with routine traffic in adjacent lanes, such as might be considered with Load Combination Strength II in Table 3.4.1-1 may be determined as:

$$G = G_p\left(\frac{g_1}{Z}\right) + G_D\left(g_m - \frac{g_1}{Z}\right) \qquad (4.6.2.2.4\text{-}1)$$

where:

$G$ = final force effect applied to a girder (kip or kip-ft.)

$G_p$ = force effect due to overload truck (kip or kip-ft.)

$g_1$ = single lane live load distribution factor

$G_D$ = force effect due to design loads (kip or kip-ft.)

$g_m$ = multiple lane live load distribution factor

$Z$ = a factor taken as 1.20 where the lever rule was not utilized, and 1.0 where the lever rule was used for a single lane live load distribution factor

*C4.6.2.2.5*

Because the number of loaded lanes used to determine the multiple lane live load distribution factor, $g_m$, is not known, the multiple lane multiple presence factor, $m$, is implicitly set equal to 1.0 in this equation, which assumes only two lanes are loaded, resulting in a conservative final force effect over using the multiple presence factors for three or more lanes loaded.

The factor $Z$ is used to distinguish between situations where the single lane live load distribution factor was determined from a specified algebraic equation and situations where the lever rule was specified for the determination of the single lane live load distribution factor. In the situation where an algebraic equation was specified, the multiple presence factor of 1.20 for a single lane loaded has been included in the algebraic equation and must be removed by using $Z$ = 1.20 in Eq. 1 so that the distribution factor can be utilized in Eq. 1 to determine the force effect resulting from a multiple lane loading.

This formula was developed from a similar formula presented without investigation by Modjeski and Masters, Inc. (*1994*) in a report to the Pennsylvania Department of Transportation in 1994, as was examined in Zokaie (*1998*).

### 4.6.2.3 Equivalent Strip Widths for Slab-Type Bridges

This Article shall be applied to the types of cross-sections shown schematically in Table 1. For the purpose of this Article, cast-in-place voided slab bridges may be considered as slab bridges.

The equivalent width of longitudinal strips per lane for both shear and moment with one lane, i.e., two lines of wheels, loaded may be determined as:

C4.6.2.3

$$E = 10.0 + 5.0\sqrt{L_1 W_1} \qquad (4.6.2.3\text{-}1)$$

The equivalent width of longitudinal strips per lane for both shear and moment with more than one lane loaded may be determined as:

$$E = 84.0 + 1.44\sqrt{L_1 W_1} \leq \frac{12.0W}{N_L} \qquad (4.6.2.3\text{-}2)$$

where:

$E$ = equivalent width (in.)

$L_1$ = modified span length taken equal to the lesser of the actual span or 60.0 (ft.)

$W_1$ = modified edge-to-edge width of bridge taken to be equal to the lesser of the actual width or 60.0 for multilane loading, or 30.0 for single-lane loading (ft.)

$W$ = physical edge-to-edge width of bridge (ft.)

$N_L$ = number of design lanes as specified in Article 3.6.1.1.1

For skewed bridges, the longitudinal force effects may be reduced by the factor $r$:

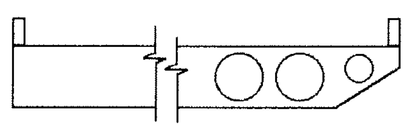
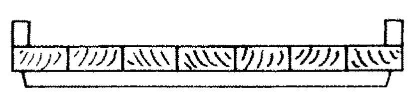
$$r = 1.05 - 0.25\tan\theta \leq 1.00 \qquad (4.6.2.3\text{-}3)$$

where:

$\theta$ = skew angle (°)

In Eq. 1, the strip width has been divided by 1.20 to account for the multiple presence effect.

**Table 4.6.2.3-1 Typical Schematic Cross-Section.**

| Supporting Components | Type of Deck | Typical Cross-Section |
|---|---|---|
| Cast-in-Place Concrete Slab or Voided Slab | Monolithic |  (a) |
| Stressed Wood Deck | Integral Wood |   (b) |
| Glued/Spiked Wood Panels with Spreader Beam | Integral Wood |  (c) |

### 4.6.2.4 Truss and Arch Bridges

The lever rule may be used for the distribution of gravity loads in trusses and arches when analyzed as planar structures. If a space analysis is used, either the lever rule or direct loading through the deck or deck system may be used.

Where loads, other than the self-weight of the members and wind loads there on, are transmitted to the truss at the panel points, the truss may be analyzed as a pin-connected assembly.

### 4.6.2.5 Effective Length Factor, *K*

Physical column lengths shall be multiplied by an effective length factor, *K*, to compensate for rotational and translational boundary conditions other than pinned ends.

In the absence of a more refined analysis, where lateral stability is provided by diagonal bracing or other suitable means, the effective length factor in the braced plane, *K*, for the compression members in triangulated trusses, trusses, and frames may be taken as:

- The remainder of this page is intentionally left blank.For bolted or welded end connections at both ends: $K = 0.750$
- For pinned connections at both ends: $K = 0.875$
- For single angles, regardless of end connection: $K = 1.0$

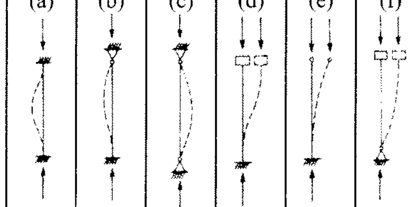
Vierendeel trusses shall be treated as unbraced frames.

### C4.6.2.5

Equations for the compressive resistance of columns and moment magnification factors for beam-columns include a factor, *K*, which is used to modify the length according to the restraint at the ends of the column against rotation and translation.

*K* is the ratio of the effective length of an idealized pin-end column to the actual length of a column with various other end conditions. *KL* represents the length between inflection points of a buckled column influenced by the restraint against rotation and translation of column ends. Theoretical values of *K*, as provided by the Structural Stability Research Council, are given in Table C1 for some idealized column end conditions.

**Table C4.6.2.5-1 Effective Length Factors, *K*.**

| | (a) | (b) | (c) | (d) | (e) | (f) |
|---|---|---|---|---|---|---|
| Buckled shape of column is shown by dashed line |  | | | | | |
| Theoretical *K* value | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Design value of *K* when ideal conditions are approximated | 0.65 | 0.80 | 1.2 | 1.0 | 2.1 | 2.0 |
| End condition code |  | Rotation fixed | | Translation fixed | | |
| | | Rotation free | | Translation fixed | | |
| | | Rotation fixed | | Translation free | | |
| | | Rotation free | | Translation free | | |

Because actual column end conditions seldom comply fully with idealized restraint conditions against rotation and translation, the design values suggested by the Structural Stability Research Council are higher than the idealized values.

Lateral stability of columns in continuous frames, unbraced by attachment to shear walls, diagonal bracing, or adjacent structures, depends on the flexural stiffness of the rigidly connected beams. Therefore, the effective length factor, *K*, is a function of the total flexural restraint provided by the beams at the ends of the column. If the stiffness of the beams is small in relation to that of the column, the value of *K* could exceed 2.0.

Single angles are loaded through one leg and are subject to eccentricity and twist, which is often not recognized. $K$ is set equal to 1.0 for these members to more closely match the strength provided in the Guide for Design of Steel Transmission Towers (*ASCE Manual No. 52, 1971*).

Assuming that only elastic action occurs and that all columns buckle simultaneously, it can be shown that (*Chen and Liu, 1991; ASCE Task Committee on Effective Length, 1997*):

For braced frames:

$$\frac{G_a G_b}{4}\left(\frac{\pi}{K}\right)^2 + \frac{G_a + G_b}{2}\left(1 - \frac{\frac{\pi}{K}}{tan\left(\frac{\pi}{K}\right)}\right) + \frac{2\tan\left(\frac{\pi}{2K}\right)}{\frac{\pi}{K}} = 1 \qquad \text{(C4.6.2.5-1)}$$

For unbraced frames:

$$\frac{G_a G_b\left(\frac{\pi}{K}\right)^2 - 36}{6\left(G_a + G_b\right)} = \frac{\frac{\pi}{K}}{\tan\left(\frac{\pi}{K}\right)} \qquad \text{(C4.6.2.5-2)}$$

where subscripts $a$ and $b$ refer to the two ends of the column under consideration

in which:

$$G = \frac{\Sigma\left(\frac{E_c I_c}{L_c}\right)}{\Sigma\left(\frac{E_g I_c}{L_g}\right)} \qquad \text{(C4.6.2.5-3)}$$

where:

$\Sigma$ = summation of the properties of components rigidly connected to an end of the column in the plane of flexure

$E_c$ = modulus of elasticity of column (ksi)

$I_c$ = moment of inertia of column (in.$^4$)

$L_c$ = unbraced length of column (in.)

$E_g$ = modulus of elasticity of beam or other restraining member (ksi)

$I_g$ = moment of inertia of beam or other restraining member (in.$^4$)

$L_g$ = unsupported length of beam or other restraining member (in.)

$K$ = effective length factor for the column under consideration

Figures C1 and C2 are graphical representations of the relationship among $K$, $G_a$, and $G_b$ for Eqs. C1 and C2, respectively. The figures can be used to obtain values of $K$ directly.

Eqs. C1, C2, and the alignment charts in Figures C1 and C2 are based on assumptions of idealized conditions. The development of the chart and formula can be found in textbooks such as Salmon and Johnson (*1990*) and Chen and Lui (*1991*). When actual conditions differ significantly from these idealized assumptions, unrealistic designs may result. Galambos (*1988*), Yura (*1971*), Disque (*1973*), Duan and Chen (*1988*), and AISC (*1993*) may be used to evaluate end conditions more accurately.

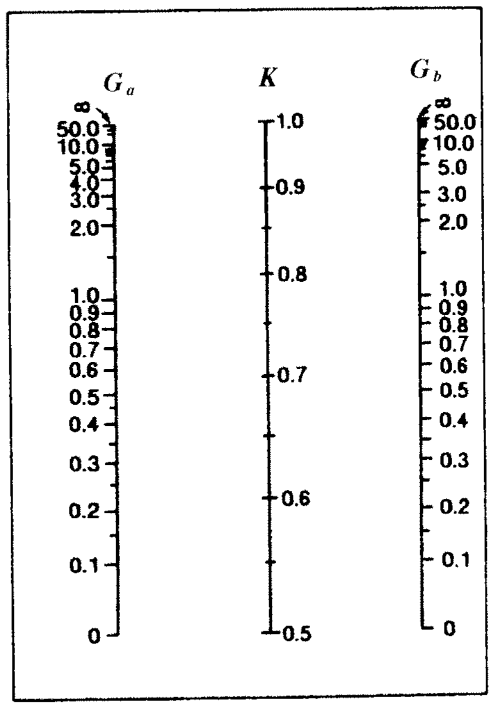



**Figure C4.6.2.5-1 Alignment Chart for Determining Effective Length Factor, *K*, for Braced Frames.**

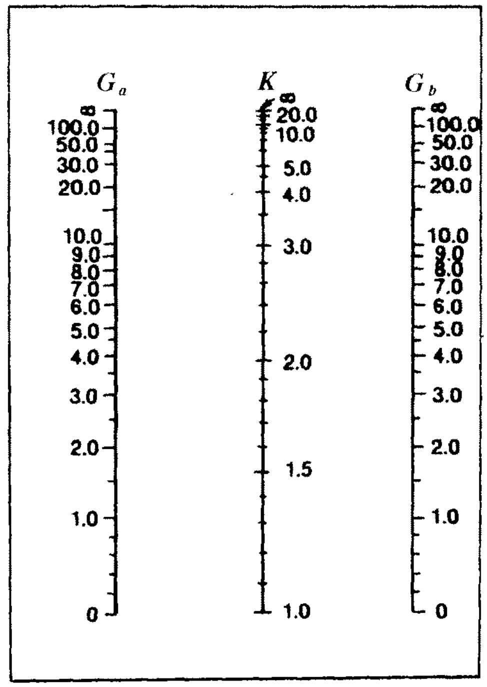



**Figure C4.6.2.5-2 Alignment Chart for Determining Effective Length Factor, *K*, for Unbraced Frames.**

The following applies to the use of Figures C1 and C2:

- For column ends supported by but not rigidly connected to a footing or foundation, $G$ is theoretically equal to infinity, but unless actually designed as a true frictionless pin, may be taken equal to 10 for practical design. If the column end is rigidly attached to a properly designed footing, $G$ may be taken equal to 1.0. Smaller values may be taken if justified by analysis.

- In computing effective length factors for members with monolithic connections, it is important to properly evaluate the degree of fixity in the foundation using engineering judgment. In absence of a more refined analysis, the following values can be used:

| Condition | $G$ |
|---|---|
| Footing anchored on rock | 1.5 |
| Footing not anchored on rock | 3.0 |
| Footing on soil | 5.0 |
| Footing on multiple rows of end bearing piles | 1.0 |

In lieu of the alignment charts, the following alternative $K$-factor equations (*Duan, King, and Chen, 1993*) may be used.

For braced frames:

$$K = 1 - \frac{1}{5+9G_a} - \frac{1}{5+9G_b} - \frac{1}{10+G_aG_b} \qquad \text{(C4.6.2.5-4)}$$

For unbraced frames:

- For $K < 2$

$$K = 4 - \frac{1}{1+0.2G_a} - \frac{1}{1+0.2G_b} - \frac{1}{1+0.01G_aG_b} \qquad \text{(C4.6.2.5-5)}$$

- For $K \geq 2$

$$K = \frac{2\pi a}{0.9+\sqrt{0.81+4ab}} \qquad \text{(C4.6.2.5-6)}$$

in which:

$$a = \frac{G_aG_b}{G_a+G_b} + 3 \qquad \text{(C4.6.2.5-7)}$$

$$b = \frac{36}{G_a+G_b} + 6 \qquad \text{(C4.6.2.5-8)}$$

Eq. C5 is used first. If the value of $K$ calculated by Eq. C5 is greater than 2, Eq. C6 is used. The values for $K$ calculated using Eqs. C5 and C6 are a good fit with results from the alignment chart Eqs. C1, C2, C3, and allow an Engineer to perform a direct noniterative solution for $K$.

### 4.6.2.6 Effective Flange Width

#### *4.6.2.6.1 General*

In the absence of a more refined analysis and/or unless otherwise specified, limits of the width of a concrete slab, taken as effective in composite action for determining resistance for all limit states, shall be as specified herein. The calculation of deflections should be based on the full flange width. For the calculation of live load deflections, where required, the provisions of Article 2.5.2.6.2 shall apply.

The effective span length used in calculating effective flange width may be taken as the actual span for simply supported spans and the distance between points of permanent load inflection for continuous spans, as appropriate for either positive or negative moments.

#### *C4.6.2.6.1*

Longitudinal stresses in the flanges are spread across the flange and the composite deck slab by in-plane shear stresses. Therefore, the longitudinal stresses are not uniform. The effective flange width is a reduced width over which the longitudinal stresses are assumed to be uniformly distributed and yet result in the same force as the nonuniform stress distribution would if integrated over the whole width.