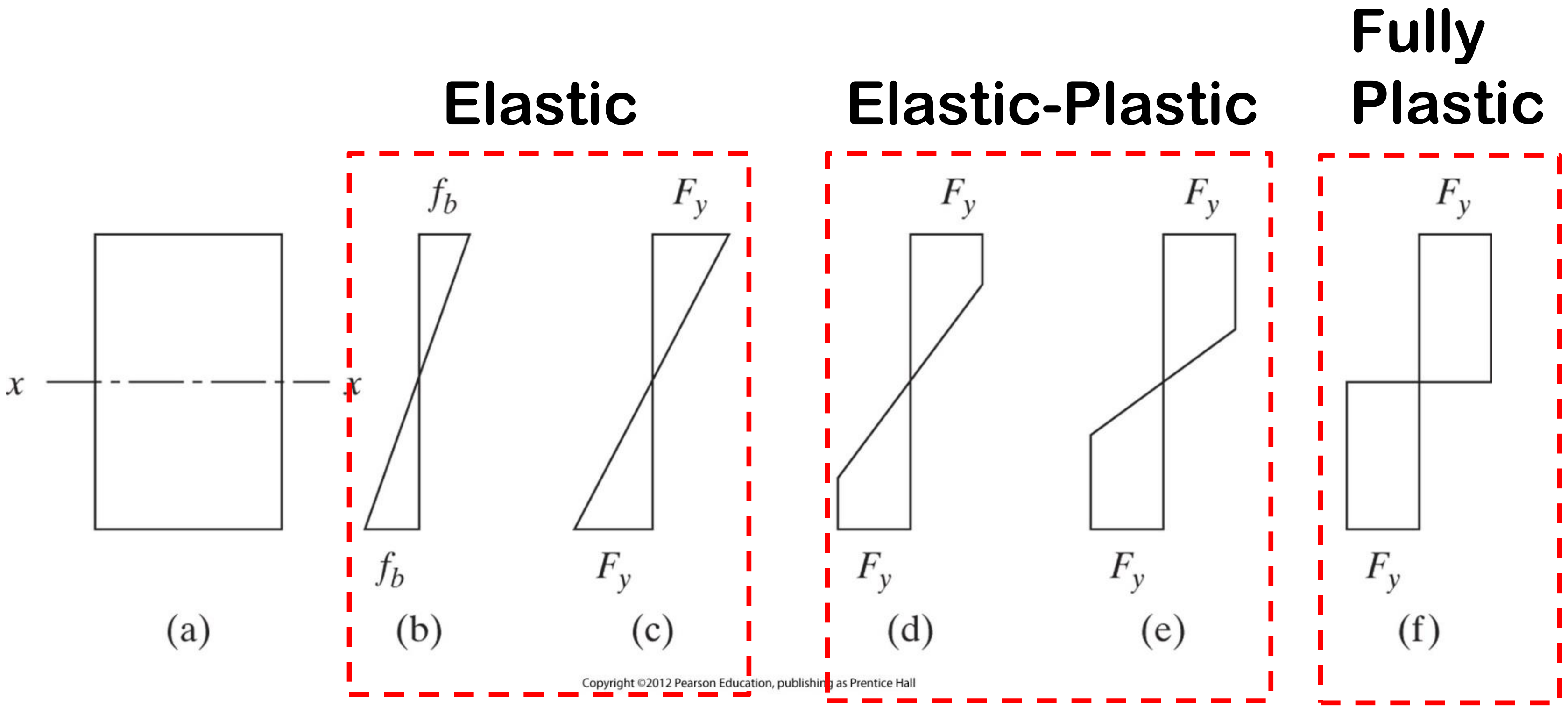
Design of Flexural Members
Chapter 8-10

Objectives are to review:

- Flexural Design [16.F]
- Shear: [16.G2]
- Serviceability: [16.L3] Deflection

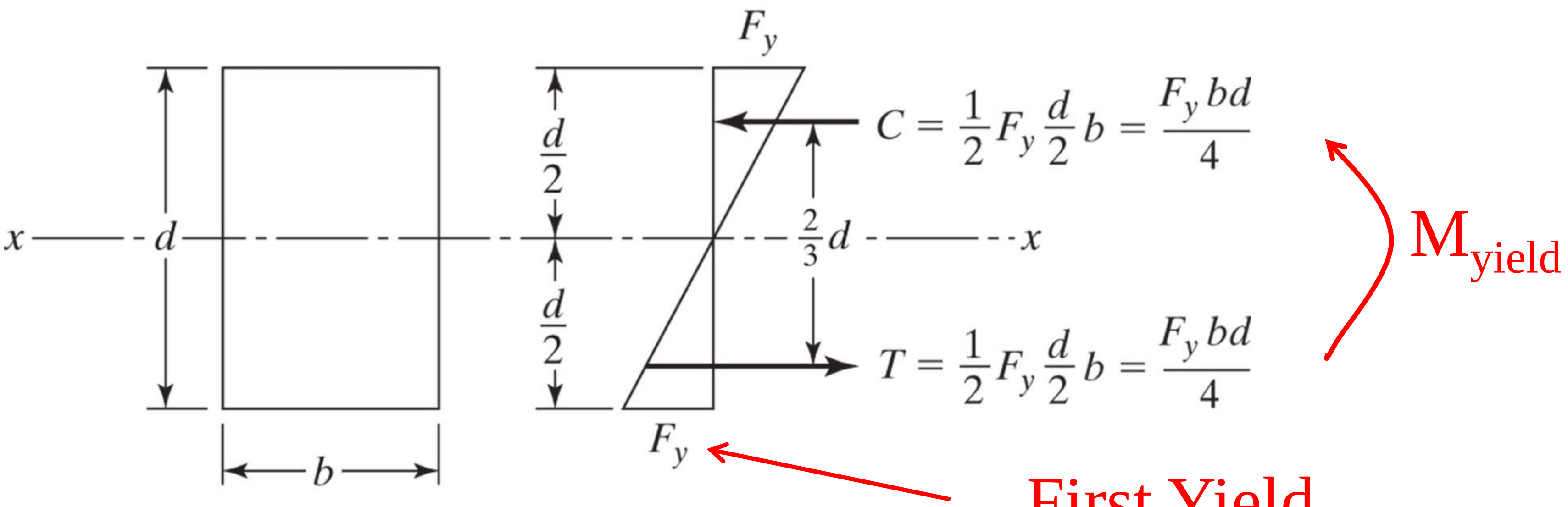
1. Briefly Discuss Vibration [16.L5] & Ponding [16.App 2]
2. Evaluate members with concentrated loads:[16.J10]
3. Evaluate members with biaxial bending: [16.H1]
4. Determine Shear Center and Torsion applied to members
5. Design Beam Bearing Plates: [16.J8]
6. Design Lateral Bracing : [16.App 6]

Flexural Stress



Flexural Stress
Elastic Section Modulus, S

Rectangular Section: $S_{rect.}$

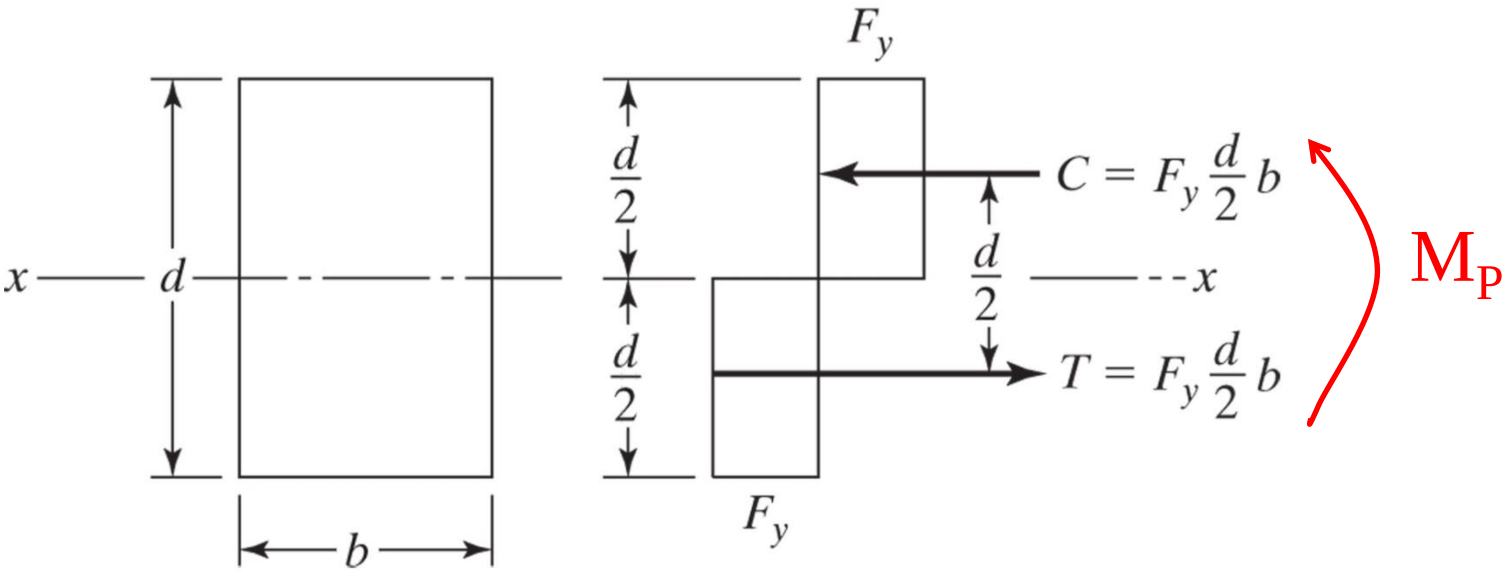


$M_y = F_y\ [(bd/4)\ (2d/3)] = F_y\ [bd^2/6]$

Therefore, elastic section modulus, $S_{rect} = bd^2/6$

Flexural Stress
Plastic Section Modulus, Z

Rectangular Section: $Z_{rect.}$



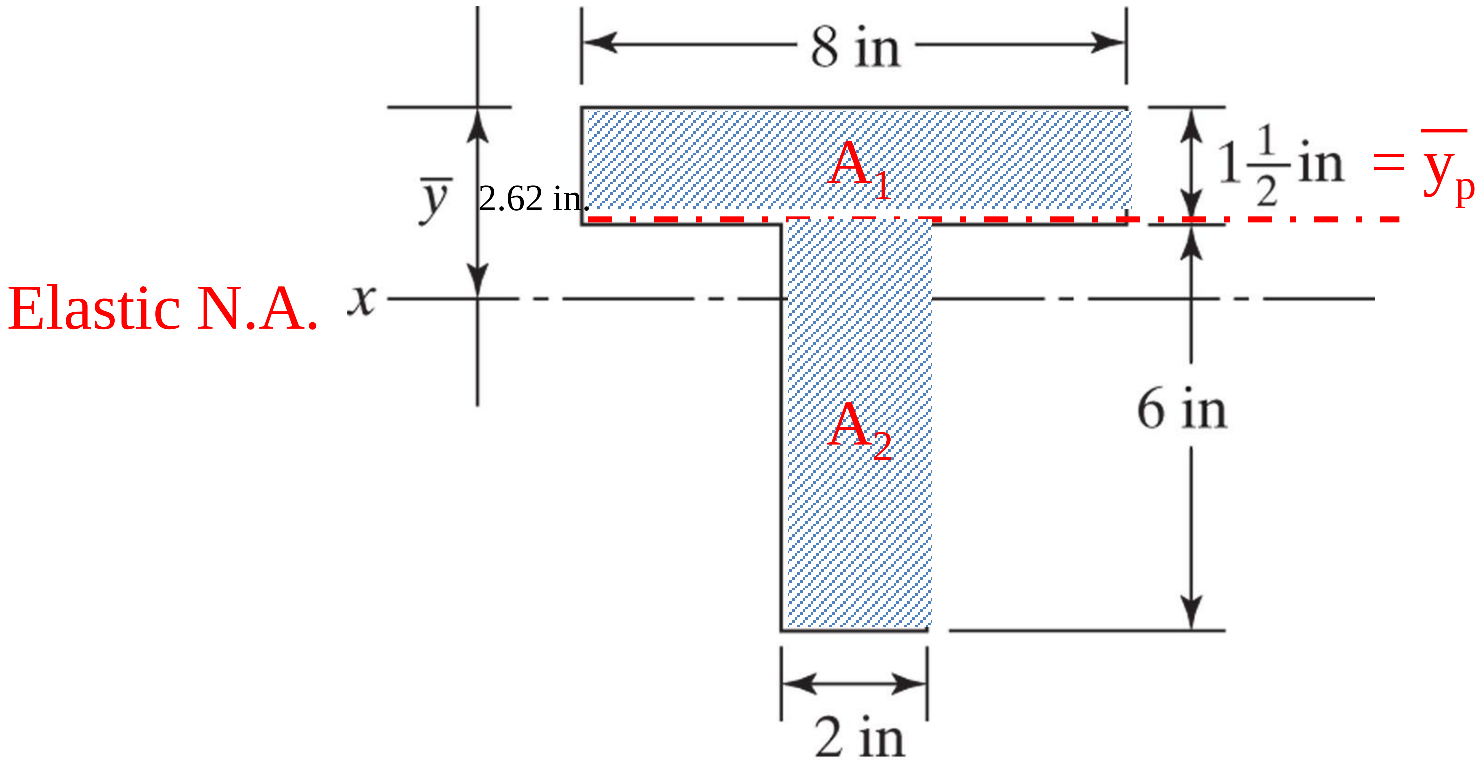
$M_P = M_N = F_y\, [(bd/2)\,(d/2)] = F_y\, [bd^2/4]$

Therefore, plastic section modulus, $Z_{rect.} = [bd^2/4]$

Plastic Section Modulus, Z

Example 8-1

First, we must find the Plastic Neutral Axis (N.A.).



To find the Plastic N.A. set $A_1 = A_2$ and solve for $\overline{y}_p = 1.5$ in

Plastic Section Modulus, $Z = \sum A_i \, |\overline{y}_i - \overline{y}_p|$

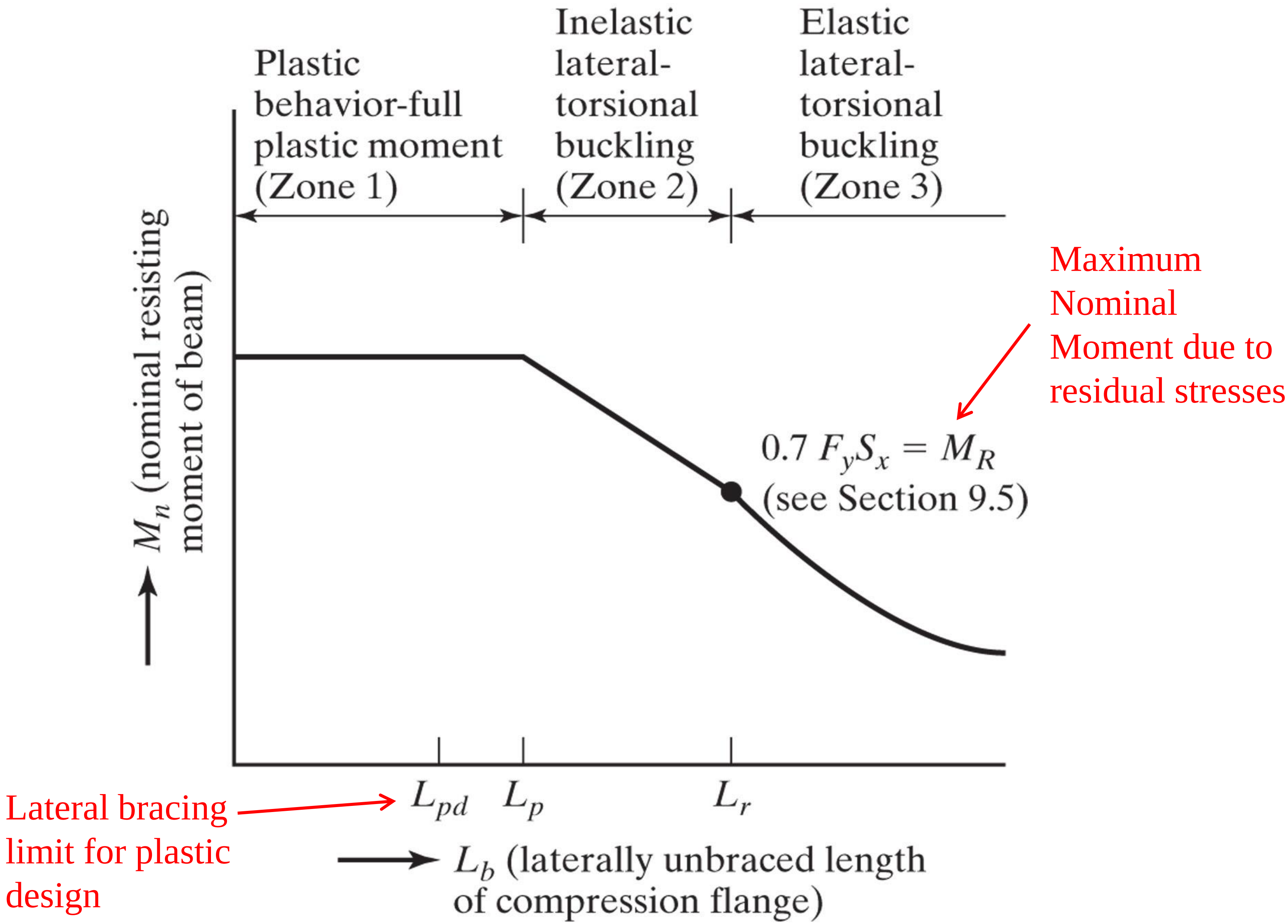
Flexure Limit States

Chapter 9-AISC Part 16.F

1. **Full Section Yielding**
   - Continuously Braced (Laterally) or $L_b < L_p$
   - Plastic Behavior Controls
   - Compact sections

2. **Inelastic Lateral-Torsional Buckling**
   - Braced at intermediate intervals, $L_p < L_b < L_r$

3. **Elastic Lateral-Torsional Buckling**
   - Braced at long intervals, $L_r < L_b$


Plastic behavior-full plastic moment (Zone 1)
Inelastic lateral-torsional buckling (Zone 2)
Elastic lateral-torsional buckling (Zone 3)
M_n (nominal resisting moment of beam)
$0.7\,F_yS_x = M_R$
(see Section 9.5)
Maximum Nominal Moment due to residual stresses
L_{pd}
L_p
L_r
Lateral bracing limit for plastic design
L_b (laterally unbraced length of compression flange)

Plastic Behavior
Full Yielding-Elastic Design

Requirements:

1. **Member must be braced such that:**

$$L_b \leq L_p$$

Where,

L_b = Distance between points in the compression flange that are either braced against lateral displacement or against twist of the section.

L_p = Limiting Braced Distance for Full Plastic Yielding and the Beginning of Inelastic Lateral-Torsional Buckling (LTB) using elastic analysis

Plastic Behavior

Full Yielding-Elastic Design

Compression Flange Bracing Limits for compact I-shaped Members and Channels:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad \text{EQ. F2-5}$$

Note: L_p (LTB) does not apply to circular or square HSS or I-shaped members and channels with flexure about weak axis.

Plastic Behavior

2. Member must be <u>compact</u>.

- <u>Compact</u> sections are capable of reaching full plastic yielding before buckling.

- Width-to-Thickness Ratio Limits for compact elements are specified in AISC Table B4.1b.

$$\lambda \le \lambda_r$$

where,

λ = Element Width-to-Thickness Ratio

λ_r = Element compact/noncompact limit

Note: Most hot-rolled shapes are compact (see user note in AISC 16.F2). Other restrictions for λ_r apply for plastic analysis (see Appendix 1-1.2.2).

Plastic Behavior

Nominal Moment Capacity:

$$M_n = M_p = F_y Z \qquad \text{EQ. F2-1}$$

$$\Phi_b = 0.9 \text{ (LRFD)} \qquad \Omega_b = 1.67 \text{ (ASD)}$$

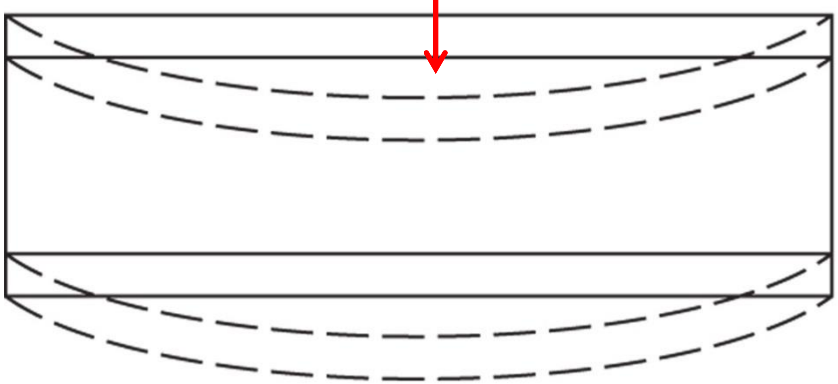
Table 3-2 AISC provides Plastic moment capacity for W shapes.

Zone 2 and 3 Behavior
Lateral Torsional Buckling (LTB)

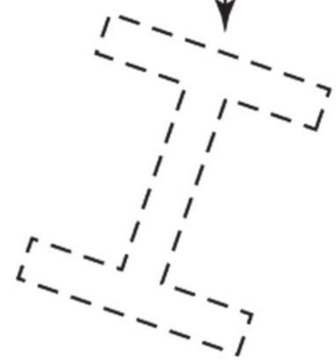
Lateral Buckling of
Compression Flange

Causes

Rotation or twisting of
cross section

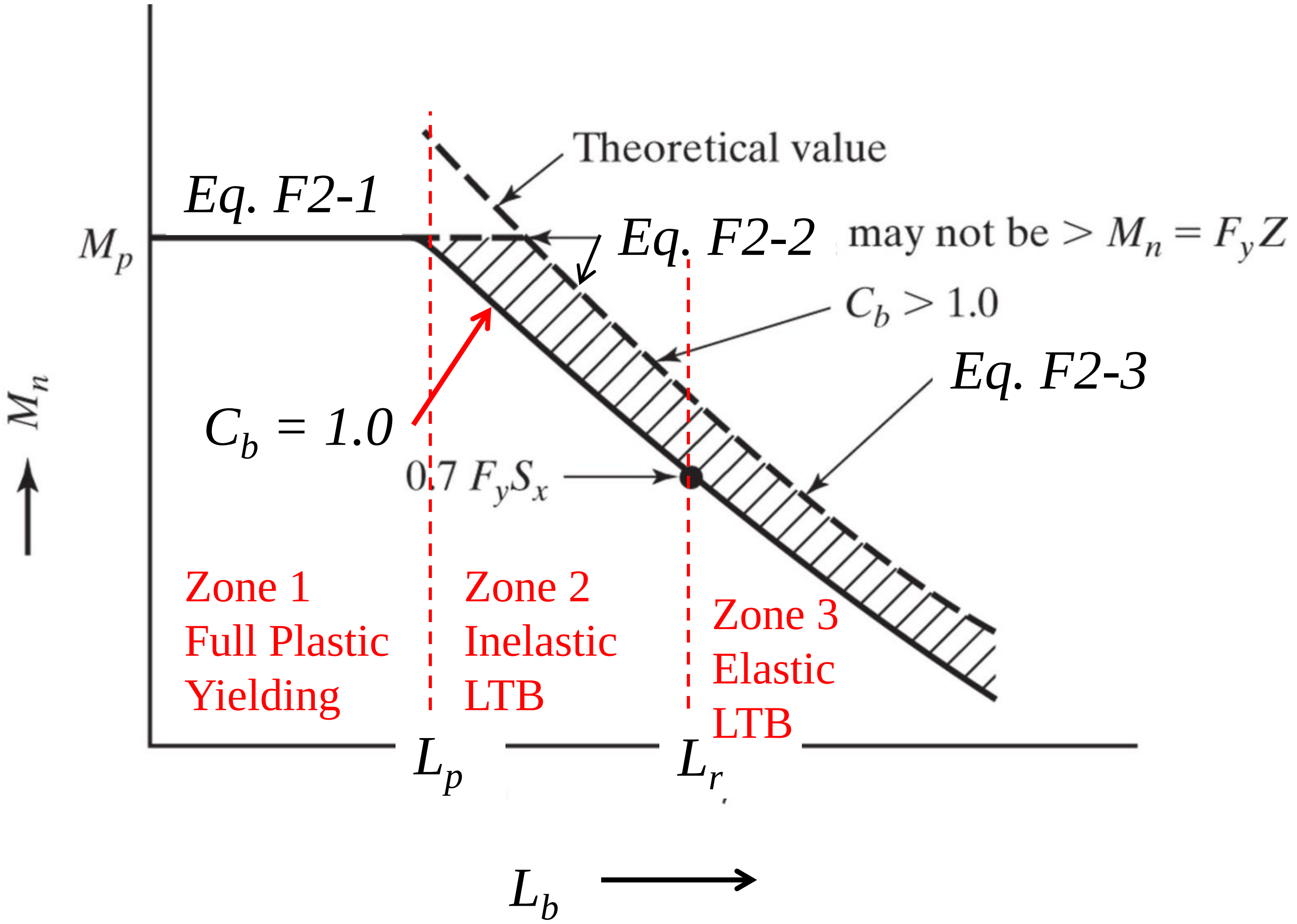


Plan View



Section View

Nominal Moment vs. Unbraced Length
Compact Sections



Nominal Moment Capacity
Zone 2-Inelastic LTB

AISC Eq. F2-2: ***I-Shaped Compact Sections***

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p$$

L_r = *Eq. F2-6 AISC.*

Note: L_p and L_r are provided in Table 3-2 for W shapes

Nominal Moment Capacity

Zone 2-Inelastic LTB

From AISC Eq. F2-2:

$$\emptyset M_n = \emptyset C_b \left[M_{px} - \left(M_{px} - 0.7 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq \emptyset M_p$$

We can simplify this equation as follows:

LRFD

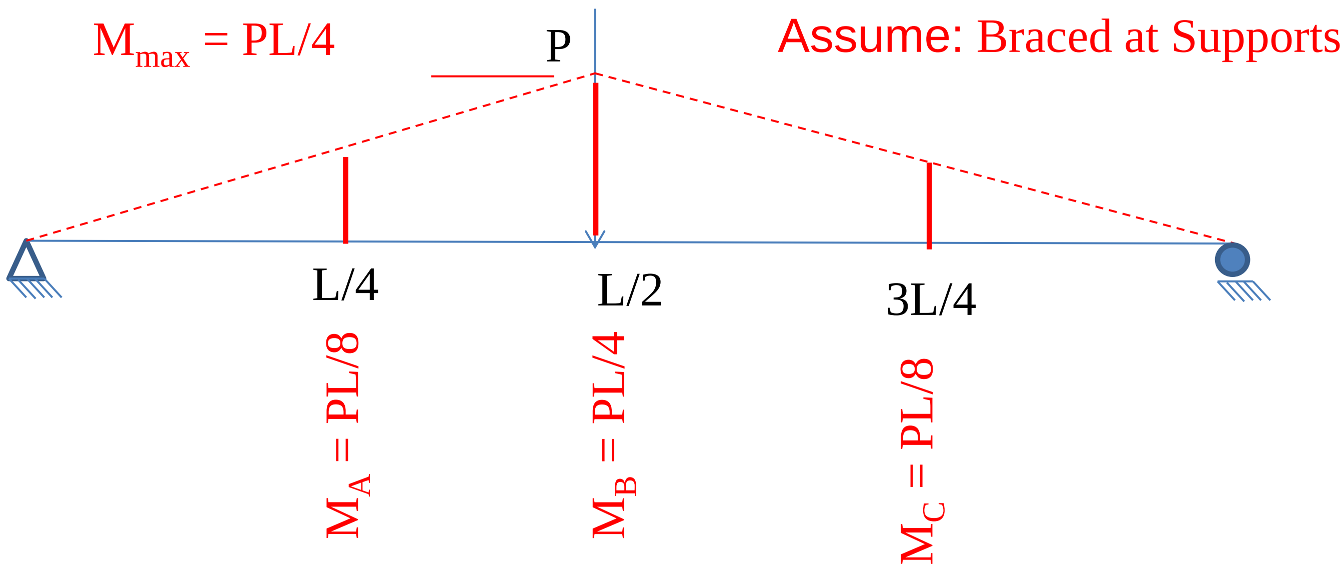
$$\emptyset M_n = C_b \left[\emptyset M_{px} - (BF)\left(L_b - L_p \right) \right] \leq \emptyset M_{px} \quad \textbf{Eq. 3-4a}$$

*Since **BF** is provided in Table 3-2, we can interpolate ΦM_n for any $L_p < L_b \leq L_r$ for all W shapes.*

Bending Coefficients, C_b

Lateral-Torsional Buckling Modification Factor

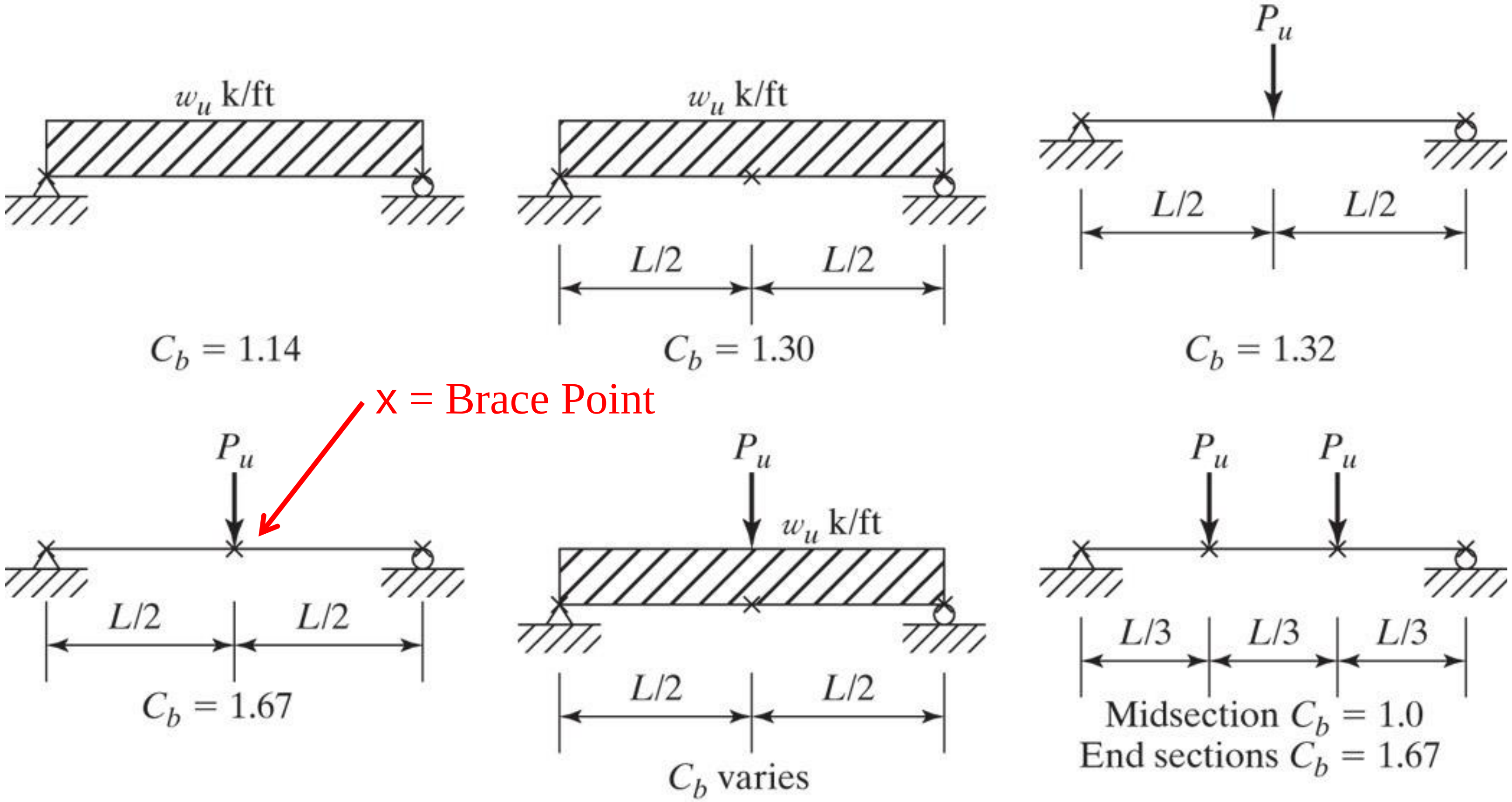
$$C_b = \frac{12.5\ M_{max}}{2.5\ M_{max} + 3\ M_A + 4\ M_B + 3\ M_C} \quad (Eq. F1-1)$$



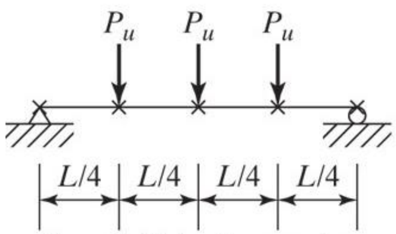
C_b = [12.5/4]/[(2.5/4+3/8+4/4+3/8) = 1.316

Beam Bending Coefficients, C_b

Table 3-1 AISC



Bending Coefficients, C_b

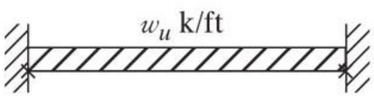


$C_b = 1.11$ for two center sections and 1.67 for end ones

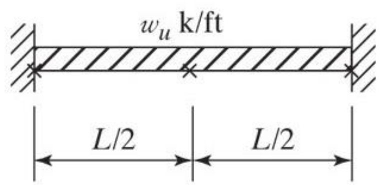
$C_b = 1.0$

1.67

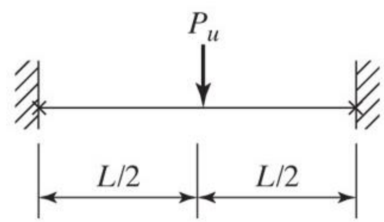
$C_b = 2.27$



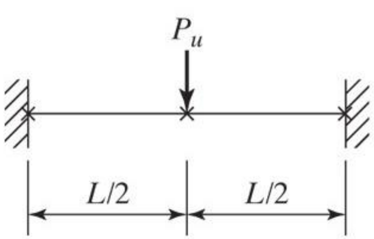
$C_b = 2.38$



$C_b = 2.38$



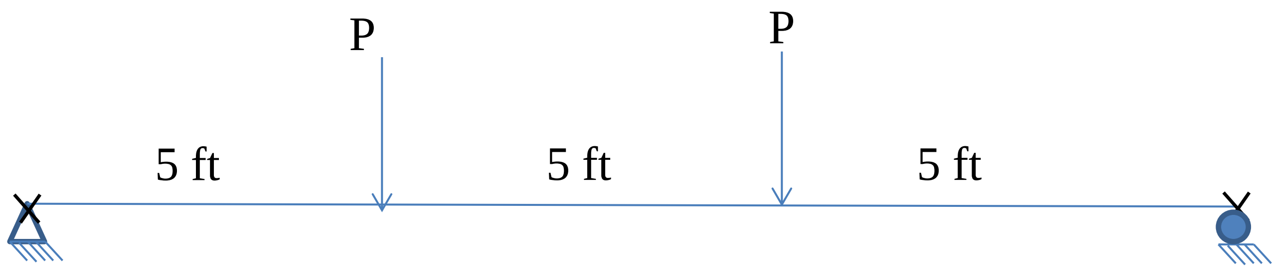
$C_b = 1.92$



$C_b = 2.27$

Example 1

Given: The simply supported W18 x 55 beam is braced at ends only. P_{LL} = 40 kips, P_{DL} = 10 kips.



Find:

1. Is the beam adequate for flexure? Check LRFD and ASD

Nominal Moment Capacity
Zone 3-Elastic LTB

AISC Eq. F2-3: ***I-Shaped Compact Sections***

$M_n = F_{cr}S_x \leq M_p$ ***Eq. F2-3***

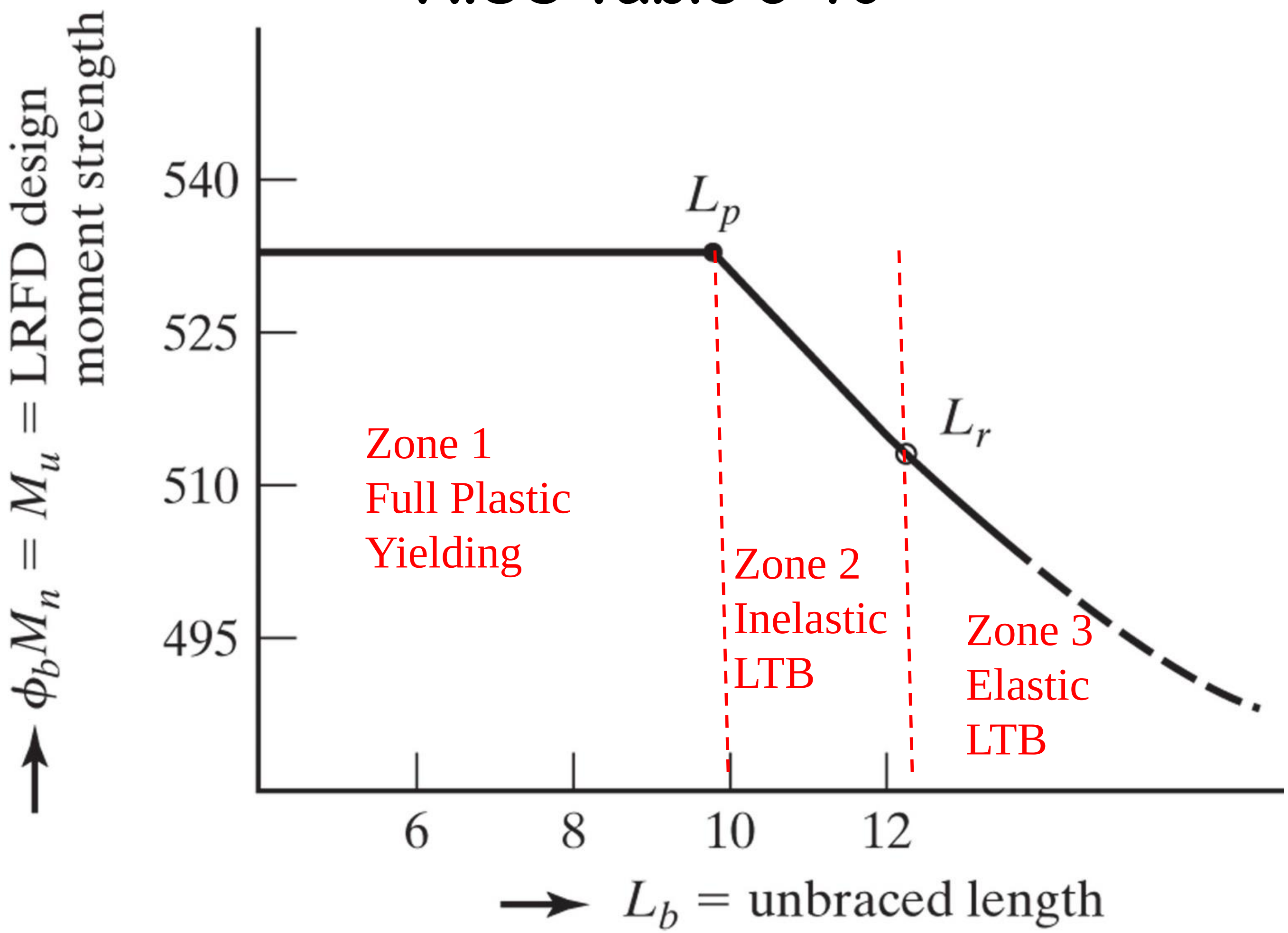
where,

F_{cr} = *Critical Elastic Buckling Stress in Beams.* (***EQ. F2-4***)

S_x = *Elastic Section Modulus about x- axis*

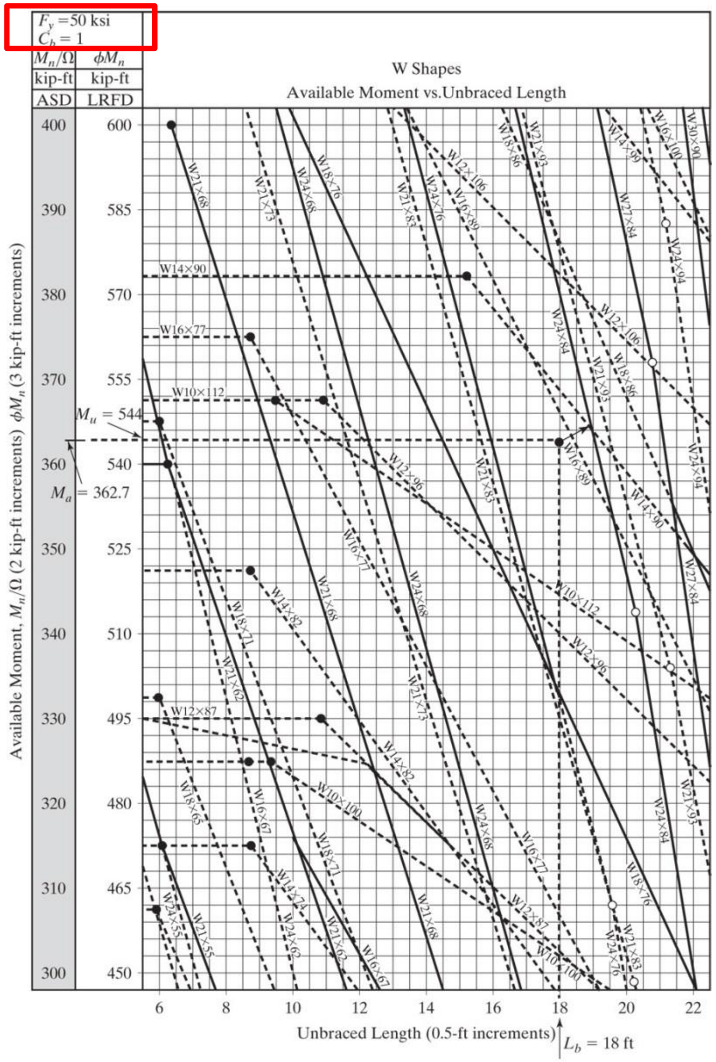
Design Chart

AISC Table 3-10



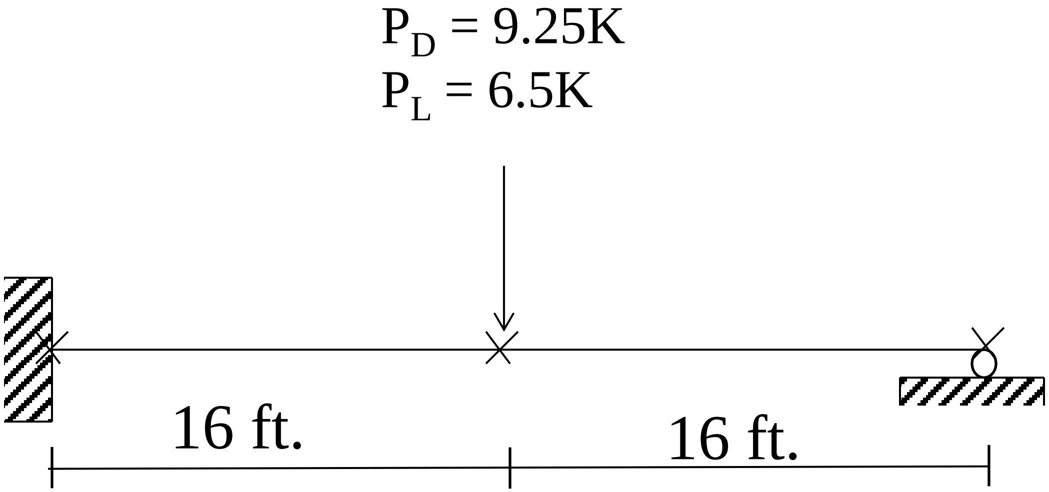
Design Chart

AISC Table 3-10



Example 2
Problem 9-27

Given: W16 x 36 A992 beam



Find :

1. Using Elastic Analysis, is beam adequate? Neglect self-wt. and assume C_b =1.0. LRFD and ASD

Nominal Moment Capacity
Non-Compact Sections

- W-shapes with non-compact compression flanges are included in Table 3-2 and have reduced strength due to local buckling and LTB.

- Section F3 of the AISC specification applies to non-compact I-shaped members and built up members with slender compression flanges.

Nominal Moment Capacity
Non-Compact Sections

Two Limit States: Full yielding does not apply

1. F3.1 Lateral Torsional Buckling: Same as F2.2

2. F3.2 Compression Flange Local Buckling:

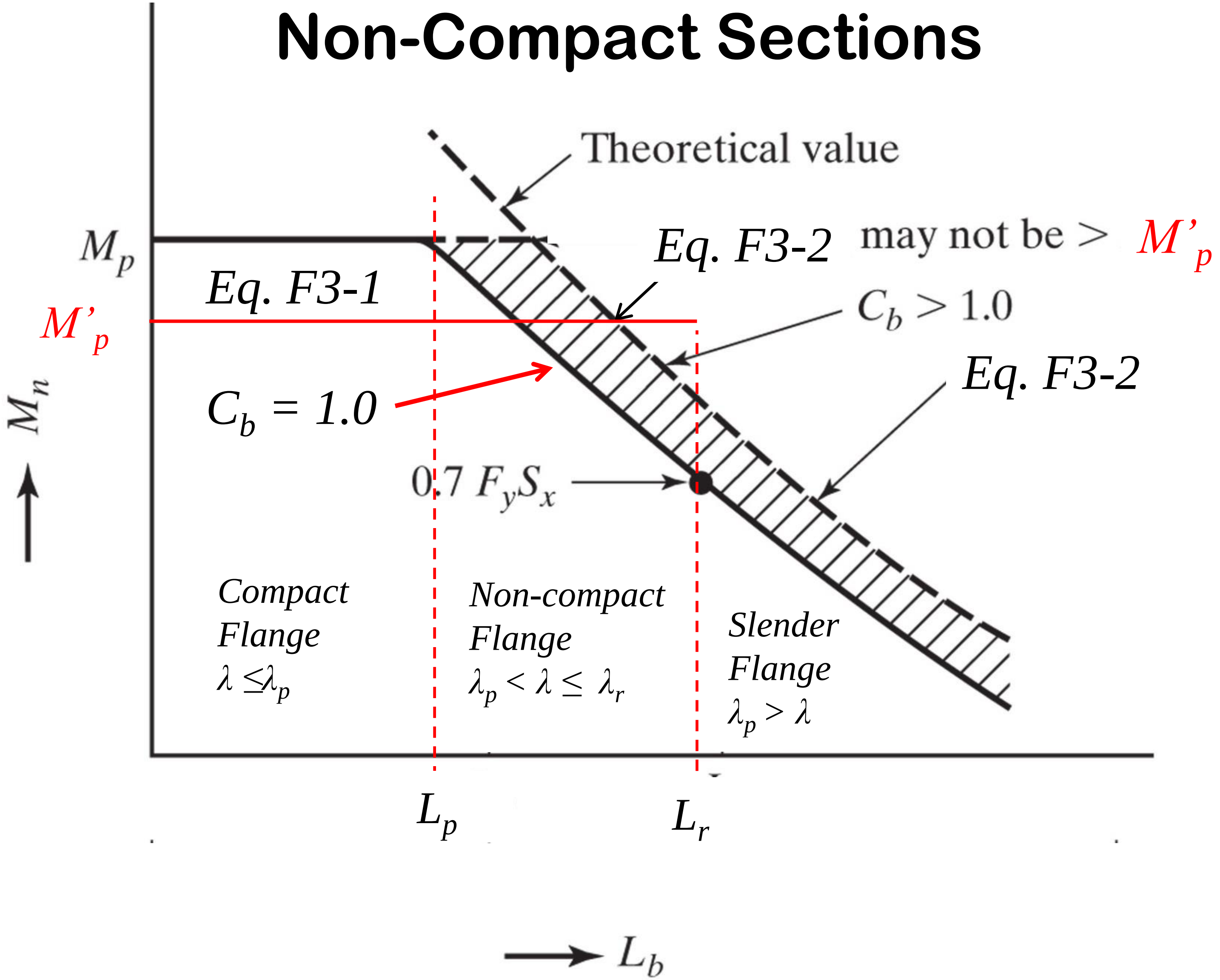
    a.) Non-compact Flanges:

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad \text{Eq. F3-1}$$

    b.) Slender Flanges:

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2} \qquad \text{Eq. F3-2}$$

Nominal Moment vs. Unbraced Length
Non-Compact Sections



Class Problem 1

Given:

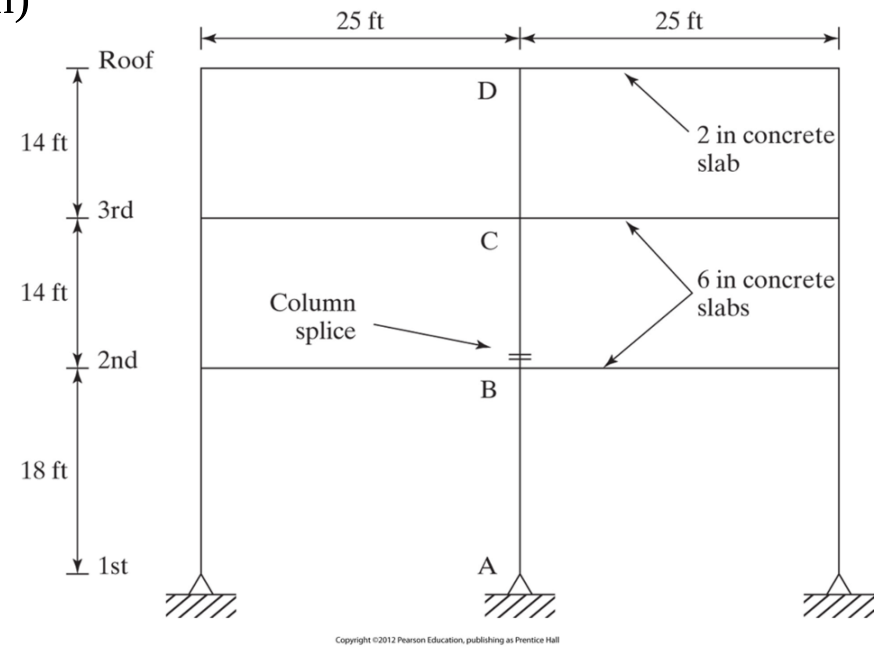
	LL(psf)	DL (psf)
Roof	30	10 (roofing only)
Floor	75	12 (superimposed)
		15 (partition)

Bay spacing = 35ft.
Density of Concrete = 150 pcf

Find: Design interior Beams (L = 35ft):

1. 2nd & 3rd Floor (beams @ 5' c.c.)
2. Roof (beams @ 6'-4" c.c)

Method:(LRFD)



Shear Design

AISC Part 16.G1: General Provisions

All beams must be designed for adequate Web Shear Strength:

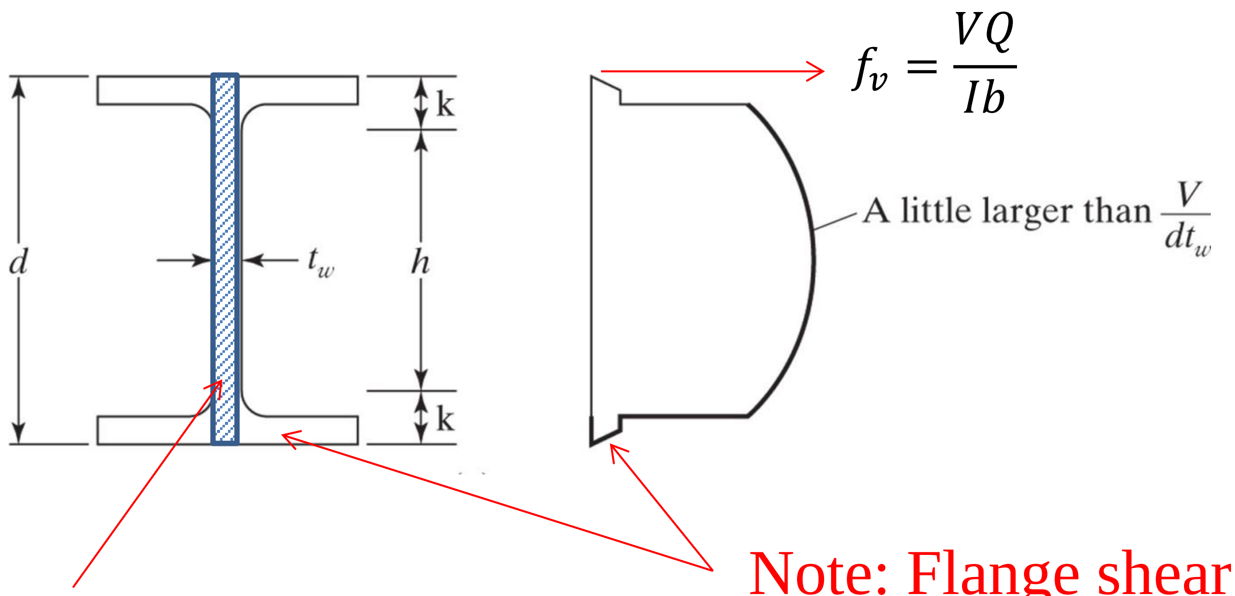
$\Phi_v V_n \geq V_u$ (LRFD) $\qquad$ $V_n / \Omega_v \geq V_u$ (ASD)

$\Phi_v = 1.0$ $\qquad$ $\Omega_v = 1.5$

- Shear strength is generally not a problem for most rolled shapes except:
  - High concentrated loads near supports
  - Notched or coped beams
  - Thin webs

- Shear strength can be a critical consideration for built-up Plate girders because of the relatively thin webs.

Web Shear Stress



Area of Web (A_w) = d t_w

Note: Flange shear strength is negligible.

Web Shear Strength

Shear Yielding and Buckling

$$V_n = 0.6F_y\, A_w\, C_v \qquad \text{G2-1}$$

G2.1 a) For all rolled I-shaped members with $\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$

$\Phi_v = 1.0$ (LRFD) , $\Omega_v = 1.5$ (ASD)

and

$C_v = 1.0$ (Web Shear Coefficient) G2-2

Few exceptions to the above requirements are listed in the user note.

Web Shear Coefficient, C_v

G2.1 b.) For all other shapes except round HSS, C_v depends on <u>web slenderness ratio h/t_w</u> and the presence of transverse web stiffeners.

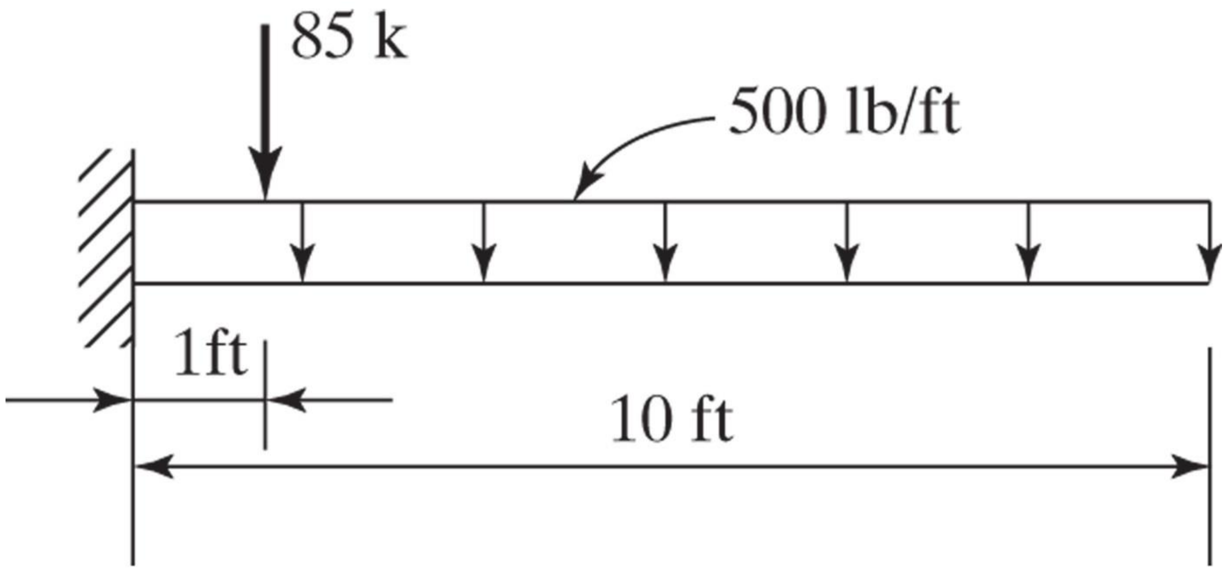
- Use AISC Equations G2-3 through G2-5 to obtain C_v

Note: Transverse web stiffeners will increase web shear strength as determined by the web plate buckling coefficient, k_v utilized in these equations.

Table 3-2 provides shear strength values for all rolled shapes without web stiffeners and includes web slenderness considerations.

Example 4
Problem 10-21

Given: W14 x 34 A992 beam in diagram below



Find: Is Beam adequate for Shear? LRFD and ASD.

Design for Serviceability

AISC Part 16 Chapter L

L1: *"Serviceability* is a state in which the function of a building, its appearance, maintainability, durability, and comfort of its occupants are preserved under normal usage."

- Camber
- Deflections
- Drift
- Vibration
- Wind-induced motion
- Thermal expansion and contraction
- Connection slip

Design for Serviceability
Beam Deflection

- Maximum beam deflections are evaluated under service load conditions for each span.

- Span deflection limits are specified by building codes and AASHTO for bridges.

Typical Requirements:

- For buildings: $\Delta_{LL} <$ **span length/360 Floors**

- For bridges : $\Delta_{LL+I} <$ **span length/800 to 1000**

Beam Deflection Limits

IBC 2009

TABLE 10.1 Deflection Limits from IBC 2009

Members	Loading conditions		
	L	D + L	S or W
For floor members	$\frac{L}{360}$	$\frac{L}{240}$	—
For roof members supporting plaster ceiling*	$\frac{L}{360}$	$\frac{L}{240}$	$\frac{L}{360}$
For roof members supporting nonplaster ceilings*	$\frac{L}{240}$	$\frac{L}{180}$	$\frac{L}{240}$
For roof members not supporting ceilings*	$\frac{L}{180}$	$\frac{L}{120}$	$\frac{L}{180}$

*All roof members should be investigated for ponding.

Beam Deflection

Steps for Checking Service Load Deflections

1. Use Beam Diagrams/Structural analysis software to calculate maximum deflection for each span under Service (un-factored) Loads.

2. Check $\Delta_{LL(max)} \leq L/360$ for Floors

   $\Delta_{LL + D(max)} \leq L/240$ for Floors

3. Increase Beam Size (Moment of Inertia) if requirements are not satisfied. Use Table 3-3.

4. Consider cambering long beams to minimize dead load deflections and permanent sag in floors.

Beams with Concentrated Loads [AISC 16J10]

1. **Forces applied normal to the flange of W-shaped sections.**

2. **Type of Forces**
   A. **Single concentrated force (tension or compression)**
   B. **Double concentrated force (force couple acting on ones side of the member)**

Members with concentrated loads applied to flanges

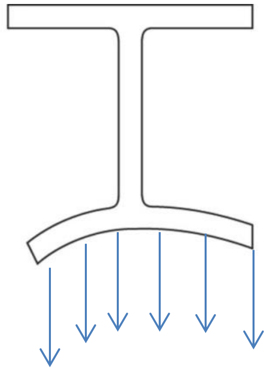
C&W Warehouse, Spartanburg, SC. (Courtesy of Britt, Peters and Associates.)

Beams with Concentrated Loads [AISC 16J10]

1. **Limit States**
   A. **Flange Local Bending (Tensile only)** [J10.1]
   B. **Web Local Yielding (All)** [J10.2]
   C. **Web Local Crippling (Compression only)** [J10.3]
   D. **Web Sidesway Buckling (Compression & unrestrained for torsion)** [J10.4]
   E. **Web Compression Buckling (Compression both flanges)** [J10.5]
   F. **Web Panel Zone Shear(Force couple applied to one or both flanges** [J10.6]

Beams with Concentrated Loads

Flange Local Bending



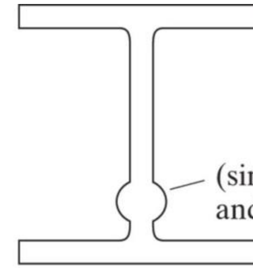
16.J10.1 : Flange Local Bending

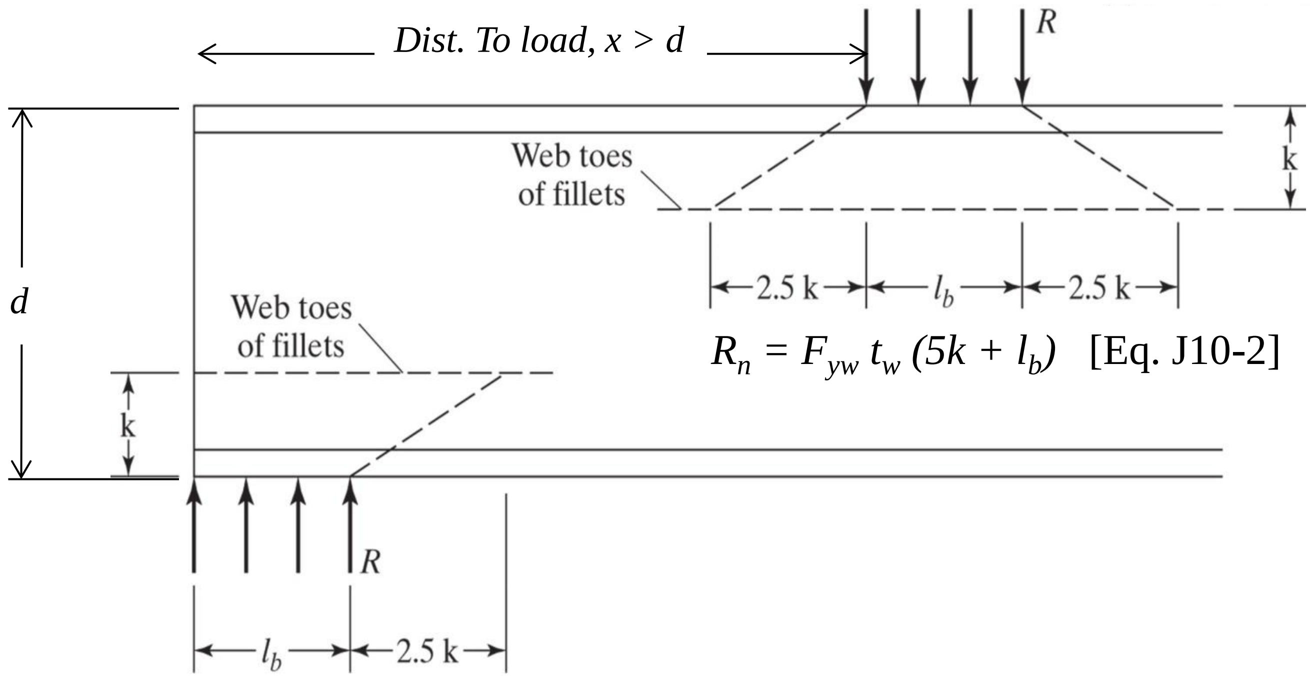
- Tensile loads applied to flanges

$R_n = 6.25F_{yf}\ t_f^2$ *(kips)* **[Eq. J10-1]**

Local Web Yielding

[AISC 16.J10.2]

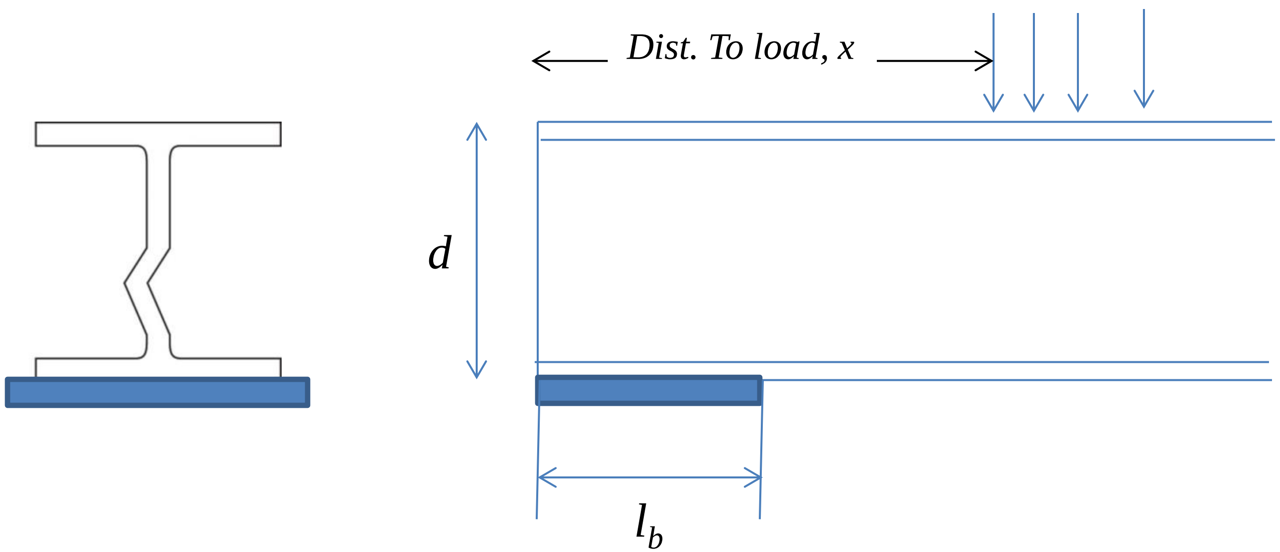




$R_n = F_{yw}\, t_w\, (5k + l_b)$ [Eq. J10-2]

$R_n = F_{yw}\, t_w\, (2.5k + l_b)$ [Eq. J10-3]

Beams with Concentrated Loads
Web Crippling



16.J10.3 : Web Local Crippling

- Compression loads applied to flanges

(a) $x \geq d/2$; R_n : **[Eq. J10-4]**

(b) $x < d/2$;

i. $l_b/d \leq 0.2$; R_n : **[Eq. J10-5a]**

ii. $l_b/d > 0.2$; R_n : **[Eq. J10-5b]**

Beams with Concentrated Loads
[AISC Tables]

1. **Use Table 9-4: "Beam Bearing Constants" for the following limit states:**

   A. Web Local Yielding

   i. $x \le d\ ;\ \Phi R_n = \Phi R_1 + l_b\,(\Phi R_2)$ or $R_n / \Omega = R_1 / \Omega + l_b\,(R_2 / \Omega)$

   ii. $x > d\ ;\ \Phi R_n = 2\Phi R_1 + l_b\,(\Phi R_2)$ or $R_n / \Omega = 2R_1 / \Omega + l_b\,(R_2 / \Omega)$

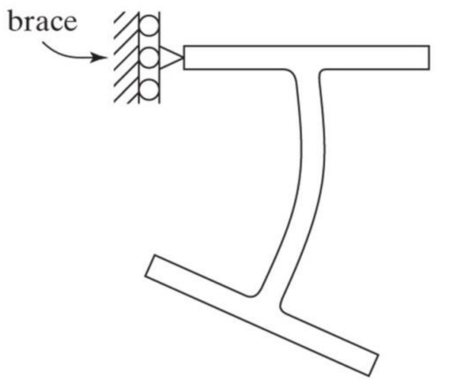
   B. Web Local Crippling

   i. $l_b / d \le 0.2\ ;\ \Phi R_n = \Phi R_3 + l_b\,(\Phi R_4)$ or $R_n / \Omega = R_3 / \Omega + l_b\,(R_4 / \Omega)$

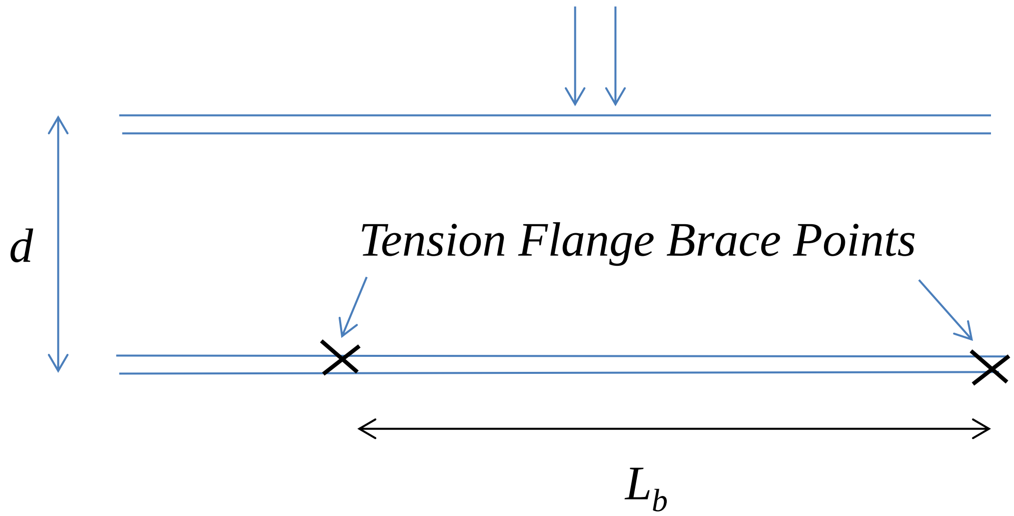
   ii. $l_b / d > 0.2;\ \Phi R_n = 2\Phi R_5 + l_b\,(\Phi R_6)$ or $R_n / \Omega = 2R_5 / \Omega + l_b\,(R_6 / \Omega)$

Beams with Concentrated Loads

Web Sidesway Buckling



(d) Sidesway web buckling



16.J10.4 : Web Sidesway Buckling

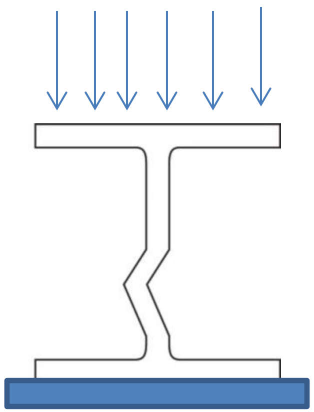
- Compression loads applied to flanges

(a) Compression Flange restrained against rotation; R_n : **[Eq. J10-6]**

(b) Compression Flange not restrained against rotation; R_n : **[Eq. J10-7]**

Beams with Concentrated Loads

Web Compression Buckling

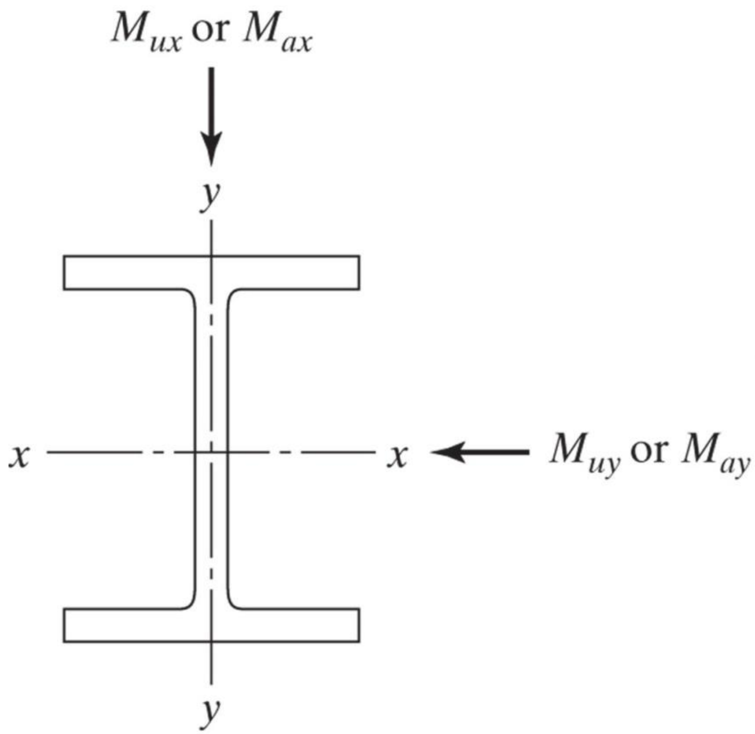


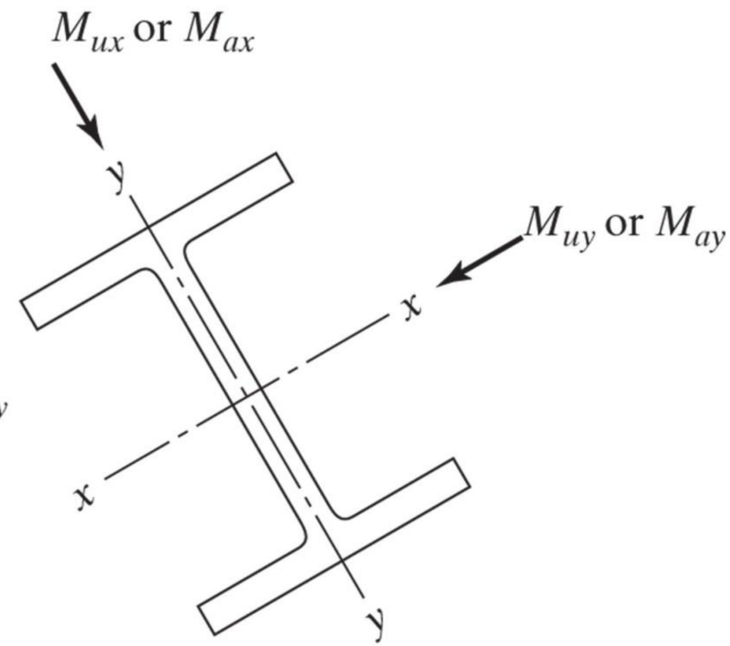
16.J10.5 : Web Compression Buckling

- Pair of Compression forces applied to flanges

$$R_n = \frac{24 t_w{}^3 \sqrt{E F_{yw}}}{h}$$ **[Eq. J10-8]**

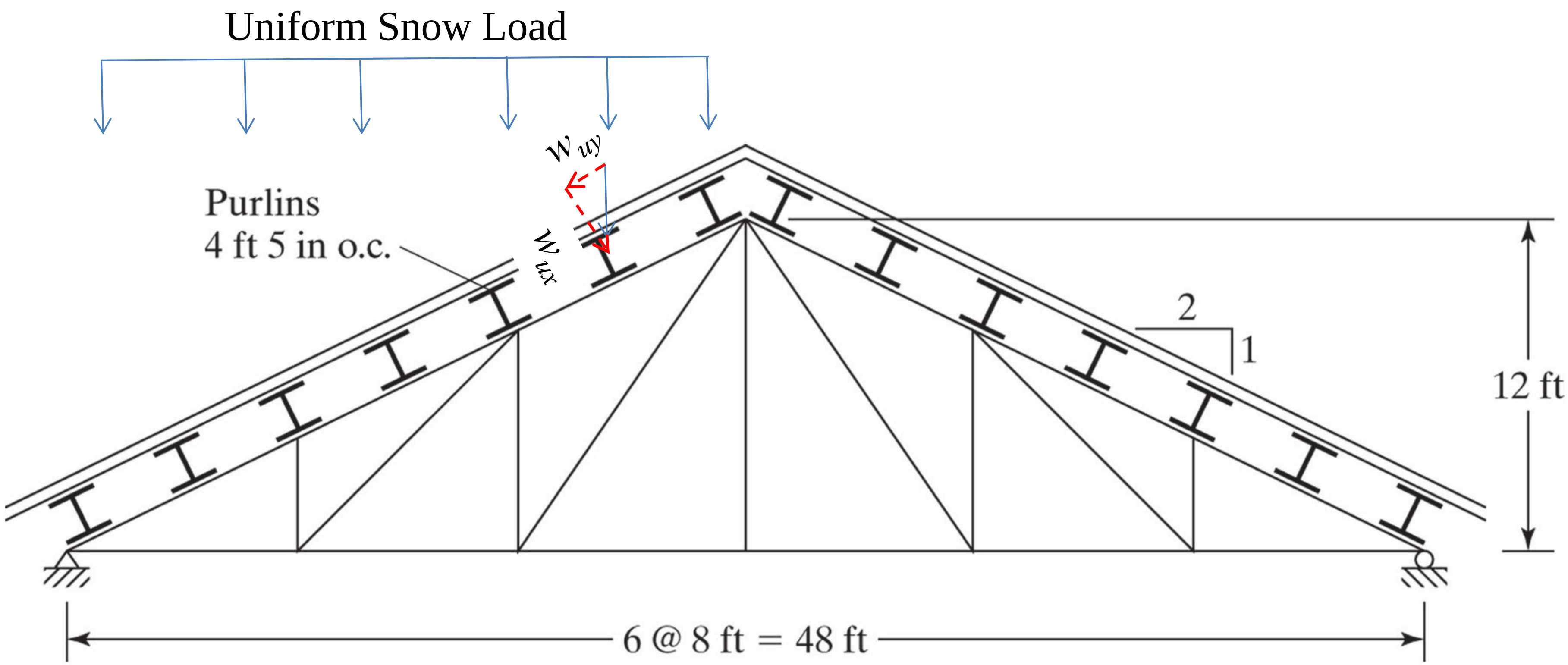
Biaxial Bending
[AISC 16H1]





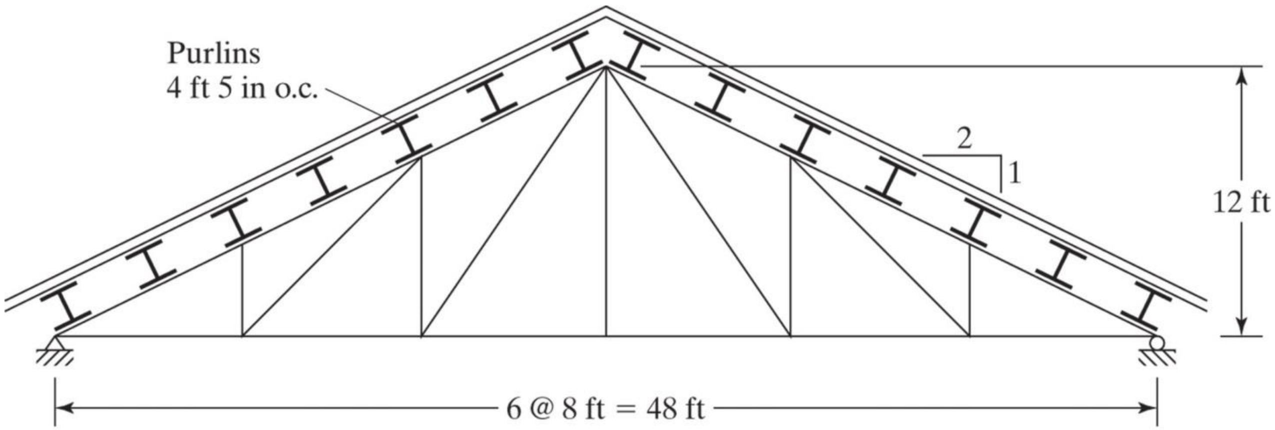
$$\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0 \qquad \text{(Eq. H1-1b)}$$

Biaxial Bending
[AISC 16H1]



Class Problem 2

Given: Snow Load = 50 psf, DL = 10 psf, Wind = 20 psf ⊥ to roof truss @ 18 ft. 6 in. c-c.



Find:

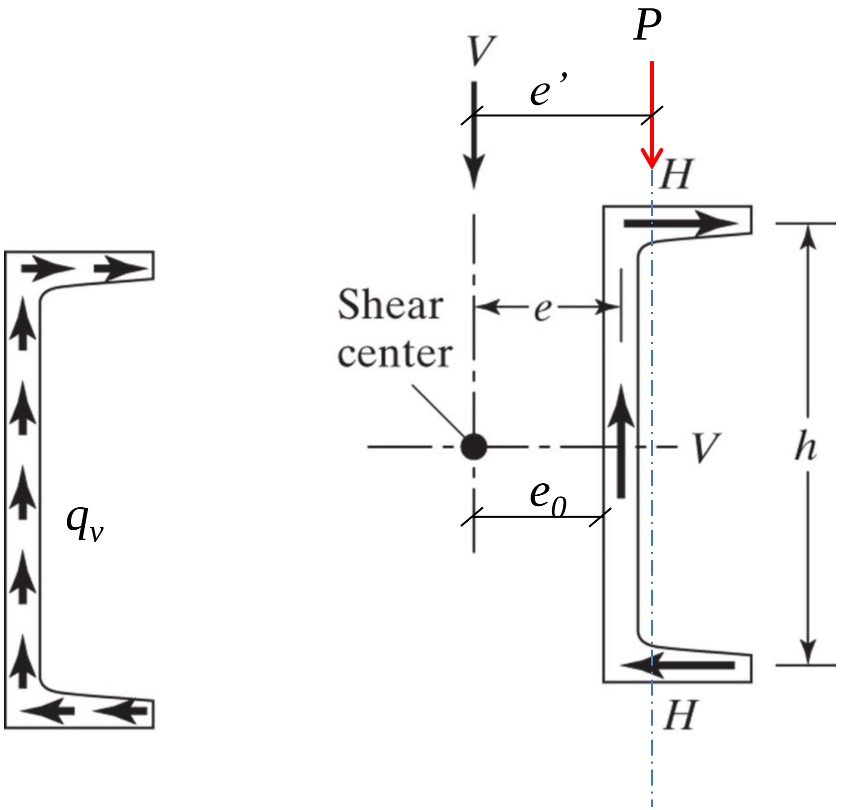
1. Determine lightest channel for purlin considering biaxial bending only. Assume fully braced top flange.
2. Maximum Torsion

Shear Center

Shear Stress: $\sigma_v = \frac{VQ}{Ib}$ (psi)

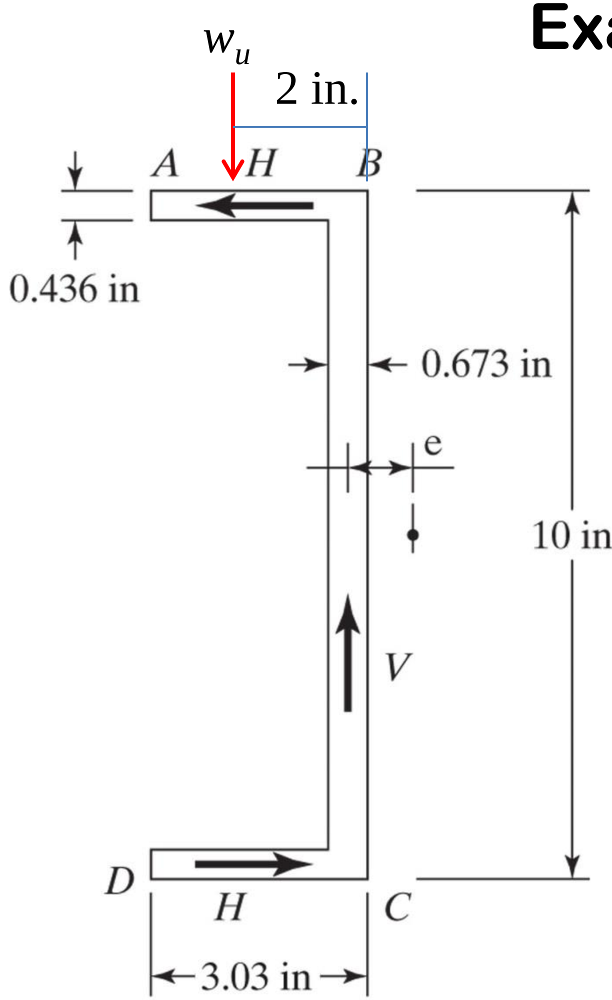
Shear Flow: $q_v = \frac{VQ}{I}$ (lb/in)

1. Determine shear flow at various points on the section.
2. Place an imaginary Vertical Force (V) a distance e from the center of web and opposite direction of the centroid.
3. Sum Moments about the center of web and set equal to zero.
4. Solve for shear center, e_0



Torsion Force = P x (horiz. dist. to shear center, e')

Shear Center
Example 10-8



Given: C10 x 30 beam, Simple Span = 15 ft. , w_u = 0.4 klf, shear center, e = 0.706

Find:

1. Maximum Torsion Force applied to the beam. Assume ends are restrained against rotation.
2. Maximum Rotation of beam

Beam Bearing Plate Design

1. Compute minimum plate area based on concrete bearing strength similar to base plate design. [16J8]

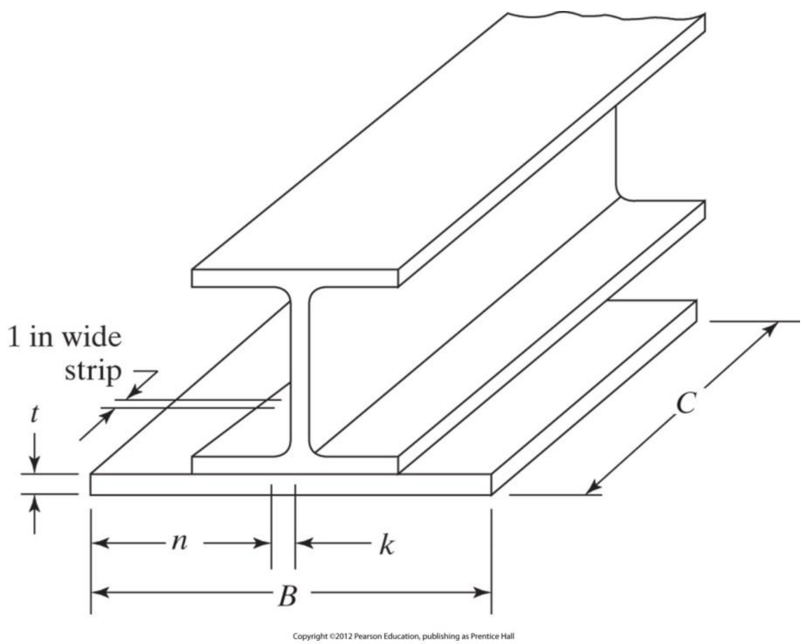
   $P_p = 0.85 f_c' A_1$ **Eq.J8-1**

   $\Phi_c = 0.65,\ \Omega_c = 2.31$

2. Assume 1in. wide strip cantilever fixed at root of fillet. Determine plate thickness assuming full plastic yield of plate.

LRFD: $t = \sqrt{\dfrac{2R_u n^2}{\varphi_b A_1 F_y}}$; $\Phi_b = 0.9$

ASD: $t = \sqrt{\dfrac{2R_a n^2 \Omega_b}{A_1 F_y}}$; $\Omega_b = 1.67$



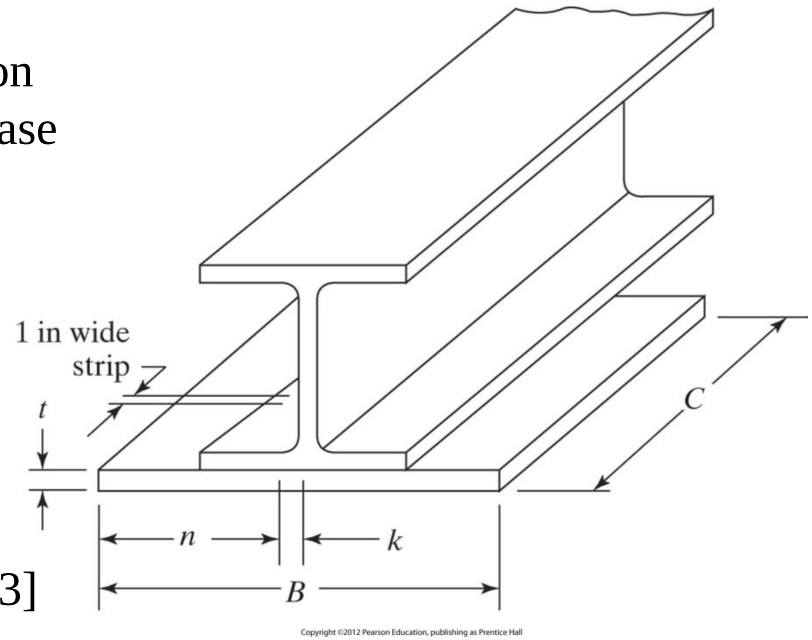
Beam Bearing Plate Design

1. Compute minimum plate area based on concrete bearing strength similar to base plate design. [16J8]

   $P_p = 0.85 f_c' A_1$ **Eq.J8-1**

   $\Phi_c = 0.65, \ \Omega_c = 2.31$

2. Check Web Local Yielding [Eq. J10-3] and Web Crippling [Eq. J10-5b]

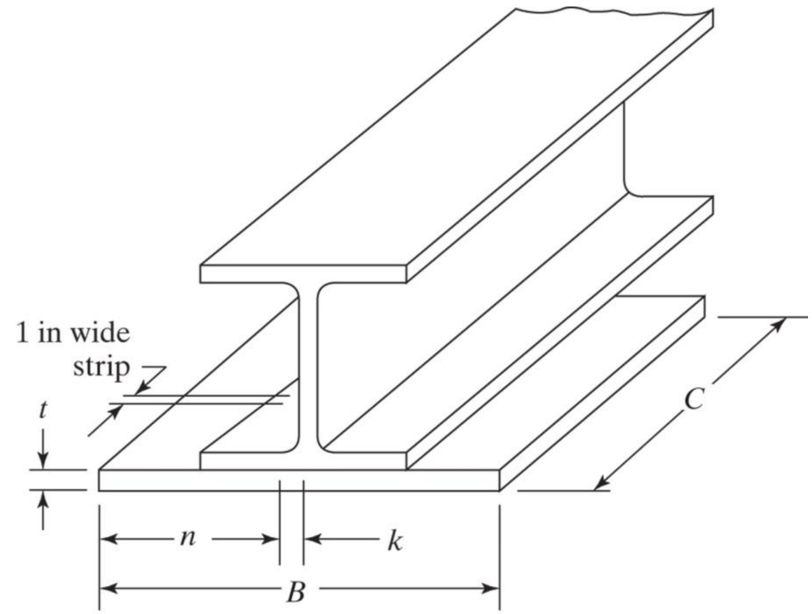


Beam Bearing Plate Design

3. Assume cantilever strip (1in. wide) fixed at root of fillet. Determine plate thickness assuming full plastic yield of plate at root.

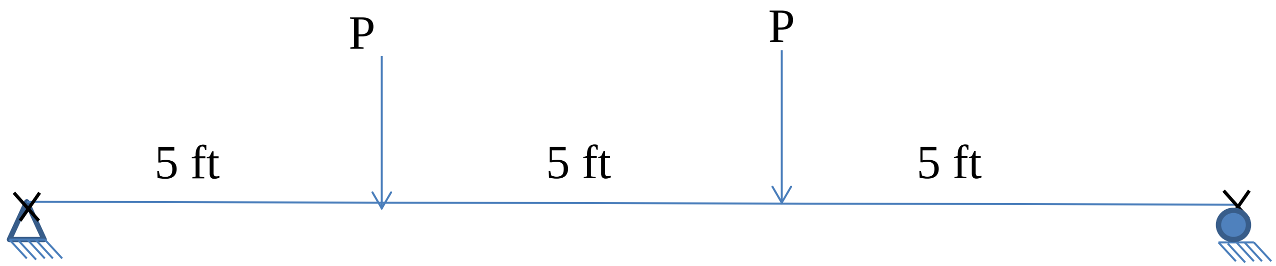
LRFD: $t = \sqrt{\dfrac{2R_u n^2}{\varphi_b A_1 F_y}}$; $\Phi_b = 0.9$

ASD: $t = \sqrt{\dfrac{2R_a n^2 \Omega_b}{A_1 F_y}}$; $\Omega_b = 1.67$



Example 5

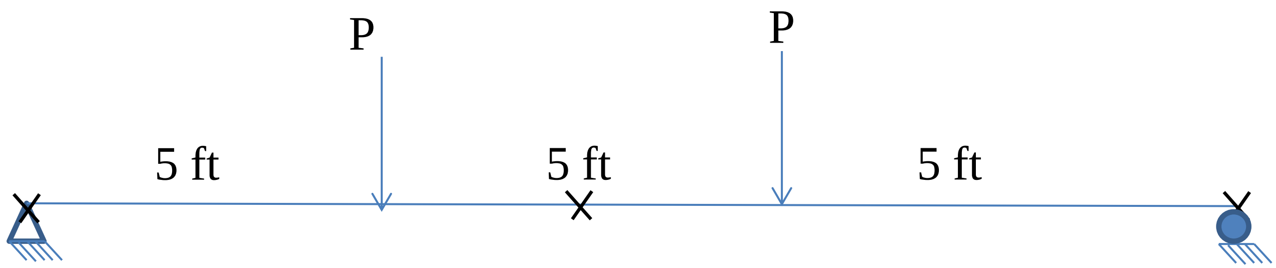
Given: The simply supported beam is braced at ends only. P_{LL} = 40 kips, P_{DL} = 10 kips. Assume Each end is supported on 8 in. wide concrete bearing pad. (f_c' = 4ksi)



Find: Determine minimum size bearing plate for W18 x 55. LRFD

Beam Lateral Bracing
Appendix 6.3

<u>Given:</u> W18 x 55 is braced at middle and ends.



<u>Find:</u> Required strength of lateral bracing. LRFD

Homework #2

Ch. 9 Problems: 25, 32

Ch10 Problems: 9, 17,24 (check LL deflection),27,28,30,31,32

Due: 09/22/14