Combined Bending and Axial Forces–Unbraced Frames

Objectives:

1. Understand and apply AISC column stability requirements for unbraced frames.
2. Use Approximate Methods to evaluate second-order P–δ and P–Δ effects.
3. Develop computer models for unbraced frames and evaluate beam-columns using second-order methods.

Design Requirements

- **Combined Forces (AISC Specifications):**
  - Chapter H: Combined Forces
    - Includes Chapters D through F (Design for Tension, Compression, and Flexure)
  - Chapter C: Stability: Direct Analysis Method
  - Appendix 7: Alternative Stability Analysis Methods
  - Appendix 8: Approximate 2^{nd} Order Analysis
  - Part 6: Design Tables for combined flexure and axial force.

16.1.H1.1: Doubly and Singly Symmetric Members: Flexure and Compression

For $\frac{P_r}{P_c} \geq 0.2$; $\quad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ Equation H1-1a

P_r = required axial compressive strength from 2nd ORDER ANALYSIS (from LRFD load combinations).
P_c = available design axial compressive strength LRFD (strength from Chapter E).

M_r = required flexural strength from 2nd ORDER ANALYSIS (from LRFD load combinations).
M_c = available design flexural strength LRFD (strength from Chapter F).

x = strong axis bending
y = weak axis bending

16.1.H1.1: Doubly and Singly Symmetric Members: Flexure and Compression

For $\frac{P_r}{P_c} < 0.2$;

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0$$

Equation H1-1b

Table 6-1: W-Shapes: Flexure and Compression

For $pP_r \geq 0.2$; $pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$ Eq. 6-1 [Eq H1-1a]

P_r = required axial compressive strength from 2nd ORDER ANALYSIS (from LRFD load combinations).

M_r = required flexural strength from 2nd ORDER ANALYSIS (from LRFD load combinations).

p = $1/(\Phi_c P_n)$ (kips^{-1})

b_x = $8/(9\,\Phi_b M_{nx})$ (kip-ft^{-1})

b_y = $8/(9\,\Phi_b M_{ny})$ (kip-ft^{-1})

x = strong axis bending
y = weak axis bending

Table 6-1: W-Shapes: Flexure and Compression

For $pP_r < 0.2$; $1/2\,pP_r + 9/8(b_x M_{rx} + b_y M_{ry}) \leq 1.0$

Eq. 6-2
[Eq H1-1b]

P_r = required axial compressive strength from 2nd ORDER ANALYSIS (from LRFD load combinations).

M_r = required flexural strength from 2nd ORDER ANALYSIS (from LRFD load combinations).

p = $1/(\Phi_c P_n)$ (kips^{-1})

b_x = $8/(9\,\Phi_b M_{nx})$ (kip-ft^{-1})

b_y = $8/(9\,\Phi_b M_{ny})$ (kip-ft^{-1})

x = strong axis bending
y = weak axis bending

Stability Analysis and Design
Chapter C

Chapter C1 General Requirements:

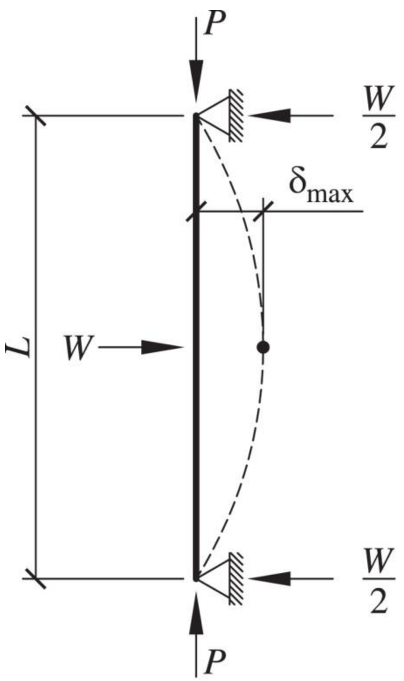
C1.1 Direct Analysis Method is the prescribed method and is applicable to any structure.

C1.2 Alternative Methods described in Appendix 7 (Effective Length and First-Order) may be used provided specified constraints are satisfied.

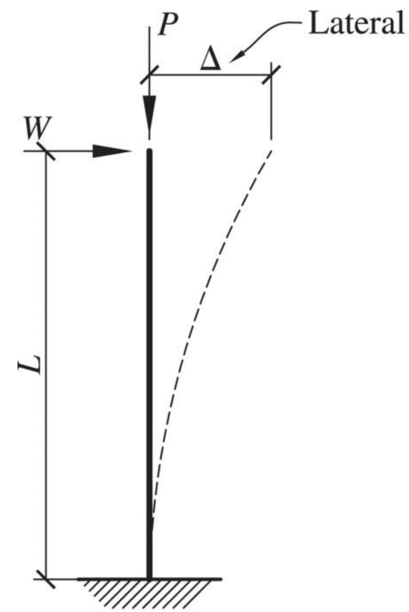
Note: Direct Analysis is REQUIRED if $\Delta_{2nd\ Order}/\Delta_{1st\ Order} > 1.5$ ($B_2 > 1.5$) (See Section Appendix 7.2.1)

Appendix 8: Approximate Second-Order Analysis

Second-Order Effects:



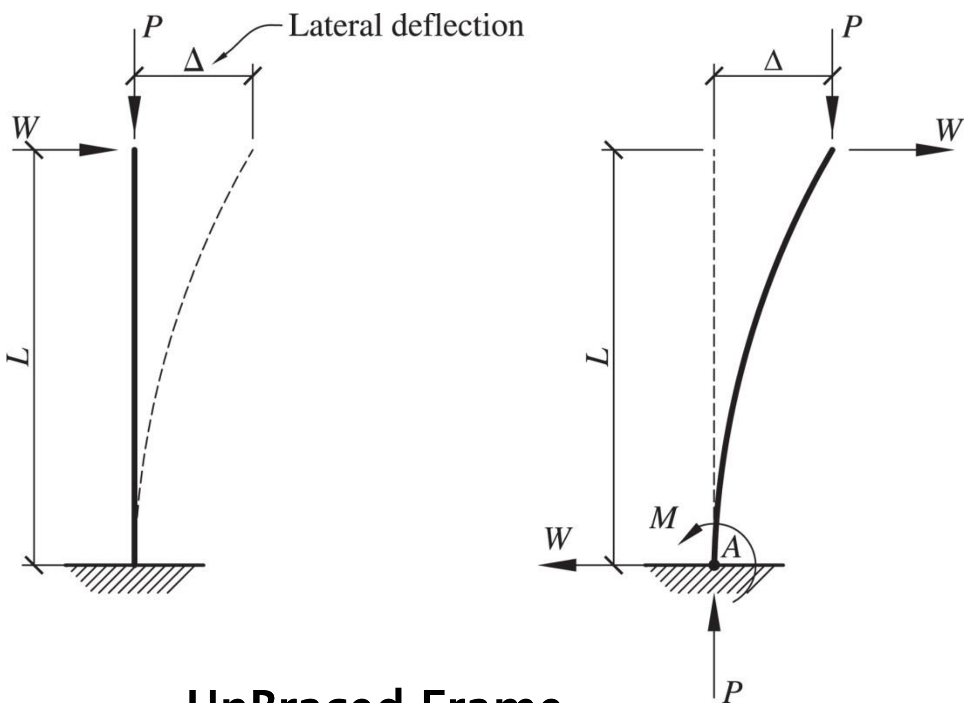
Braced Frame (*nt*)



Unbraced Frame (*lt*)

Appendix 8: Approximate Second-Order Analysis

Second-Order *lt* Effects:



UnBraced Frame

Appendix 8: Approximate Second-Order Analysis

Amplified First Order Moments

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{Eq. A-8-1}$$

M_r = second order required flexural strength

B_1 = amplification factor to account for second order effects caused by displacements along member length (P-δ effects).

M_{nt} = first order moment, assuming no lateral translation of frame (from load combinations).

B_2 = amplification factor to account for second order effects caused by displacements of member ends (P-Δ effects).

M_{lt} = first order moment <u>caused by lateral translation of frame</u> (from factored load combinations).

Appendix 8: Approximate Second-Order Analysis

Amplified First Order Axial Forces:

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{Eq A-8-2}$$

P_r = second order required axial strength.
P_{nt} = first order axial force, assuming no lateral translation of frame (from load combinations).
B_2 = amplification factor to account for second order effects caused by displacements of member ends (P-Δ effects).
P_{lt} = first order axial force caused by lateral translation of frame (from factored load combinations).

Amplified First Order Analysis
Unbraced Frames: Translation Effects

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{estory}}}$$ Eq. A-8-6

Where,

$\boldsymbol{P_{story}}$ = Total vertical load supported by the story, LRFD or ASD, and

$\boldsymbol{P_{e\,story}}$ = Elastic critical buckling strength for the story in the direction of translation being considered.

α=1 for LRFD design

α=1.6 for ASD design

Amplified First Order Analysis
Unbraced Frames: Translation Effects

$$P_{e\,story} = R_M \frac{HL}{\Delta_H}$$ Eq. A-8-7

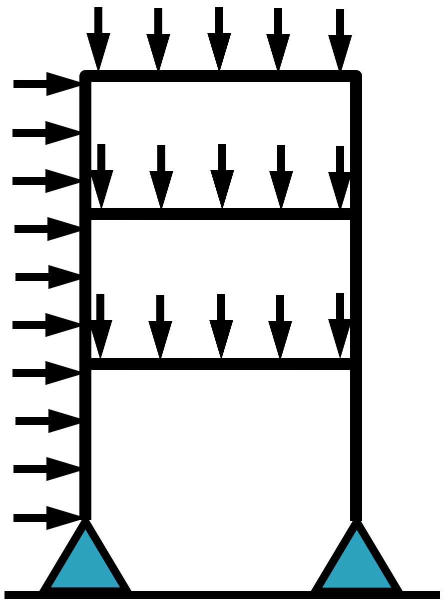
Where,

$$R_M = 1 - 0.15 \frac{P_{mf}}{P_{e\,story}}$$ Eq. A=8-8

H = story shear in the direction of translation considered

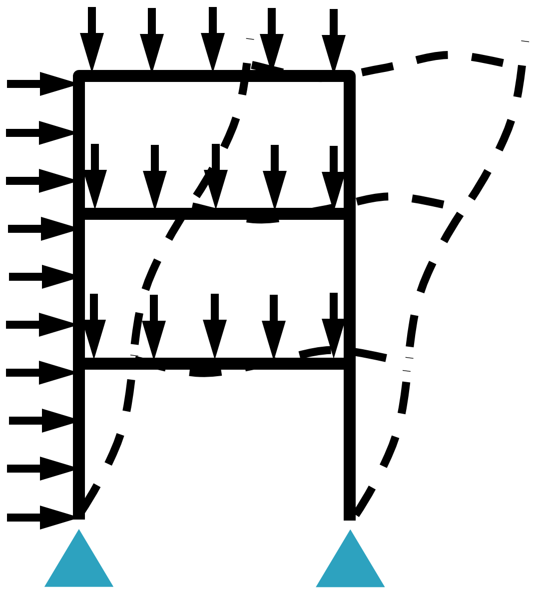
$\frac{\Delta_H}{L}$ = First order interstory drift/story height, (typically *1/500* for service loads, *1/300* for factored loads)

Amplified First Order Analysis
Unbraced Frames



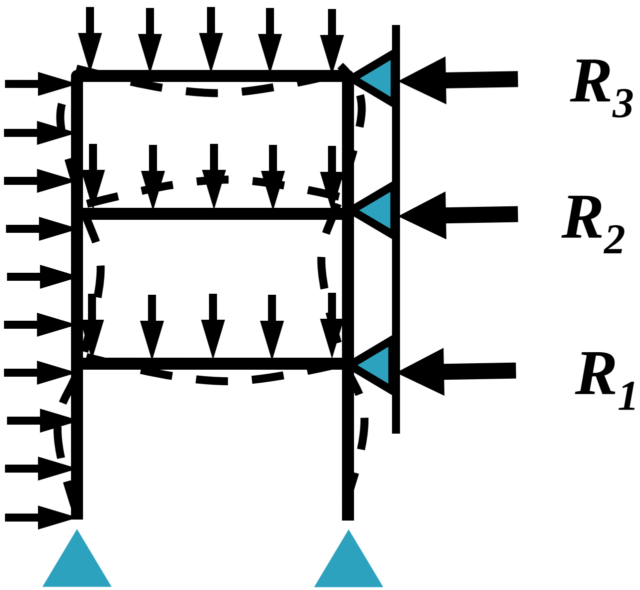
Amplified First Order Analysis
Unbraced Frames

Combined forces will cause P_{nt} and P_{lt} & M_{nt} and M_{lt} in a structure.



Amplified First Order Analysis
Unbraced Frames

To determine M_{nt} and P_{nt}, restrain lateral movement at each story.

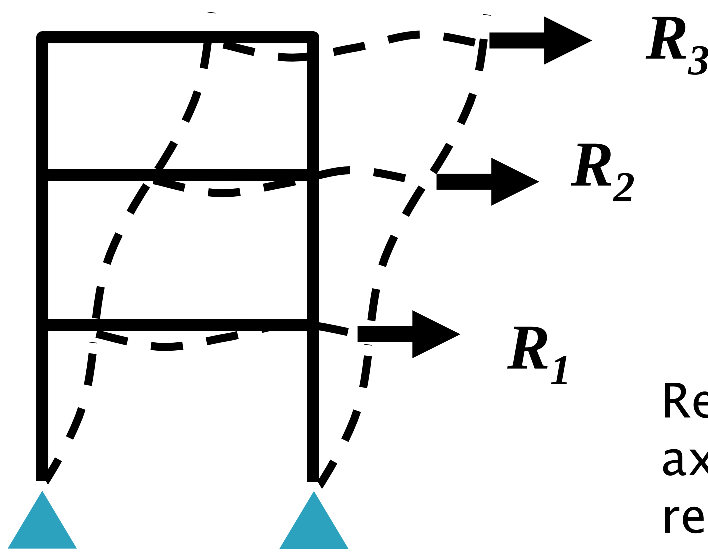


Resulting reactions at each story are required to resist the lateral translation of the structure.

Resulting Moments and Axial Forces are M_{nt} and P_{nt} respectively.

Amplified First Order Analysis
Unbraced Frames

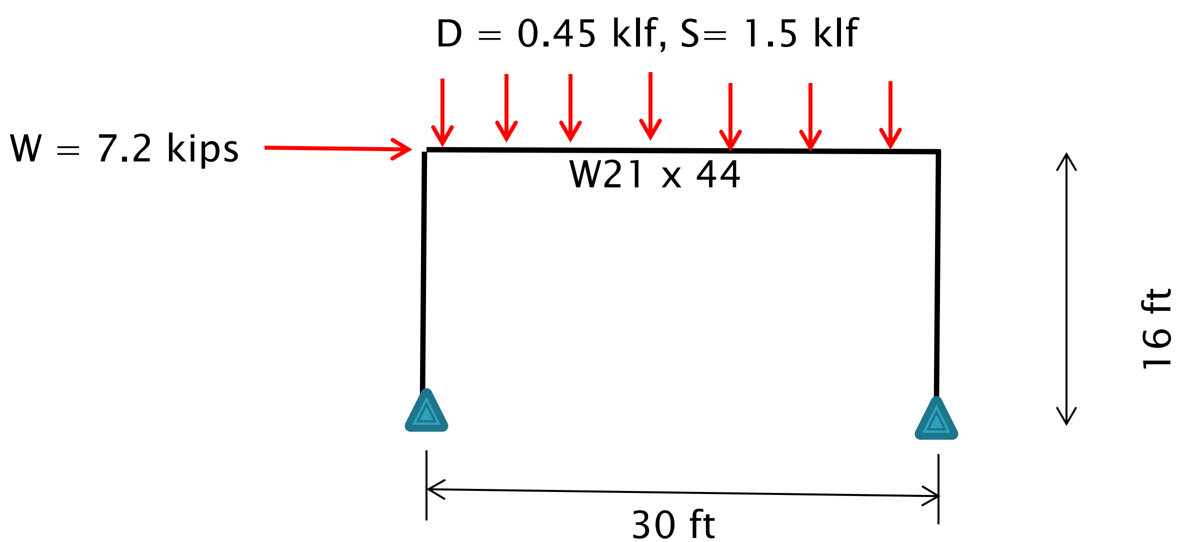
To determine M_{lt} and P_{lt}, apply the resulting reaction at each story.



Resulting moments and axial forces are M_{lt} and P_{lt} respectively.

Example 4-Unbraced Frame

Given: Unbraced Rigid Frame, $P_{story} = 100 \text{ kips}$



Find:

1. Evaluate W10x33 Columns.

Method: LRFD, ELM, include *l/500* out-of-plumbness, Appendix 8-Approximate 2nd order

MODELING ISSUES

Second Order Effects

Final deflections and moments can be calculated in several different ways.

Closed form mathematical solutions:
Derivations exist for standard results, but very difficult, if not impossible, for a full structure.

Approximate Methods:
Amplification factor applied to first order displacement and moments by simple method

OR

approximate computer methods will provide results within a given tolerance.

Computer Analysis

Most structural analysis programs will include some form of "second order" analysis.

To satisfy a "rigorous 2nd order analysis" a program must include both P-Δ and P-δ analysis, or the designer must verify that P-δ effects are minimal in the structure.

Computer software uses approximate solutions rather than exact closed form solutions, and iterate until a specified error tolerance is reached.

Computer Analysis

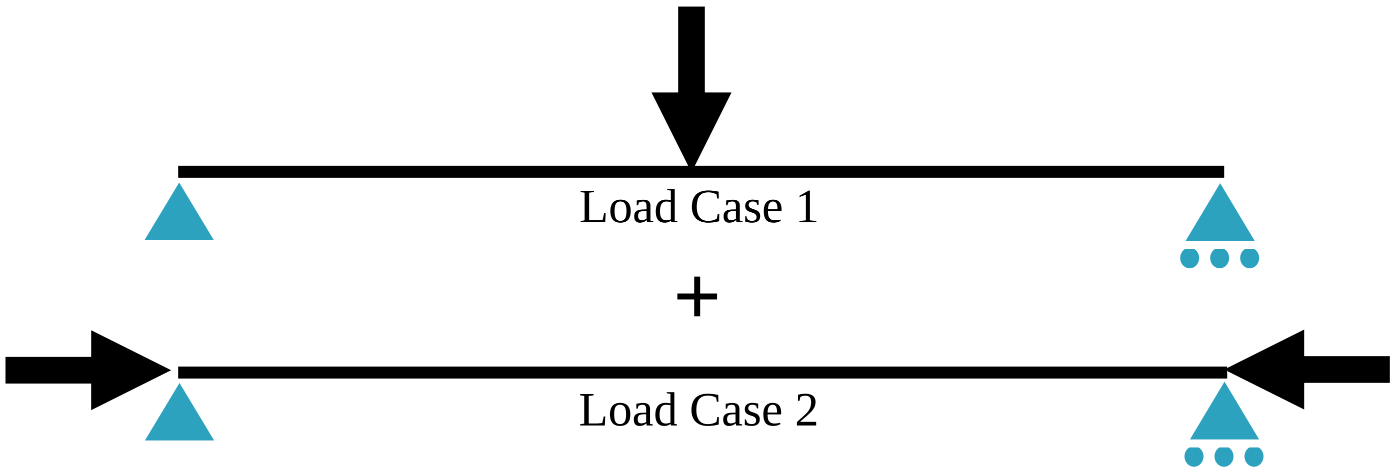
Designers must verify that second order effects are correctly handled by the programs being used.

Verify that load combinations include second order effects. Some programs analyze individual load cases only. Load case results are factored and summed to produce load combinations, but are not re-analyzed.

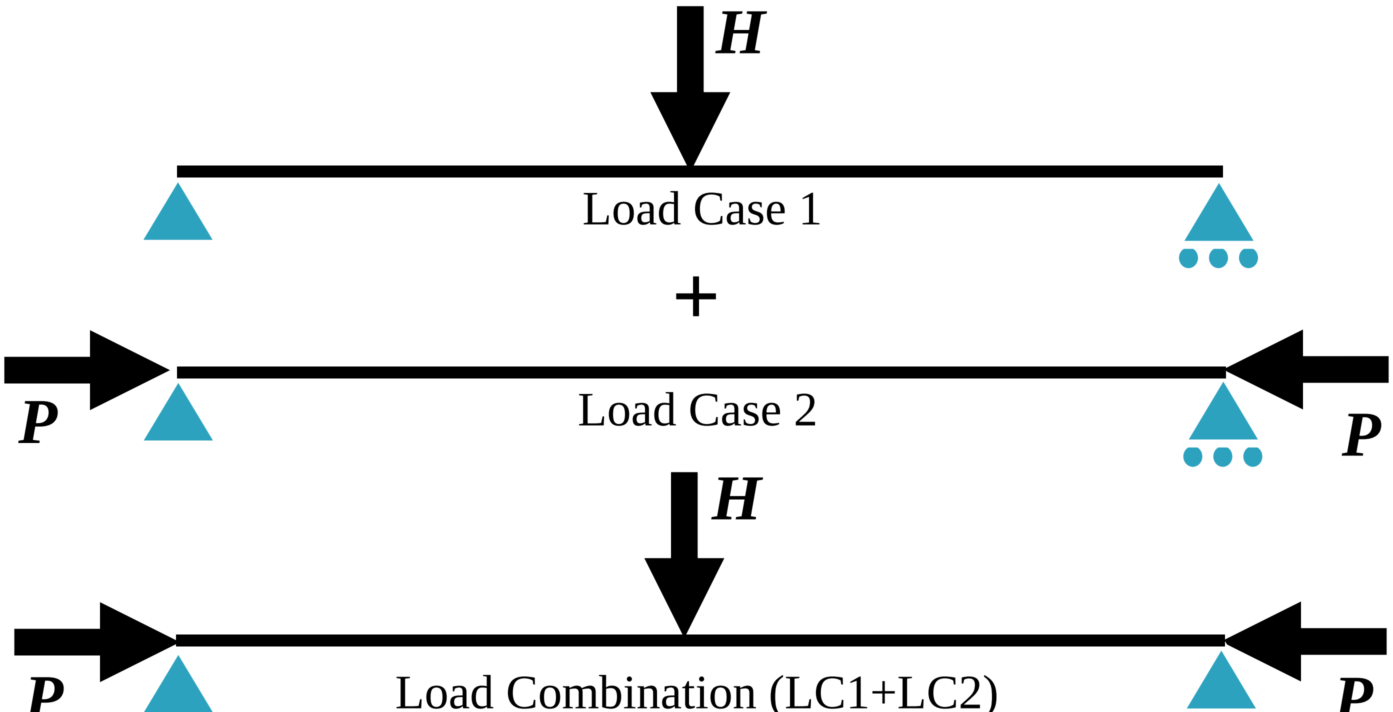
Therefore, moments resulting from lateral loads in one load case may not be correctly amplified by axial load in a separate load case when combined in a load combination.

Computer Analysis

How can we verify computer programs correctly analyze load combinations.



Verify Computer Analysis



Load Combination (LC1+LC2)

OR

Load Case 3

Compare 2nd order analysis from the load combination and load case 3. If results are identical, the program correctly includes 2nd order analysis in load combinations.

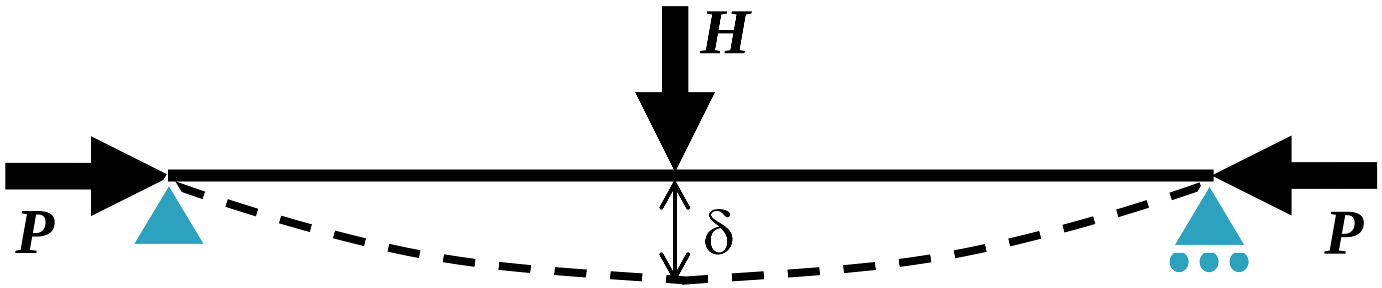
Computer Analysis

Designers must verify that second order effects are correctly handled by the programs being used.

To determine the capabilities of a specific program compare first and second order analysis results for known load cases – typically a flagpole (P–Δ) and simple beam (P–δ) with axial and lateral loads applied. (Figures C-C2-2 and C-C2-3)

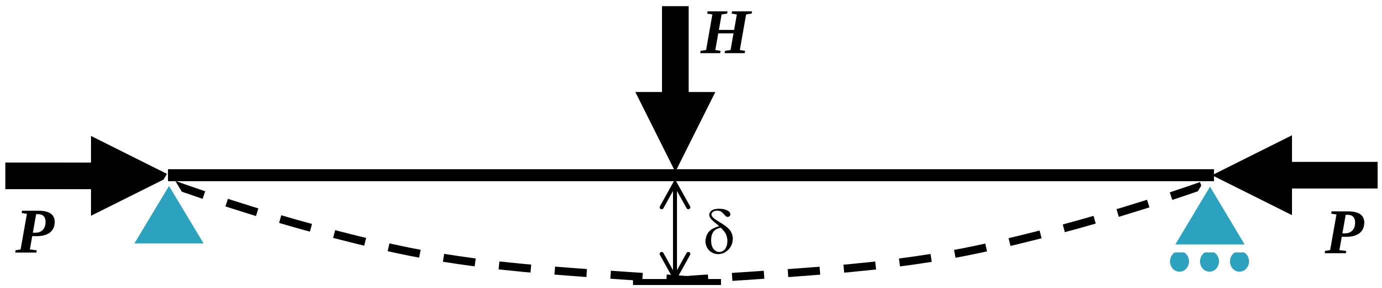
Most programs only include P–Δ. If P–δ effects are significant the designer can use multiple elements to provide equivalent P–Δ effects within the original element.

Computer Analysis

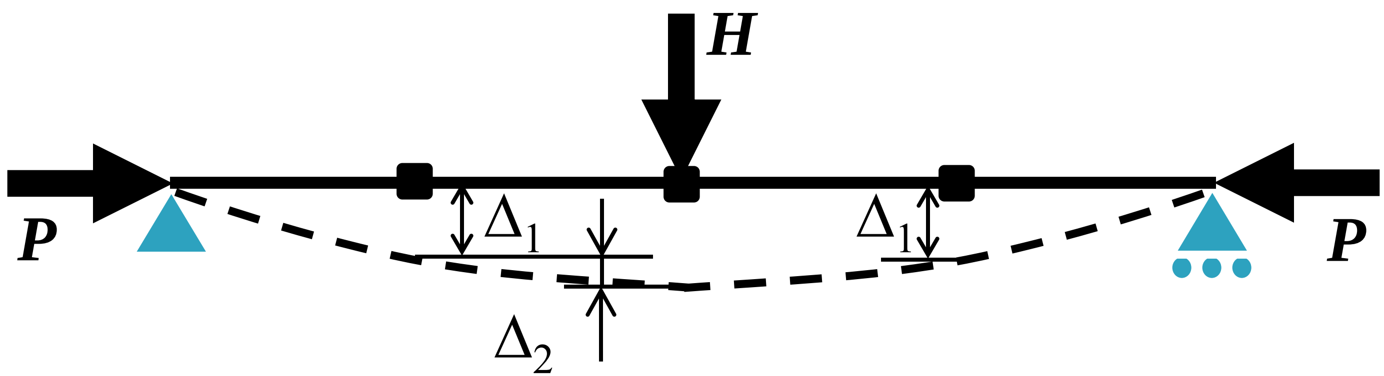


Some computer programs do not analyze *P*–δ effects such as the case shown.

Computer Analysis



Provide additional joints in the member.
Member shown is now made up of 4 shorter members.



P–δ analysis of the overall member is now analyzed as *P*–Δ analysis of each individual member, which most programs will be able to analyze.

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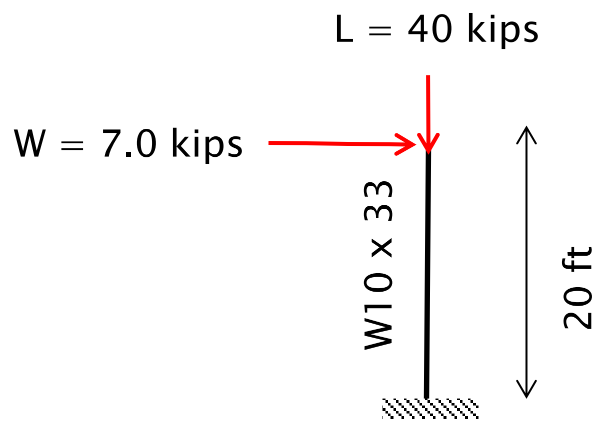
Example 5–Model Verification

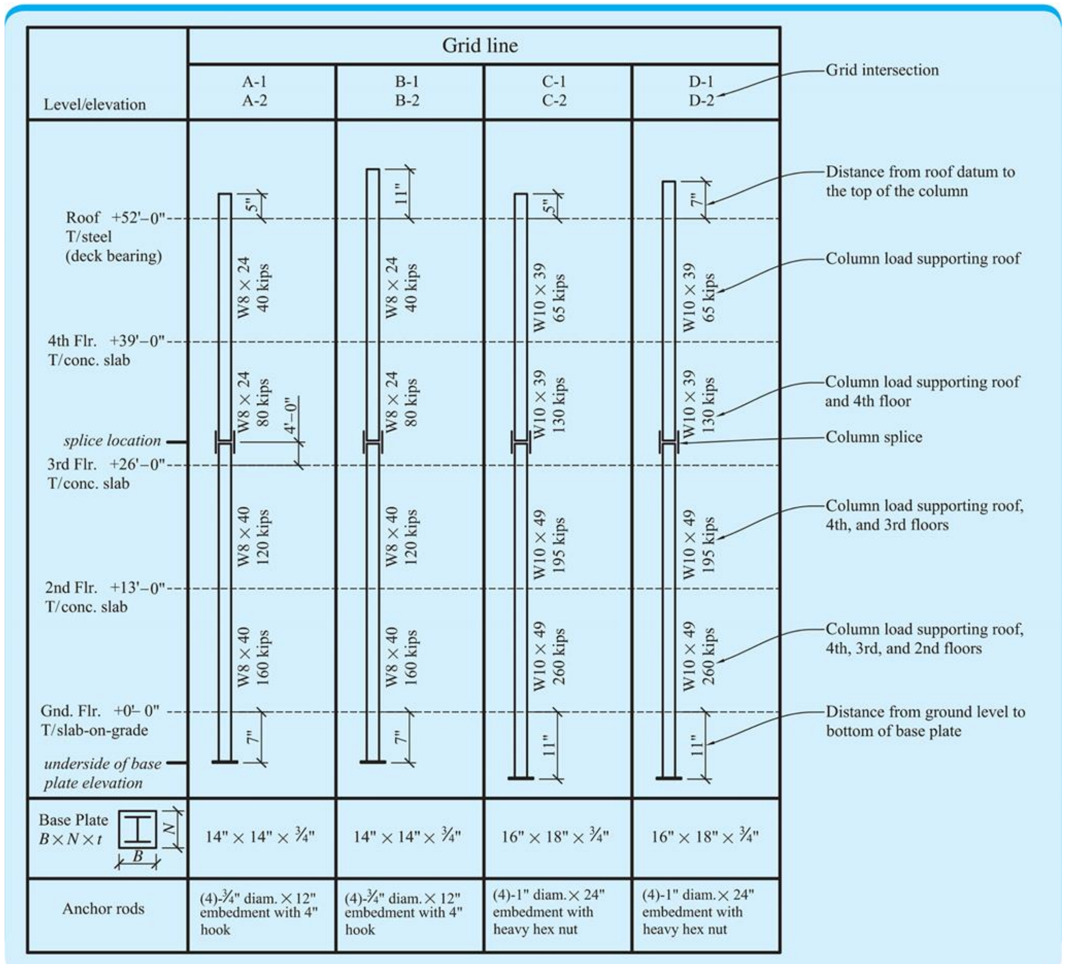
Given: Single column



Find:

1. Validate 2^{nd} order computer analysis

Typical Column Schedule



HW#5

Handout Due 10/20/14