4.6 STATIC ANALYSIS

4.6.1 Influence of Plan Geometry

4.6.1.1 Plan Aspect Ratio

If the span length of a superstructure with torsionally stiff closed cross-sections exceeds 2.5 times its width, the superstructure may be idealized as a single-spine beam. The following dimensional definitions shall be used to apply this criterion:

- Width—the core width of a monolithic deck or the average distance between the outside faces of exterior webs.
- Length for rectangular simply supported bridges—the distance between deck joints.

  Length for continuous and/or skewed bridges—the length of the longest side of the rectangle that can be drawn within the plan view of the width of the smallest span, as defined herein.

This restriction does not apply to cast-in-place multicell box girders.

C4.6.1.1

Where transverse distortion of a superstructure is small in comparison with longitudinal deformation, the former does not significantly affect load distribution, hence, an equivalent beam idealization is appropriate. The relative transverse distortion is a function of the ratio between structural width and height, the latter, in turn, depending on the length. Hence, the limits of such idealization are determined in terms of the width-to-effective length ratio.

Simultaneous torsion, moment, shear, and reaction forces and the attendant stresses are to be superimposed as appropriate. The equivalent beam idealization does not alleviate the need to investigate warping effects in steel structures. In all equivalent beam idealizations, the eccentricity of loads should be taken with respect to the centerline of the equivalent beam.

4.6.1.2 Structures Curved in Plan

4.6.1.2.1 General

The moments, shears, and other force effects required to proportion the superstructure components shall be based on a rational analysis of the entire superstructure.

The entire superstructure, including bearings, shall be considered as an integral structural unit. Boundary conditions shall represent the articulations provided by the bearings and/or integral connections used in the design. Analyses may be based on elastic small-deflection theory, unless more rigorous approaches are deemed necessary by the Engineer.

Analyses shall consider bearing orientation and restraint of bearings afforded by the substructure. These load effects shall be considered in designing bearings, cross-frames, diaphragms, bracing, and the deck.

Distortion of the cross-section need not be considered in the structural analysis.

Centrifugal force effects shall be considered in accordance with Article 3.6.3.

C4.6.1.2.1

Since equilibrium of horizontally curved I-girders is developed by the transfer of load between the girders, the analysis must recognize the integrated behavior of all structural components. Equilibrium of curved box girders may be less dependent on the interaction between girders. Bracing members are considered primary members in curved bridges since they transmit forces necessary to provide equilibrium.

The deck acts in flexure, vertical shear, and horizontal shear. Torsion increases the horizontal deck shear, particularly in curved box girders. The lateral restraint of the bearings may also cause horizontal shear in the deck.

Small-deflection theory is adequate for the analysis of most curved-girder bridges. However, curved I-girders are prone to deflect laterally when the girders are insufficiently braced during erection. This behavior may not be well recognized by small-deflection theory.

Classical methods of analysis usually are based on strength of materials assumptions that do not recognize cross-section deformation. Finite element analyses that model the actual cross-section shape of the I- or box girders can recognize cross-section distortion and its effect on structural behavior. Cross-section deformation of steel box girders may have a significant effect on torsional behavior, but this effect is limited by the provision of sufficient internal cross bracing.

4.6.1.2.2 Single-Girder Torsionally Stiff Superstructures

A horizontally curved, torsionally stiff single-girder superstructure meeting the requirements of Article 4.6.1.1 may be analyzed for global force effects as a curved spine beam.

The location of the centerline of such a beam shall be taken at the center of gravity of the cross-section, and the eccentricity of dead loads shall be established by volumetric consideration.

C4.6.1.2.2

In order to apply the aspect ratio provisions of Article 4.6.1.1, as specified, the plan needs to be hypothetically straightened. Force effects should be calculated on the basis of the actual curved layout.

With symmetrical cross-sections, the center of gravity of permanent loads falls outside the center of gravity. Shear center of the cross-section and the resulting eccentricity need to be investigated.

4.6.1.2.3 Multicell Concrete Box Girders

Horizontally curved cast-in-place multicell box girders may be designed as single-spine beams with straight segments, for central angles up to 34° within one span, unless concerns about other force effects dictate otherwise.

C4.6.1.2.3

A parameter study conducted by Song, Chai, and Hida (*2003*) indicated that the distribution factors from the LRFD formulae compared well with the distribution factors from grillage analyses when using straight segments on spans with central angles up to 34° in one span.

4.6.1.2.4 Steel Multiple-Beam Superstructures

4.6.1.2.4a General

Horizontally curved superstructures may be analyzed as grids or continuums in which the segments of the longitudinal beams are assumed to be straight between nodes. The actual eccentricity of the segment between the nodes shall not exceed 2.5 percent of the length of the segment.

C4.6.1.2.4a

An eccentricity of 2.5 percent of the length of the segment corresponds to a central angle subtended by a curved segment of about 12°.

This Article applies only to major-axis bending moment and does not apply to lateral flange bending, or torsion, which should always be examined with respect to curvature.

Bridges with even slight curvature may develop large radial forces at the abutment bearings. Therefore, thermal analysis of all curved bridges is recommended.

4.6.1.2.4b I-Girders

The effect of curvature on stability shall be considered for all curved I-girders.

Where I-girder bridges meet the following four conditions, the effects of curvature may be ignored in the analysis for determining the major-axis bending moments and bending shears:

- Girders are concentric;
- Bearing lines are not skewed more than 10° from radial;
- The stiffnesses of the girders are similar;
- For all spans, the arc span divided by the girder radius in feet is less than 0.06 radians where the arc span, L_{as}, shall be taken as follows:

C4.6.1.2.4b

The requirement for similar stiffness among the girders is intended to avoid large and irregular changes in stiffness which could alter transverse distribution of load. Under such conditions, a refined analysis would be appropriate. Noncomposite dead load preferably is to be distributed uniformly to the girders since the cross-frames provide restoring forces that prevent the girders from deflecting independently. Certain dead loads applied to the composite bridge may be distributed uniformly to the girders as provided in Article 4.6.2.2.1. However, heavier concentrated line loads such as parapets, sidewalks, barriers, or sound walls should not be distributed equally to the girders. Engineering judgment must be used in determining the distribution of these loads. Often the largest portion of the load on an overhang is assigned to the exterior girder, or to the exterior girder and the first interior girder. The exterior girder on the outside of the curve is often critical in curved girder bridges.

For simple spans:

L_{as} = arc length of the girder (ft.)

For end spans of continuous members:

L_{as} = 0.9 times the arc length of the girder (ft.)

For interior spans of continuous members:

L_{as} = 0.8 times the arc length of the girder (ft.)

An I-girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Lateral flange bending effects should then be determined from an appropriate approximation and considered in the design.

Cross-frame or diaphragm members shall be designed in accordance with Articles 6.7.4 and 6.13 for forces computed by rational means.

Cross-frame spacing shall be set to limit flange lateral bending in the girders.

The effect of curvature on the torsional behavior of a girder must be considered regardless of the amount of curvature since stability and strength of curved girders is different from that of straight girders (*Hall and Yoo, 1996*).

In lieu of a refined analysis, Eq. C1 may be appropriate for determining the lateral bending moment in I-girder flanges due to curvature (*Richardson, Gordon, and Associates, 1976; United States Steel, 1984*).

$$M_{lat} = \frac{M\ell^2}{NRD} \tag{C4.6.1.2.4b-1}$$

where:

M_{lat} = flange lateral bending moment (kip-ft.)

M = major-axis bending moment (kip-ft.)

ℓ = unbraced length (ft.)

R = girder radius (ft.)

D = web depth (ft.)

N = a constant taken as 10 or 12 in past practice

Although the depth to be used in computing the flange lateral moment from Eq. C1 is theoretically equal to the depth, h, between the midthickness of the top and bottom flanges, for simplicity, the web depth, D, is conservatively used in Eq. C1. The Engineer may substitute the depth, h, for D in Eq. C1, if desired. Eq. C1 assumes the presence of a cross-frame at the point under investigation, that the cross-frame spacing is relatively uniform, and that the major-axis bending moment, M, is constant between brace points. Therefore, at points not actually located at cross-frames, flange lateral moments from Eq. C1 may not be strictly correct. The constant, N, in Eq. C1 has been taken as either 10 or 12 in past practice and either value is considered acceptable depending on the level of conservatism that is desired.

Other conditions that produce torsion, such as skew, should be dealt with by other analytical means which generally involve a refined analysis.

4.6.1.2.4c Closed Box and Tub Girders

The effect of curvature on strength and stability shall be considered for all curved box girders.

Where box girder bridges meet the following three conditions, the effect of curvature may be ignored in the analysis for determination of the major-axis bending moments and bending shears:

- Girders are concentric,
- Bearings are not skewed, and

C4.6.1.2.4c

Although box-shaped girders have not been examined as carefully as I-girders with regard to approximate methods, bending moments in closed girders are less affected by curvature than are I-girders (*Tung and Fountain, 1970*). However, in a box shape, torsion is much greater than in an open shape so that web shears are affected by torsion due to curvature, skew or loads applied away from the shear center of the box. Double bearings resist significant torque compared to a box-centered single bearing.

- For all spans, the arc span divided by the girder radius is less than 0.3 radians, and the girder depth is less than the width of the box at middepth where the arc span, L_{as}, shall be taken as defined in Article 4.6.1.2.4b.

A box girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Lateral flange bending effects should then be found from an appropriate approximation and considered in the design.

Cross-frame or diaphragm members shall be designed in accordance with the provisions of Articles 6.7.4 and 6.13 and lateral bracing members shall be designed in accordance with Articles 6.7.5 and 6.13 for forces computed by rational means.

If the box is haunched or tapered, the shallowest girder depth should be used in conjunction with the narrowest width of the box at middepth in determining whether the effects of curvature may be ignored in calculating the major axis bending moments and bending shears.

4.6.2 Approximate Methods of Analysis

4.6.2.1 Decks

4.6.2.1.1 General

An approximate method of analysis in which the deck is subdivided into strips perpendicular to the supporting components shall be considered acceptable for decks other than:

- fully filled and partially filled grids for which the provisions of Article 4.6.2.1.8 shall apply, and
- top slabs of segmental concrete box girders for which the provisions of 4.6.2.9.4 shall apply.

Where the strip method is used, the extreme positive moment in any deck panel between girders shall be taken to apply to all positive moment regions. Similarly, the extreme negative moment over any beam or girder shall be taken to apply to all negative moment regions.

C4.6.2.1.1

This model is analogous to past AASHTO Specifications.

In determining the strip widths, the effects of flexure in the secondary direction and of torsion on the distribution of internal force effects are accounted for to obtain flexural force effects approximating those that would be provided by a more refined method of analysis.

Depending on the type of deck, modeling and design in the secondary direction may utilize one of the following approximations:

- Secondary strip designed in a manner like the primary strip, with all the limit states applicable;
- Resistance requirements in the secondary direction determined as a percentage of that in the primary one as specified in Article 9.7.3.2 (i.e., the traditional approach for reinforced concrete slab in the previous editions of the AASHTO Standard Specifications); or
- Minimum structural and/or geometry requirements specified for the secondary direction independent of actual force effects, as is the case for most wood decks.

The approximate strip model for decks is based on rectangular layouts. Currently about two-thirds of all bridges nationwide are skewed. While skew generally tends to decrease extreme force effects, it produces negative moments at corners, torsional moments in the end zones, substantial redistribution of reaction forces, and a number of other structural phenomena that should be considered in the design.

4.6.2.1.2 Applicability

The use of design aids for decks containing prefabricated elements may be permitted in lieu of analysis if the performance of the deck is documented and supported by sufficient technical evidence. The Engineer shall be responsible for the accuracy and implementation of any design aids used.

For slab bridges and concrete slabs spanning more than 15.0 ft. and which span primarily in the direction parallel to traffic, the provisions of Article 4.6.2.3 shall apply.

4.6.2.1.3 Width of Equivalent Interior Strips

The width of the equivalent strip of a deck may be taken as specified in Table 1. Where decks span primarily in the direction parallel to traffic, strips supporting an axle load shall not be taken to be greater than 40.0 in. for open grids and not greater than 144 in. for all other decks where multilane loading is being investigated. For deck overhangs, where applicable, the provisions of Article 3.6.1.3.4 may be used in lieu of the strip width specified in Table 1 for deck overhangs. The equivalent strips for decks that span primarily in the transverse direction shall not be subject to width limits. The following notation shall apply to Table 1:

S = spacing of supporting components (ft.)

h = depth of deck (in.)

L = span length of deck (ft.)

P = axle load (kip)

S_b = spacing of grid bars (in.)

$+M$ = positive moment

$-M$ = negative moment

X = distance from load to point of support (ft.)

C4.6.2.1.3

Values provided for equivalent strip widths and strength requirements in the secondary direction are based on past experience. Practical experience and future research work may lead to refinement.

To get the load per unit width of the equivalent strip, divide the total load on one design traffic lane by the calculated strip width.

Table 4.6.2.1.3-1 Equivalent Strips.

Type of Deck	Direction of Primary Strip Relative to Traffic	Width of Primary Strip (in.)
Concrete:		
• Cast-in-place	Overhang	$45.0 + 10.0X$
	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
• Cast-in-place with stay-in-place concrete formwork	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
• Precast, post-tensioned	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
Steel:		
• Open grid	Main Bars	$1.25P + 4.0S_b$
• Filled or partially filled grid	Main Bars	Article 4.6.2.1.8 applies
• Unfilled, composite grids	Main Bars	Article 4.6.2.1.8 applies
Wood:		
• Prefabricated glulam ○ Noninterconnected	Parallel Perpendicular	$2.0h + 30.0$ $2.0h + 40.0$
○ Interconnected	Parallel Perpendicular	$90.0 + 0.84L$ $4.0h + 30.0$
• Stress-laminated	Parallel Perpendicular	$0.8S + 108.0$ $10.0S + 24.0$
• Spike-laminated ○ Continuous decks or interconnected panels	Parallel Perpendicular	$2.0h + 30.0$ $4.0h + 40.0$
○ Noninterconnected panels	Parallel Perpendicular	$2.0h + 30.0$ $2.0h + 40.0$

Wood plank decks shall be designed for the wheel load of the design truck distributed over the tire contact area. For transverse planks, i.e., planks perpendicular to traffic direction:

- If $w_p \geq 10.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 10.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 10.0 in.

Only the wheel load is specified for plank decks. Addition of lane load will cause a negligible increase in force effects, however, it may be added for uniformity of the Code.

For longitudinal planks:

- If $w_p \geq 20.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 20.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 20.0 in.

where:

w_p = plank width (in.)

4.6.2.1.4 Width of Equivalent Strips at Edges of Slabs

4.6.2.1.4a General

For the purpose of design, the notional edge beam shall be taken as a reduced deck strip width specified herein. Any additional integral local thickening or similar protrusion acting as a stiffener to the deck that is located within the reduced deck strip width can be assumed to act with the reduced deck strip width as the notional edge beam.

4.6.2.1.4b Longitudinal Edges

Edge beams shall be assumed to support one line of wheels and, where appropriate, a tributary portion of the design lane load.

Where decks span primarily in the direction of traffic, the effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the edge of the deck and the inside face of the barrier, plus 12.0 in., plus one-quarter of the strip width, specified in either Article 4.6.2.1.3, Article 4.6.2.3, or Article 4.6.2.10, as appropriate, but not exceeding either one-half the full strip width or 72.0 in.

4.6.2.1.4c Transverse Edges

Transverse edge beams shall be assumed to support one axle of the design truck in one or more design lanes, positioned to produce maximum load effects. Multiple presence factors and the dynamic load allowance shall apply.

The effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the transverse edge of the deck and the centerline of the first line of support for the deck, usually taken as a girder web, plus one-half of the width of strip as specified in Article 4.6.2.1.3. The effective width shall not exceed the full strip width specified in Article 4.6.2.1.3.

C4.6.2.1.4c

For decks covered by Table A4-1, the total moment acting on the edge beam, including the multiple presence factor and the dynamic load allowance, may be calculated by multiplying the moment per unit width, taken from Table A4-1, by the corresponding full strip width specified in Article 4.6.2.1.3.

4.6.2.1.5 Distribution of Wheel Loads

If the spacing of supporting components in the secondary direction exceeds 1.5 times the spacing in the primary direction, all of the wheel loads shall be considered to be applied to the primary strip, and the provisions of Article 9.7.3.2 may be applied to the secondary direction.

If the spacing of supporting components in the secondary direction is less than 1.5 times the spacing in the primary direction, the deck shall be modeled as a system of intersecting strips.

The width of the equivalent strips in both directions may be taken as specified in Table 4.6.2.1.3-1. Each wheel load shall be distributed between two intersecting strips. The distribution shall be determined as the ratio between the stiffness of the strip and the sum of stiffnesses of the intersecting strips. In the absence of more precise calculations, the strip stiffness, k_s, may be estimated as:

$$k_s = \frac{EI_s}{S^3} \qquad (4.6.2.1.5\text{-}1)$$

where:

I_s = moment of inertia of the equivalent strip (in.^{4})

S = spacing of supporting components (in.)

C4.6.2.1.5

This Article attempts to clarify the application of the traditional AASHTO approach with respect to continuous decks.

4.6.2.1.6 Calculation of Force Effects

The strips shall be treated as continuous beams or simply supported beams, as appropriate. Span length shall be taken as the center-to-center distance between the supporting components. For the purpose of determining force effects in the strip, the supporting components shall be assumed to be infinitely rigid.

The wheel loads may be modeled as concentrated loads or as patch loads whose length along the span shall be the length of the tire contact area, as specified in Article 3.6.1.2.5, plus the depth of the deck. The strips should be analyzed by classical beam theory.

The design section for negative moments and shear forces, where investigated, may be taken as follows:

- For monolithic construction, closed steel boxes, closed concrete boxes, open concrete boxes without top flanges, and stemmed precast beams, i.e., Cross-sections (b), (c), (d), (e), (f), (g), (h), (i), and (j) from Table 4.6.2.2.1-1, at the face of the supporting component,

- For steel I-beams and steel tub girders, i.e., Cross-sections (a) and (c) from Table 4.6.2.2.1-1, one-quarter the flange width from the centerline of support,

C4.6.2.1.6

This is a deviation from the traditional approach based on a continuity correction applied to results obtained for analysis of simply supported spans. In lieu of more precise calculations, the unfactored design live load moments for many practical concrete deck slabs can be found in Table A4-1.

For short-spans, the force effects calculated using the footprint could be significantly lower, and more realistic, than force effects calculated using concentrated loads.

Reduction in negative moment and shear replaces the effect of reduced span length in the current code. The design sections indicated may be applied to deck overhangs and to portions of decks between stringers or similar lines of support.

Past practice has been to not check shear in typical decks. A design section for shear is provided for use in nontraditional situations. It is not the intent to investigate shear in every deck.

- For precast I-shaped concrete beams and open concrete boxes with top flanges, i.e., Cross-sections (c) and (k) from Table 4.6.2.2.1-1, one-third the flange width, but not exceeding 15.0 in., from the centerline of support,

- For wood beams, i.e., Cross-section (l) from Table 4.6.2.2.1-1, one-fourth the top beam width from centerline of beam.

For open box beams, each web shall be considered as a separate supporting component for the deck. The distance from the centerline of each web and the adjacent design sections for negative moment shall be determined based on the type of construction of the box and the shape of the top of the web using the requirements outlined above.

4.6.2.1.7 Cross-Sectional Frame Action

Where decks are an integral part of box or cellular cross-sections, flexural and/or torsional stiffnesses of supporting components of the cross-section, i.e., the webs and bottom flange, are likely to cause significant force effects in the deck. Those components shall be included in the analysis of the deck.

If the length of a frame segment is modeled as the width of an equivalent strip, provisions of Articles 4.6.2.1.3, 4.6.2.1.5, and 4.6.2.1.6 may be used.

C4.6.2.1.7

The model used is essentially a transverse segmental strip, in which flexural continuity provided by the webs and bottom flange is included. Such modeling is restricted to closed cross-sections only. In open-framed structures, a degree of transverse frame action also exists, but it can be determined only by complex, refined analysis.

In normal beam-slab superstructures, cross-sectional frame action may safely be neglected. If the slab is supported by box beams or is integrated into a cellular cross-section, the effects of frame action could be considerable. Such action usually decreases positive moments, but may increase negative moments resulting in cracking of the deck. For larger structures, a three-dimensional analysis may be appropriate. For smaller structures, the analysis could be restricted to a segment of the bridge whose length is the width of an equivalent strip.

Extreme force effects may be calculated by combining the:

- Longitudinal response of the superstructure approximated by classical beam theory, and

- Transverse flexural response modeled as a cross-sectional frame.

4.6.2.1.8 Live Load Force Effects for Fully and Partially Filled Grids and for Unfilled Grid Decks Composite with Reinforced Concrete Slabs

Moments in kip-in./in. of deck due to live load may be determined as:

- Main bars perpendicular to traffic:

  For $L \leq 120$ in.

$$M_{transverse} = 1.28 D^{0.197} L^{0.459} C \qquad (4.6.2.1.8\text{-}1)$$

  For $L > 120$ in.

$$M_{transverse} = \frac{D^{0.188}\left(3.7L^{1.35} - 956.3\right)}{L}(C) \qquad (4.6.2.1.8\text{-}2)$$

- Main bars parallel to traffic:

  For $L \leq 120$ in.

$$M_{parallel} = 0.73 D^{0.123} L^{0.64} C \qquad (4.6.2.1.8\text{-}3)$$

  For $L > 120$ in.

$$M_{parallel} = \frac{D^{0.138}\left(3.1L^{1.429} - 1088.5\right)}{L}(C) \qquad (4.6.2.1.8\text{-}4)$$

where:

L = span length from center-to-center of supports (in.)

C = continuity factor; 1.0 for simply supported and 0.8 for continuous spans

D = D_x/D_y

D_x = flexural rigidity of deck in main bar direction (kip-in.^{2}/in.)

D_y = flexural rigidity of deck perpendicular to main bar direction (kip-in.^{2}/in.)

For grid decks, D_x and D_y should be calculated as EI_x and EI_y where E is the modulus of elasticity and I_x and I_y are the moment of inertia per unit width of deck, considering the section as cracked and using the transformed area method for the main bar direction and perpendicular to main bar direction, respectively.

Moments for fatigue assessment may be estimated for all span lengths by reducing Eq. 1 for main bars perpendicular to traffic or Eq. 3 for main bars parallel to traffic by a factor of 3.

C4.6.2.1.8

The moment equations are based on orthotropic plate theory considering vehicular live loads specified in Article 3.6. The equations take into account relevant factored load combinations including truck and tandem loads. The moment equations also account for dynamic load allowance, multiple presence factors, and load positioning on the deck surface to produce the largest possible moment.

Negative moment can be determined as maximum simple span positive moment times the continuity factor, *C*.

The reduction factor of 3.0 in the last sentence of Article 4.6.2.1.8 accounts for smaller dynamic load allowance (15 percent vs. 33 percent), smaller load factor (0.75 vs. 1.75) and no multiple presence (1.0 vs. 1.2) when considering fatigue. Use of Eqs. 1 and 3 for all spans is appropriate as Eqs. 1 and 3 reflect an individual design truck on short-span lengths while Eqs. 2 and 4 reflect the influence of multiple design tandems that control moment envelope on longer span lengths. The approximation produces reasonable estimates of fatigue moments, however, improved estimates can be determined using fatigue truck patch loads in the infinite series formula provided by Higgins (*2003*).

Actual D_x and D_y values can vary considerably depending on the specific deck design, and using assumed values based only on the general type of deck can lead to unconservative design moments. Flexural rigidity in each direction should be calculated analytically as EI considering the section as cracked and using the transformed area method.

Deflection in units of in. due to vehicular live load may be determined as:

- Main bars perpendicular to traffic:

$$\Delta_{transverse} = \frac{0.0052 D^{0.19} L^3}{D_x} \quad (4.6.2.1.8\text{-}5)$$

- Main bars parallel to traffic:

$$\Delta_{parallel} = \frac{0.0072 D^{0.11} L^3}{D_x} \quad (4.6.2.1.8\text{-}6)$$

The deflection equations permit calculation of the midspan displacement for a deck under service load. The equations are based on orthotropic plate theory and consider both truck and tandem loads on a simply supported deck.

Deflection may be reduced for decks continuous over three or more supports. A reduction factor of 0.8 is conservative.

4.6.2.1.9 Inelastic Analysis

The inelastic finite element analysis or yield line analysis may be permitted by the Owner.

4.6.2.2 Beam-Slab Bridges

4.6.2.2.1 Application

The provisions of this Article may be applied to straight girder bridges and horizontally curved concrete bridges, as well as horizontally curved steel girder bridges complying with the provisions of Article 4.6.1.2.4. The provisions of this Article may also be used to determine a starting point for some methods of analysis to determine force effects in curved girders of any degree of curvature in plan.

Except as specified in Article 4.6.2.2.5, the provisions of this Article shall be taken to apply to bridges being analyzed for:

- A single lane of loading, or
- Multiple lanes of live load yielding approximately the same force effect per lane.

If one lane is loaded with a special vehicle or evaluation permit vehicle, the design force effect per girder resulting from the mixed traffic may be determined as specified in Article 4.6.2.2.5.

For beam spacing exceeding the range of applicability as specified in tables in Articles 4.6.2.2.2 and 4.6.2.2.3, the live load on each beam shall be the reaction of the loaded lanes based on the lever rule unless specified otherwise herein.

The provisions of Article 3.6.1.1.2 specify that multiple presence factors shall not be used with the approximate load assignment methods other than statical moment or lever arm methods because these factors are already incorporated in the distribution factors.

C4.6.2.2.1

The V-load method is one example of a method of curved bridge analysis which starts with straight girder distribution factors (*United States Steel, 1984*).

The lever rule involves summing moments about one support to find the reaction at another support by assuming that the supported component is hinged at interior supports.

When using the lever rule on a three-girder bridge, the notional model should be taken as shown in Figure C1. Moments should be taken about the assumed, or notional, hinge in the deck over the middle girder to find the reaction on the exterior girder.

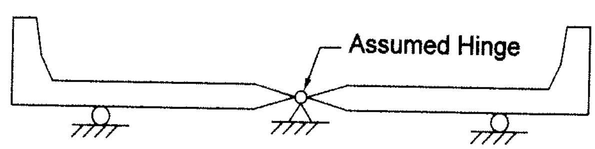


Figure C4.6.2.2.1-1 Notional Model for Applying Lever Rule to Three-Girder Bridges.

Provisions in Articles 4.6.2.2.2 and 4.6.2.2.3 that do not appear in earlier editions of the Standard Specifications come primarily from Zokaie et al. (*1991*). Correction factors for continuity have been deleted for two reasons:

Bridges not meeting the requirements of this Article shall be analyzed as specified in Article 4.6.3.

The distribution of live load, specified in Articles 4.6.2.2.2 and 4.6.2.2.3, may be used for girders, beams, and stringers, other than multiple steel box beams with concrete decks that meet the following conditions and any other conditions identified in tables of distribution factors as specified herein:

- Width of deck is constant;
- Unless otherwise specified, the number of beams is not less than four;
- Beams are parallel and have approximately the same stiffness;
- Unless otherwise specified, the roadway part of the overhang, d_e, does not exceed 3.0 ft.;
- Curvature in plan is less than the limit specified in Article 4.6.1.2.4, or where distribution factors are required in order to implement an acceptable approximate or refined analysis method satisfying the requirements of Article 4.4 for bridges of any degree of curvature in plan; and
- Cross-section is consistent with one of the cross-sections shown in Table 1.

Where moderate deviations from a constant deck width or parallel beams exist, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value for beam spacing.

- Correction factors dealing with 5 percent adjustments were thought to imply misleading levels of accuracy in an approximate method, and
- Analyses of many continuous beam-slab-type bridges indicate that the distribution coefficients for negative moments exceed those obtained for positive moments by approximately 10 percent. On the other hand, it has been observed that stresses at or near an internal bearing are reduced due to the fanning of the reaction force. This reduction is about the same magnitude as the increase in distribution factors, hence the two tend to cancel each other out, and thus are omitted from these Specifications.

In Strength Load Combination II, applying a distribution factor procedure to a loading involving a heavy permit load can be overly conservative unless lane-by-lane distribution factors are available. Use of a refined method of analysis will circumvent this situation.

A rational approach may be used to extend the provisions of this Article to bridges with splayed girders. The distribution factor for live load at any point along the span may be calculated by setting the girder spacing in the equations of this Article equal to half the sum of the center-to-center distance between the girder under consideration and the two girders to either side. This will result in a variable distribution factor along the length of the girder. While the variable distribution factor is theoretically correct, it is not compatible with existing line girder computer programs that only allow constant distribution factor. Further simplifications may be used to allow the use of such computer programs. One such simplification involves running the computer program a number of times equal to the number of spans in the bridge. For each run, the girder spacing is set equal to the maximum girder spacing in one span and the results from this run are applied to this span. This approach is guaranteed to result in conservative design. In the past, some jurisdictions applied the latter approach, but used the girder spacing at the 2/3 or 3/4 points of the span; which will also be an acceptable approximation.

Most of the equations for distribution factors were derived for constant deck width and parallel beams. Past designs with moderate exceptions to these two assumptions have performed well when the S/D distribution factors were used. While the distribution factors specified herein are more representative of actual bridge behavior, common sense indicates that some exceptions are still possible, especially if the parameter S is chosen with prudent judgment, or if the factors are appropriately varied at selected locations along the span.

Cast-in-place multicell concrete box girder bridge types may be designed as whole-width structures. Such cross-sections shall be designed for the live load distribution factors in Articles 4.6.2.2.2 and 4.6.2.2.3 for interior girders, multiplied by the number of girders, i.e., webs.

Additional requirements for multiple steel box girders with concrete decks shall be as specified in Article 4.6.2.2.2b.

Where bridges meet the conditions specified herein, permanent loads of and on the deck may be distributed uniformly among the beams and/or stringers.

Live load distribution factors, specified herein, may be used for permit and rating vehicles whose overall width is comparable to the width of the design truck.

The following notation shall apply to tables in Articles 4.6.2.2.2 and 4.6.2.2.3:

A = area of stringer, beam or girder (in.^{2})
b = width of beam (in.)
C = stiffness parameter
D = width of distribution per lane (ft.)
d = depth of beam or stringer (in.)
d_e = distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (ft.)
e = correction factor
g = distribution factor
I_p = polar moment of inertia (in.^{4})
J = St. Venant's torsional inertia (in.^{4})
K = constant for different types of construction
K_g = longitudinal stiffness parameter (in.^{4})
L = span of beam (ft.)
N_b = number of beams, stringers or girders
N_c = number of cells in a concrete box girder
N_L = number of design lanes as specified in Article 3.6.1.1.1
S = spacing of beams or webs (ft.)
t_g = depth of steel grid or corrugated steel plank including integral concrete overlay or structural concrete component, less a provision for grinding, grooving, or wear (in.)
t_o = depth of structural overlay (in.)
t_s = depth of concrete slab (in.)
W = edge-to-edge width of bridge (ft.)
W_e = half the web spacing, plus the total overhang (ft.)
θ = skew angle (°)
μ = Poisson's ratio

Unless otherwise stated, the stiffness parameters for area, moments of inertia and torsional stiffness used herein and in Articles 4.6.2.2.2 and 4.6.2.2.3 shall be taken as those of the cross-section to which traffic will be applied, i.e., usually the composite section.

Whole-width design is appropriate for torsionally-stiff cross-sections where load-sharing between girders is extremely high and torsional loads are hard to estimate. Prestressing force should be evenly distributed between girders. Cell width-to-height ratios should be approximately 2:1.

In lieu of more refined information, the St. Venant torsional inertia, J, may be determined as:

- For thin-walled open beam:

$$J = \frac{1}{3}\Sigma bt^3 \qquad \text{(C4.6.2.2.1-1)}$$

- For stocky open sections, e.g., prestressed I-beams, T-beams, etc., and solid sections:

$$J = \frac{A^4}{40.0 I_p} \qquad \text{(C4.6.2.2.1-2)}$$

- For closed thin-walled shapes:

$$J = \frac{4A_o^2}{\Sigma \frac{s}{t}} \qquad \text{(C4.6.2.2.1-3)}$$

where:

b = width of plate element (in.)
t = thickness of plate-like element (in.)
A = area of cross-section (in.^{2})
I_p = polar moment of inertia (in.^{4})
A_o = area enclosed by centerlines of elements (in.^{2})
s = length of a side element (in.)

Eq. C2 has been shown to substantially underestimate the torsional stiffness of some concrete I-beams and a more accurate, but more complex, approximation can be found in Eby et al. (*1973*).

The transverse post-tensioning shown for some cross-sections herein is intended to make the units act together. A minimum 0.25 ksi prestress is recommended.

For beams with variable moment of inertia, K_g may be based on average properties.

For bridge types "f," "g," "h," "i," and "j," longitudinal joints between precast units of the cross-section are shown in Table 1. This type of construction

The longitudinal stiffness parameter, K_g, shall be taken as:

$$K_g = n\left(I + Ae_g^2\right) \quad (4.6.2.2.1\text{-}1)$$

in which:

$$n = \frac{E_B}{E_D} \quad (4.6.2.2.1\text{-}2)$$

where:

E_B = modulus of elasticity of beam material (ksi)

E_D = modulus of elasticity of deck material (ksi)

I = moment of inertia of beam (in.^{4})

e_g = distance between the centers of gravity of the basic beam and deck (in.)

The parameters A and I in Eq. 1 shall be taken as those of the noncomposite beam.

The bridge types indicated in tables in Articles 4.6.2.2.2 and 4.6.2.2.3, with reference to Table 1, may be taken as representative of the type of bridge to which each approximate equation applies.

acts as a monolithic unit if sufficiently interconnected. In Article 5.14.4.3.3f, a fully interconnected joint is identified as a flexural shear joint. This type of interconnection is enhanced by either transverse post-tensioning of an intensity specified above or by a reinforced structural overlay, which is also specified in Article 5.14.4.3.3f, or both. The use of transverse mild steel rods secured by nuts or similar unstressed dowels should not be considered sufficient to achieve full transverse flexural continuity unless demonstrated by testing or experience. Generally, post-tensioning is thought to be more effective than a structural overlay if the intensity specified above is achieved.

In some cases, the lower limit of deck slab thickness, t_s, shown in the range of applicability column in tables in Articles 4.6.2.2.2 and 4.6.2.2.3 is less than 7.0 in. The research used to develop the equations in those tables reflects the range of slab thickness shown. Article 9.7.1.1 indicates that concrete decks less than 7.0 in. in thickness should not be used unless approved by the Owner. Lesser values shown in tables in Articles 4.6.2.2.2 and 4.6.2.2.3 are not intended to override Article 9.7.1.1.

The load distribution factor equations for bridge type "d", cast-in-place multicell concrete box girders, were derived by first positioning the vehicle longitudinally, and then transversely, using an I-section of the box. While it would be more appropriate to develop an algorithm to find the peak of an influence surface, using the present factor for the interior girders multiplied by the number of girders is conservative in most cases.

Table C1 describes how the term L (length) may be determined for use in the live load distribution factor equations given in Articles 4.6.2.2.2 and 4.6.2.2.3.

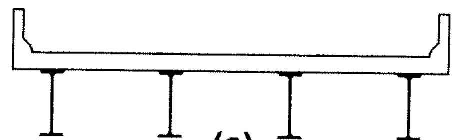
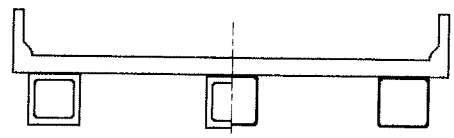
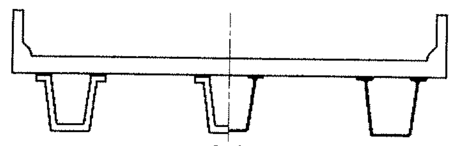
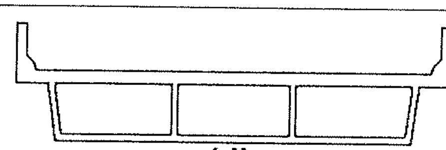
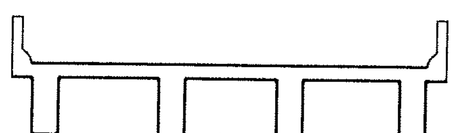
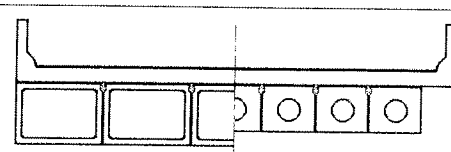
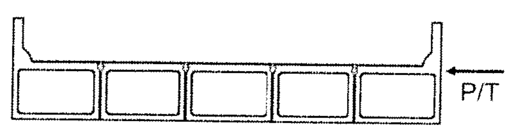
Table C4.6.2.2.1-1 *L* for Use in Live Load Distribution Factor Equations.

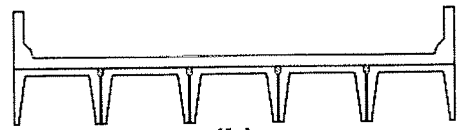
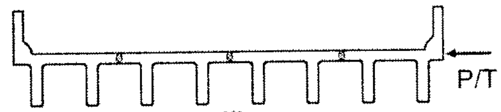
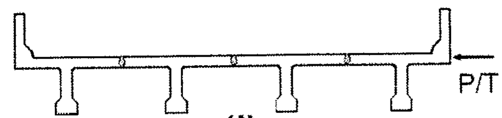
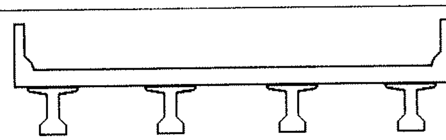
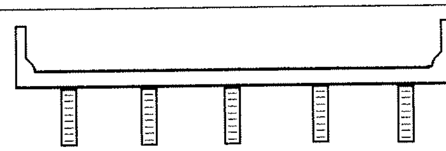
Force Effect	L (ft.)
Positive Moment	The length of the span for which moment is being calculated
Negative Moment—Near interior supports of continuous spans from point of contraflexure to point of contraflexure under a uniform load on all spans	The average length of the two adjacent spans
Negative Moment—Other than near interior supports of continuous spans	The length of the span for which moment is being calculated
Shear	The length of the span for which shear is being calculated
Exterior Reaction	The length of the exterior span
Interior Reaction of Continuous Span	The average length of the two adjacent spans

Except as permitted by Article 2.5.2.7.1, regardless of the method of analysis used, i.e., approximate or refined, exterior girders of multibeam bridges shall not have less resistance than an interior beam.

In the rare occasion when the continuous span arrangement is such that an interior span does not have any positive uniform load moment (i.e., no uniform load points of contraflexure), the region of negative moment near the interior supports would be increased to the centerline of the span, and the *L* used in determining the live load distribution factors would be the average of the two adjacent spans.

Table 4.6.2.2.1-1 Common Deck Superstructures Covered in Articles 4.6.2.2.2 and 4.6.2.2.3.

Supporting Components	Type Of Deck	Typical Cross-Section
Steel Beam	Cast-in-place concrete slab, precast concrete slab, steel grid, glued/spiked panels, stressed wood	 (a)
Closed Steel or Precast Concrete Boxes	Cast-in-place concrete slab	 (b)
Open Steel or Precast Concrete Boxes	Cast-in-place concrete slab, precast concrete deck slab	 (c)
Cast-in-Place Concrete Multicell Box	Monolithic concrete	 (d)
Cast-in-Place Concrete Tee Beam	Monolithic concrete	 (e)
Precast Solid, Voided or Cellular Concrete Boxes with Shear Keys	Cast-in-place concrete overlay	 (f)
Precast Solid, Voided, or Cellular Concrete Box with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	 (g)

Supporting Components	Type Of Deck	Typical Cross-Section
Precast Concrete Channel Sections with Shear Keys	Cast-in-place concrete overlay	 (h)
Precast Concrete Double Tee Section with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	 (i)
Precast Concrete Tee Section with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	 (j)
Precast Concrete I or Bulb-Tee Sections	Cast-in-place concrete, precast concrete	 (k)
Wood Beams	Cast-in-place concrete or plank, glued/spiked panels or stressed wood	 (l)

For cast-in-place concrete multicell box shown as cross-section Type "d" in Table 1, the distribution factors in Article 4.6.2.2.2 and 4.6.2.2.3 shall be taken to apply to a notional shape consisting of a web, overhangs of an exterior web, and the associated half flanges between a web under consideration and the next adjacent web or webs.

4.6.2.2.2 Distribution Factor Method for Moment and Shear

4.6.2.2.2a Interior Beams with Wood Decks

The live load flexural moment and shear for interior beams with transverse wood decks may be determined by applying the lane fraction specified in Table 1 and Eq. 1.

When investigation of shear parallel to the grain in wood components is required, the distributed live load shear shall be determined by the following expression:

$$V_{LL} = 0.50\left[\left(0.60V_{LU}\right) + V_{LD}\right] \quad (4.6.2.2.2a\text{-}1)$$

where:

V_{LL} = distributed live load vertical shear (kips)

V_{LU} = maximum vertical shear at $3d$ or $L/4$ due to undistributed wheel loads (kips)

V_{LD} = maximum vertical shear at $3d$ or $L/4$ due to wheel loads distributed laterally as specified herein (kips)

For undistributed wheel loads, one line of wheels is assumed to be carried by one bending member.

Table 4.6.2.2.2a-1 Distribution of Live Load Per Lane for Moment and Shear in Interior Beams with Wood Decks.

Type of Deck	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Plank	a, l	$S/6.7$	$S/7.5$	$S \leq 5.0$
Stressed Laminated	a, l	$S/9.2$	$S/9.0$	$S \leq 6.0$
Spike Laminated	a, l	$S/8.3$	$S/8.5$	$S \leq 6.0$
Glued Laminated Panels on Glued Laminated Stringers	a, l	$S/10.0$	$S/10.0$	$S \leq 6.0$
Glue Laminated Panels on Steel Stringers	a, l	$S/8.8$	$S/9.0$	$S \leq 6.0$

4.6.2.2.2b Interior Beams with Concrete Decks

C4.6.2.2.2b

The live load flexural moment for interior beams with concrete decks may be determined by applying the lane fraction specified in Table 1.

For preliminary design, the terms $K_g/(12.0Lt_s^3)$ and I/J may be taken as 1.0.

For the concrete beams, other than box beams, used in multibeam decks with shear keys:

- Deep, rigid end diaphragms shall be provided to ensure proper load distribution; and

- If the stem spacing of stemmed beams is less than 4.0 ft. or more than 10.0 ft., a refined analysis complying with Article 4.6.3 shall be used.

For multiple steel box girders with a concrete deck in bridges satisfying the requirements of Article 6.11.2.3, the live load flexural moment may be determined using the appropriate distribution factor specified in Table 1.

Where the spacing of the box girders varies along the length of the bridge, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value of N_L. In either case, the value of N_L shall be determined as specified in Article 3.6.1.1.1, using the width, w, taken at the section under consideration.

The results of analytical and model studies of simple span multiple box section bridges, reported in Johnston and Mattock (*1967*), showed that folded plate theory could be used to analyze the behavior of bridges of this type. The folded plate theory was used to obtain the maximum load per girder, produced by various critical combinations of loading on 31 bridges having various spans, numbers of box girders, and numbers of traffic lanes.

Multiple presence factors, specified in Table 3.6.1.1.2-1, are not applied because the multiple factors in past editions of the Standard Specifications were considered in the development of the equation in Table 1 for multiple steel box girders.

The lateral load distribution obtained for simple spans is also considered applicable to continuous structures.

The bridges considered in the development of the equations had interior end diaphragms only, i.e., no interior diaphragms within the spans, and no exterior diaphragms anywhere between boxes. If interior or exterior diaphragms are provided within the span, the transverse load distribution characteristics of the bridge will be improved to some degree. This improvement can be evaluated, if desired, using the analysis methods identified in Article 4.4.

Table 4.6.2.2.2b-1 Distribution of Live Loads Per Lane for Moment in Interior Beams.

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	See Table 4.6.2.2.2a-1	
Concrete Deck on Wood Beams	l	One Design Lane Loaded: $S/12.0$ Two or More Design Lanes Loaded: $S/10.0$	$S \le 6.0$
Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	One Design Lane Loaded: $0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$ Two or More Design Lanes Loaded: $0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0\,Lt_s^3}\right)^{0.1}$	$3.5 \le S \le 16.0$ $4.5 \le t_s \le 12.0$ $20 \le L \le 240$ $N_b \ge 4$ $10{,}000 \le K_g \le 7{,}000{,}000$
		use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	One Design Lane Loaded: $\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$ Two or More Design Lanes Loaded: $\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$	$7.0 \le S \le 13.0$ $60 \le L \le 240$ $N_c \ge 3$ If $N_c > 8$ use $N_c = 8$
Concrete Deck on Concrete Spread Box Beams	b, c	One Design Lane Loaded: $\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25}$ Two or More Design Lanes Loaded: $\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$	$6.0 \le S \le 18.0$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
		Use Lever Rule	$S > 18.0$
Concrete Beams used in Multibeam Decks	f	One Design Lane Loaded: $k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$ where: $k = 2.5(N_b)^{-0.2} \ge 1.5$	$35 \le b \le 60$ $20 \le L \le 120$ $5 \le N_b \le 20$
	g if sufficiently connected to act as a unit	Two or More Design Lanes Loaded: $k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.06}$	

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
	h g, i, j if connected only enough to prevent relative vertical displacement at the interface	Regardless of Number of Loaded Lanes: S/D where: $C = K(W/L) \leq K$ $D = 11.5 - N_L + 1.4N_L(1-0.2C)^2$ when $C \leq 5$ $D = 11.5 - N_L$ when $C > 5$ $K = \sqrt{\frac{(1+\mu)I}{J}}$ for preliminary design, the following values of K may be used: Beam Type — K Nonvoided rectangular beams — 0.7 Rectangular beams with circular voids: — 0.8 Box section beams — 1.0 Channel beams — 2.2 T-beam — 2.0 Double T-beam — 2.0	Skew $\leq 45°$ $N_L \leq 6$
Open Steel Grid Deck on Steel Beams	a	One Design Lane Loaded: $S/7.5$ If $t_g < 4.0$ $S/10.0$ If $t_g \geq 4.0$ Two or More Design Lanes Loaded: $S/8.0$ If $t_g < 4.0$ $S/10.0$ If $t_g \geq 4.0$	$S \leq 6.0$ $S \leq 10.5$
Concrete Deck on Multiple Steel Box Girders	b, c	Regardless of Number of Loaded Lanes: $0.05 + 0.85\frac{N_L}{N_b} + \frac{0.425}{N_L}$	$0.5 \leq \frac{N_L}{N_b} \leq 1.5$

4.6.2.2.2c Interior Beams with Corrugated Steel Decks

The live load flexural moment for interior beams with corrugated steel plank deck may be determined by applying the lane fraction, g, specified in Table 1.

Table 4.6.2.2.2c-1 Distribution of Live Load Per Lane for Moment in Interior Beams with Corrugated Steel Plank Decks.

One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
$S/9.2$	$S/9.0$	$S \leq 5.5$ $t_g \geq 2.0$

4.6.2.2.2d Exterior Beams

The live load flexural moment for exterior beams may be determined by applying the lane fraction, g, specified in Table 1.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier.

In beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

C4.6.2.2.2d

This additional investigation is required because the distribution factor for girders in a multigirder cross-section, Types "a," "e," and "k" in Table 4.6.2.2.1-1, was determined without consideration of diaphragm or cross-frames. The recommended procedure is an interim provision until research provides a better solution.

The procedure outlined in this section is the same as the conventional approximation for loads on piles.

$$R = \frac{N_L}{N_b} + \frac{X_{ext}\sum^{N_L} e}{\sum^{N_b} x^2} \qquad \text{(C4.6.2.2.2d-1)}$$

where:

R = reaction on exterior beam in terms of lanes

N_L = number of loaded lanes under consideration

e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft.)

x = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.)

X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft.)

N_b = number of beams or girders

Table 4.6.2.2.2d-1 Distribution of Live Loads Per Lane for Moment in Exterior Longitudinal Beams.

<table>
<tr><th>Type of Superstructure</th><th>Applicable Cross-Section from Table 4.6.2.2.1-1</th><th>One Design Lane Loaded</th><th>Two or More Design Lanes Loaded</th><th>Range of Applicability</th></tr>
<tr><td>Wood Deck on Wood or Steel Beams</td><td>a, l</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td>Concrete Deck on Wood Beams</td><td>l</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td rowspan="2">Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections</td><td rowspan="2">a, e, k and also i, j if sufficiently connected to act as a unit</td><td rowspan="2">Lever Rule</td><td>$g = e\, g_{interior}$
$e = 0.77 + \frac{d_e}{9.1}$</td><td>$-1.0 \le d_e \le 5.5$</td></tr>
<tr><td>use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule</td><td>$N_b = 3$</td></tr>
<tr><td rowspan="2">Cast-in-Place Concrete Multicell Box</td><td rowspan="2">d</td><td>$g = \frac{W_e}{14}$</td><td>$g = \frac{W_e}{14}$</td><td rowspan="2">$W_e \le S$</td></tr>
<tr><td colspan="2">or the provisions for a whole-width design specified in Article 4.6.2.2.1</td></tr>
<tr><td rowspan="2">Concrete Deck on Concrete Spread Box Beams</td><td rowspan="2">b, c</td><td rowspan="2">Lever Rule</td><td>$g = e\, g_{interior}$
$e = 0.97 + \frac{d_e}{28.5}$</td><td>$0 \le d_e \le 4.5$
$6.0 < S \le 18.0$</td></tr>
<tr><td>Use Lever Rule</td><td>$S > 18.0$</td></tr>
<tr><td>Concrete Box Beams Used in Multibeam Decks</td><td>f, g</td><td>$g = e\, g_{interior}$
$e = 1.125 + \frac{d_e}{30} \ge 1.0$</td><td>$g = e\, g_{interior}$
$e = 1.04 + \frac{d_e}{25} \ge 1.0$</td><td>$d_e \le 2.0$</td></tr>
<tr><td rowspan="2">Concrete Beams Other than Box Beams Used in Multibeam Decks</td><td>h</td><td rowspan="2">Lever Rule</td><td rowspan="2">Lever Rule</td><td rowspan="2">N/A</td></tr>
<tr><td>i, j if connected only enough to prevent relative vertical displacement at the interface</td></tr>
<tr><td>Open Steel Grid Deck on Steel Beams</td><td>a</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td>Concrete Deck on Multiple Steel Box Girders</td><td>b, c</td><td colspan="3">As specified in Table 4.6.2.2.2b-1</td></tr>
</table>

4.6.2.2.2e Skewed Bridges

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10°, the bending moment in the beams may be reduced in accordance with Table 1.

C4.6.2.2.2e

Accepted reduction factors are not currently available for cases not covered in Table 1.

Table 4.6.2.2.2e-1 Reduction of Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports.

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Any Number of Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$1-c_1(\tan\theta)^{1.5}$ $c_1 = 0.25\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}$ If $\theta < 30°$ then $c_1 = 0.0$ If $\theta > 60°$ use $\theta = 60°$	$30° \leq \theta \leq 60°$ $3.5 \leq S \leq 16.0$ $20 \leq L \leq 240$ $N_b \geq 4$
Concrete Deck on Concrete Spread Box Beams, Cast-in-Place Multicell Box Concrete Box Beams and Double T-Sections used in Multibeam Decks	b, c, d, f, g	$1.05 - 0.25 \tan\theta \leq 1.0$ If $\theta > 60°$ use $\theta = 60°$	$0° \leq \theta \leq 60°$

4.6.2.2.2f Flexural Moments and Shear in Transverse Floorbeams

If the deck is supported directly by transverse floorbeams, the floorbeams may be designed for loads determined in accordance with Table 1.

The fractions provided in Table 1 shall be used in conjunction with the 32.0-kip design axle load alone. For spacings of floorbeams outside the given ranges of applicability, all of the design live loads shall be considered, and the lever rule may be used.

Table 4.6.2.2.2f-1 Distribution of Live Load per Lane for Transverse Beams for Moment and Shear.

Type of Deck	Fraction of Wheel Load to Each Floorbeam	Range of Applicability
Plank	$\frac{S}{4}$	N/A
Laminated Wood Deck	$\frac{S}{5}$	$S \leq 5.0$
Concrete	$\frac{S}{6}$	$S \leq 6.0$
Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab	$\frac{S}{4.5}$	$t_g \leq 4.0$ $S \leq 5.0$
Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab	$\frac{S}{6}$	$t_g > 4.0$ $S \leq 6.0$
Steel Bridge Corrugated Plank	$\frac{S}{5.5}$	$t_g \geq 2.0$

4.6.2.2.3 Distribution Factor Method for Shear

4.6.2.2.3a Interior Beams

The live load shear for interior beams may be determined by applying the lane fractions specified in Table 1. For interior beam types not listed in Table 1, lateral distribution of the wheel or axle adjacent to the end of span shall be that produced by use of the lever rule.

For preliminary design, the term I/J may be taken as 1.0.

For concrete box beams used in multibeam decks, if the values of I or J do not comply with the limitations in Table 1, the distribution factor for shear may be taken as that for moment.

Table 4.6.2.2.3a-1 Distribution of Live Load per Lane for Shear in Interior Beams.

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	See Table 4.6.2.2.2a-1		
Concrete Deck on Wood Beams	l	Lever Rule	Lever Rule	N/A
Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T-and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$0.36+\dfrac{S}{25.0}$	$0.2+\dfrac{S}{12}-\left(\dfrac{S}{35}\right)^{2.0}$	$3.5 \le S \le 16.0$ $20 \le L \le 240$ $4.5 \le t_s \le 12.0$ $N_b \ge 4$
		Lever Rule	Lever Rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	$\left(\dfrac{S}{9.5}\right)^{0.6}\left(\dfrac{d}{12.0L}\right)^{0.1}$	$\left(\dfrac{S}{7.3}\right)^{0.9}\left(\dfrac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Concrete Spread Box Beams	b, c	$\left(\dfrac{S}{10}\right)^{0.6}\left(\dfrac{d}{12.0L}\right)^{0.1}$	$\left(\dfrac{S}{7.4}\right)^{0.8}\left(\dfrac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 18.0$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
		Lever Rule	Lever Rule	$S > 18.0$
Concrete Box Beams Used in Multibeam Decks	f, g	$\left(\dfrac{b}{130L}\right)^{0.15}\left(\dfrac{I}{J}\right)^{0.05}$	$\left(\dfrac{b}{156}\right)^{0.4}\left(\dfrac{b}{12.0L}\right)^{0.1}\left(\dfrac{I}{J}\right)^{0.05}\left(\dfrac{b}{48}\right)$ $\dfrac{b}{48} \ge 1.0$	$35 \le b \le 60$ $20 \le L \le 120$ $5 \le N_b \le 20$ $25,000 \le J \le 610,000$ $40,000 \le I \le 610,000$
Concrete Beams Other Than Box Beams Used in Multibeam Decks	h i, j if connected only enough to prevent relative vertical displacement at the interface	Lever Rule	Lever Rule	N/A
Open Steel Grid Deck on Steel Beams	a	Lever Rule	Lever Rule	N/A
Concrete Deck on Multiple Steel Box Beams	b, c	As specified in Table 4.6.2.2.2b-1		

4.6.2.2.3b Exterior Beams

The live load shear for exterior beams shall be determined by applying the lane fractions specified in Table 1. For cases not addressed in Table 4.6.2.2.3a-1 and Table 1, the live load distribution to exterior beams shall be determined by using the lever rule.

The parameter d_e shall be taken as positive if the exterior web is inboard of the curb or traffic barrier and negative if it is outboard.

The additional provisions for exterior beams in beam-slab bridges with cross-frames or diaphragms, specified in Articles 4.6.2.2.2d, shall apply.

Table 4.6.2.2.3b-1 Distribution of Live Load per Lane for Shear in Exterior Beams.

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	Lever Rule	Lever Rule	N/A
Concrete Deck on Wood Beams	l	Lever Rule	Lever Rule	N/A
Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Beams	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = e\,g_{interior}$ $e = 0.6 + \frac{d_e}{10}$	$-1.0 \le d_e \le 5.5$
			Lever Rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	Lever Rule	$g = e\,g_{interior}$ $e = 0.64 + \frac{d_e}{12.5}$	$-2.0 \le d_e \le 5.0$
		or the provisions for a whole-width design specified in Article 4.6.2.2.1		
Concrete Deck on Concrete Spread Box Beams	b, c	Lever Rule	$g = e\,g_{interior}$ $e = 0.8 + \frac{d_e}{10}$	$0 \le d_e \le 4.5$
			Lever Rule	$S > 18.0$
Concrete Box Beams Used in Multibeam Decks	f, g	$g = e\,g_{interior}$ $e = 1.25 + \frac{d_e}{20} \ge 1.0$	$g = e\,g_{interior}\left(\frac{48}{b}\right)$ $\frac{48}{b} \le 1.0$ $e = 1 + \left(\frac{d_e + \frac{b}{12} - 2.0}{40}\right)^{0.5} \ge 1.0$	$d_e \le 2.0$ $35 \le b \le 60$
Concrete Beams Other Than Box Beams Used in Multibeam Decks	h i, j if connected only enough to prevent relative vertical displacement at the interface	Lever Rule	Lever Rule	N/A
Open Steel Grid Deck on Steel Beams	a	Lever Rule	Lever Rule	N/A
Concrete Deck on Multiple Steel Box Beams	b, c	As specified in Table 4.6.2.2.2b-1		

4.6.2.2.3c Skewed Bridges

Shear in the exterior beam at the obtuse corner of the bridge shall be adjusted when the line of support is skewed. The value of the correction factor shall be obtained from Table 1. It is applied to the lane fraction specified in Table 4.6.2.2.3a-1 for interior beams and in Table 4.6.2.2.3b-1 for exterior beams.

In determining the end shear in multibeam bridges, the skew correction at the obtuse corner shall be applied to all the beams.

C4.6.2.2.3c

Verifiable correction factors are not available for cases not covered in Table 1.

The equal treatment of all beams in a multibeam bridge is conservative regarding positive reaction and shear. However, it is not necessarily conservative regarding uplift in the case of large skew and short exterior spans of continuous beams. A supplementary investigation of uplift should be considered using the correction factor from Table 1, i.e., the terms other than 1.0, taken as negative for the exterior beam at the acute corner.

Table 4.6.2.2.3c-1 Correction Factors for Load Distribution Factors for Support Shear of the Obtuse Corner.

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Correction Factor	Range of Applicability
Concrete Deck, Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Section	a, e, k and also i, j if sufficiently connected to act as a unit	$1.0+0.20\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3}\tan\theta$	$0° \le \theta \le 60°$ $3.5 \le S \le 16.0$ $20 \le L \le 240$ $N_b \ge 4$
Cast-in-Place Concrete Multicell Box	d	$1.0+\left(0.25+\frac{12.0L}{70d}\right)\tan\theta$	$0° < \theta \le 60°$ $6.0 < S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Spread Concrete Box Beams	b, c	$1.0+\frac{\sqrt{\frac{Ld}{12.0}}}{6S}\tan\theta$	$0° < \theta \le 60°$ $6.0 \le S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
Concrete Box Beams Used in Multibeam Decks	f, g	$1.0+\frac{12.0L}{90d}\sqrt{\tan\theta}$	$0° < \theta \le 60°$ $20 \le L \le 120$ $17 \le d \le 60$ $35 \le b \le 60$ $5 \le N_b \le 20$

4.6.2.2.4 Curved Steel Bridges

Approximate analysis methods may be used for analysis of curved steel bridges. The Engineer shall ascertain that the approximate analysis method used is appropriate by confirming that the method satisfies the requirements stated in Article 4.4.

In curved systems, consideration should be given to placing parapets, sidewalks, barriers and other heavy line loads at their actual location on the bridge. Wearing surface and other distributed loads may be assumed uniformly distributed to each girder in the cross-section.

C4.6.2.2.4

The V-load method (*United States Steel, 1984*) has been a widely used approximate method for analyzing horizontally curved steel I-girder bridges. The method assumes that the internal torsional load on the bridge—resulting solely from the curvature—is resisted by self-equilibrating sets of shears between adjacent girders. The V-load method does not directly account for sources of torque other than curvature and the method does not account for the horizontal shear stiffness of the concrete deck. The method is only valid for loads such as normal highway loadings. For exceptional loadings, a more refined analysis is required. The method assumes a linear distribution of girder shears across the bridge section; thus, the girders at a given cross-section should have approximately the same vertical stiffness. The V-load method is also not directly applicable to structures with reverse curvature or to a closed-framed system with horizontal lateral bracing near, or in the plane of one or both flanges. The V-load method does not directly account for girder twist; thus, lateral deflections, which become important on bridges with large spans and/or sharp skews and vertical deflections, may be significantly underestimated. In certain situations, the V-load method may not detect uplift at end bearings. The method is best suited for preliminary design, but may also be suitable for final design of structures with radial supports or supports skewed less than approximately 10°.

The M/R method provides a means to account for the effect of curvature in curved box girder bridges. The method and suggested limitations on its use are discussed by Tung and Fountain (*1970*).

Vertical reactions at interior supports on the concave side of continuous-span bridges may be significantly underestimated by both the V-load and M/R methods.

Live load distribution factors for use with the V-load and M/R methods may be determined using the appropriate provisions of Article 4.6.2.2.

Strict rules and limitations on the applicability of both of these approximate methods do not exist. The Engineer must determine when approximate methods of analysis are appropriate.

4.6.2.2.5 Special Loads with Other Traffic

Except as specified herein, the provisions of this Article may be applied where the approximate methods of analysis for the analysis of beam-slab bridges specified in Article 4.6.2.2 and slab-type bridges specified in Article 4.6.2.3 are used. The provisions of this Article shall not be applied where either:

- the lever rule has been specified for both single lane and multiple lane loadings, or
- the special requirement for exterior girders of beam-slab bridge cross-sections with diaphragms specified in Article 4.6.2.2.2d has been utilized for simplified analysis.

Force effects resulting from heavy vehicles in one lane with routine traffic in adjacent lanes, such as might be considered with Load Combination Strength II in Table 3.4.1-1 may be determined as:

$$G = G_p\left(\frac{g_1}{Z}\right) + G_D\left(g_m - \frac{g_1}{Z}\right) \qquad (4.6.2.2.4\text{-}1)$$

where:

G = final force effect applied to a girder (kip or kip-ft.)

G_p = force effect due to overload truck (kip or kip-ft.)

g_1 = single lane live load distribution factor

G_D = force effect due to design loads (kip or kip-ft.)

g_m = multiple lane live load distribution factor

Z = a factor taken as 1.20 where the lever rule was not utilized, and 1.0 where the lever rule was used for a single lane live load distribution factor

C4.6.2.2.5

Because the number of loaded lanes used to determine the multiple lane live load distribution factor, g_m, is not known, the multiple lane multiple presence factor, m, is implicitly set equal to 1.0 in this equation, which assumes only two lanes are loaded, resulting in a conservative final force effect over using the multiple presence factors for three or more lanes loaded.

The factor Z is used to distinguish between situations where the single lane live load distribution factor was determined from a specified algebraic equation and situations where the lever rule was specified for the determination of the single lane live load distribution factor. In the situation where an algebraic equation was specified, the multiple presence factor of 1.20 for a single lane loaded has been included in the algebraic equation and must be removed by using Z = 1.20 in Eq. 1 so that the distribution factor can be utilized in Eq. 1 to determine the force effect resulting from a multiple lane loading.

This formula was developed from a similar formula presented without investigation by Modjeski and Masters, Inc. (*1994*) in a report to the Pennsylvania Department of Transportation in 1994, as was examined in Zokaie (*1998*).

4.6.2.3 Equivalent Strip Widths for Slab-Type Bridges

This Article shall be applied to the types of cross-sections shown schematically in Table 1. For the purpose of this Article, cast-in-place voided slab bridges may be considered as slab bridges.

The equivalent width of longitudinal strips per lane for both shear and moment with one lane, i.e., two lines of wheels, loaded may be determined as:

C4.6.2.3