ACI 224.2R-92

(Reapproved 2004)

Cracking of Concrete Members in Direct Tension

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This report is concerned with cracking in reinforced concrete caused primarily by direct tension rather than bending. Causes of direct tension cracking are reviewed, and equations for predicting crack spacing and crack width are presented. As cracking progresses with increasing load, axial stiffness decreases. Methods for estimating post-cracking axial stiffness are discussed. The report concludes with a review of methods for controlling cracking caused by direct tension.

Keywords: cracking (fracturing); crack width and spacing; loads (forces); reinforced concrete; restraints; stiffness; strains; stresses; tensile stress; tension; volume change.

CONTENTS

The 1992 revisions became effective Mar. 1, 1992. The revisions consisted of removing year designations of the recommended references of standards-producing organizations so that they refer to current editions.

CHAPTER l-INTRODUCTION

Because concrete is relatively weak and brittle in tension, cracking is expected when significant tensile stress is induced in a member. Mild reinforcement and/or prestressing steel can be used to provide the necessary tensile strength of a tension member. However, a number of factors must be considered in both design and construction to insure proper control of cracking that may occur.

A separate report by ACI Committee 224 (ACI 224R) covers control of cracking in concrete members in general, but contains only a brief reference to tension cracking. This report deals specifically with cracking in members subjected to direct tension.

Chapter 2 reviews the primary causes of direct tension cracking, applied loads, and restraint of volume change. Chapter 3 discusses crack mechanisms in tension members and presents methods for predicting crack spacing and width. The effect of cracking on axial stiffness is discussed in Chapter 4. As cracks develop, a progressive reduction in axial stiffness takes place. Methods for estimating the reduced stiffness in the post-cracking range are presented for both one-dimensional members and more complex systems. Chapter 5 reviews measures that should be taken in both design and construction to control cracking in direct tension members.

CHAPTER 2-CAUSES OF CRACKING

2.1-Introduction

Concrete members and structures that transmit loads primarily by direct tension rather than bending include bins and silos, tanks, shells, ties of arches, roof and bridge trusses, and braced frames and towers. Members such as floor and roof slabs, walls, and tunnel linings may also be subjected to direct tension as a result of the restraint of volume change. In many instances, cracking may be attributed to a combination of stresses due to applied load and restraint of volume change. In the following sections, the effects of applied loads and restraint of volume change are discussed in relation to the formation of direct tension cracks.

2.2-Applied loads

Axial forces caused by applied loads can usually be obtained by standard analysis procedures, particularly if the structure is statically determinate. If the structure is statically indeterminate, the member forces are affected by changes in stiffness due to cracking. Methods for estimating the effect of cracking on axial stiffness are presented in Chapter 4.

Cracking occurs when the concrete tensile stress in a member reaches the tensile strength. The load carried by the concrete before cracking is transferred to the reinforcement crossing the crack. For a symmetrical member, the force in the member at cracking is

$$P = (1 - \rho + n\rho) A_g f_t' \qquad (2.1)$$

in which

A_g = gross area
= steel area
f_t' = tensile strength of concrete
n = the ratio of modulus of elasticity of the steel to that of concrete
ρ = reinforcing ratio = A_s/A_g

After cracking, if the applied force remains unchanged, the steel stress at a crack is

$$f_s = \frac{P}{A_s} = \left(\frac{1}{\rho} - 1 + n\right) f_t \qquad (2.2)$$

For n = 10, f_t' = 500 psi (3.45 MPa). Table 2.1 gives the steel stress after cracking for a range of steel ratios ρ, assuming that the yield strength of the steel f_y has not been exceeded.

Table 2.1-Steel stress after cracking for various steel ratios

ρ	$\frac{1}{\rho} - 1 + n$	f_s,* ksi (MPa)
0.005	209	105 (724)
0.010	109	55 (379)
0.030	42	21 (145)
0.050	29	15 (103)

*Assumes $f_s < f_y$.

Table 3.1-Variability of concrete tensile strength: Typical results[5]

Type of test	Mean strength, psi (MPa)	Standard deviation within batches, psi (MPa)	Coefficient of variation, percent
Splitting test	405 (2.8)	20 (0.14)	5
Direct tensile test	275 (1.9)	19 (0.13)	7
Modulus of rupture	605 (4.2)	36 (0.25)	6
Compression cube test	5980 (42)	207 (1.45)	$3^1/_2$

Table 3.2-Relation between compressive strength and tensile strengths of concrete[6]

Compressive strength of cylinders, psi (MPa)	Strength ratio: Modulus of rupture* to compressive strength	Strength ratio: Direct tensile strength to compressive strength	Direct tensile strength to modulus of rupture*
1000 (6.9)	0.23	0.11	0.48
2000 (13.8)	0.19	0.10	0.53
3000 (20.7)	0.16	0.09	0.57
4000 (27.6)	0.15	0.09	0.59
5000 (34.5)	0.14	0.08	0.59
6000 (41.4)	0.13	0.08	0.60
7000 (48.2)	0.12	0.07	0.61
8000 (55.1)	0.12	0.07	0.62
9000 (62.0)	0.11	0.07	0.63

*Determined under third-point loading.

For low steel ratios, depending on the grade of steel, yielding occurs immediately after cracking if the force in the member remains the same. The force in the cracked member at steel yield is $A_s f_y$.

2.3-Restraint

When volume change due to drying shrinkage, thermal contraction, or another cause is restrained, tensile stresses develop and often lead to cracking. The restraint may be provided by stiff supports or reinforcing bars. Restraint may also be provided by other parts of the member when volume change takes place at different rates within a member. For example, tensile stresses occur when drying takes place more rapidly at the exterior than in the interior of a member. A detailed discussion of cracking related to drying shrinkage and temperature effects is given in ACI 224R for concrete structures in general.

Axial forces due to restraint may occur not only in tension members but also in flexural members such as floor and roof slabs. Unanticipated cracking due to axial restraint may lead to undesirable structural behavior such as excessive deflection of floor slabs[1] and reduction in buckling capacity of shell structures.[2] Both are direct results of the reduced flexural stiffness caused by restraint cracking. In addition, the formation of cracks due to restraint can lead to leaking and unsightly conditions when water can penetrate the cracks, as in parking structures.

Cracking due to restraint causes a reduction in axial stiffness, which in turn leads to a reduction (or relaxation) of the restraint force in the member. Therefore, the high level stresses indicated in Table 2.1 for small steel ratios may not develop if the cracking is due to restraint. This point is demonstrated in Tam and Scanlon's numerical analysis of time-dependent restraint force due to drying shrinkage.[3]

CHAPTER 3-CRACK BEHAVIOR AND PREDICTION EQUATIONS

3.1-Introduction

This chapter reviews the basic behavior of reinforced concrete elements subjected to direct tension. Methods for determining tensile strength of plain concrete are discussed and the effect of reinforcement on development of cracks and crack geometry is examined.

3.2-Tensile strength

Methods to determine tensile strength of plain concrete can be classified into one of the following categories: 1) direct tension, 2) flexural tension, and 3) indirect tension[4]. Because of difficulties associated with applying a pure tensile force to a plain concrete specimen, there are no standard tests for direct tension. Following ASTM C 292 and C 78 the modulus of rupture, a measure of tensile strength, can be obtained by testing a plain concrete beam in flexure. An indirect measure of direct tensile strength is obtained from the splitting test (described in ASTM C 496). As indicated in Reference 4, tensile strength measured from the flexure test is usually 40 to 80 percent higher than that measured from the splitting test.

Representative values of tensile strength obtained from tests and measures of variability are shown in Tables 3.1 and 3.2.

ACI 209R suggests the following expressions to estimate tensile strength as a function of compressive strength

$$\text{modulus of rupture: } f_r = g_r [w_c (f_c')]^{1/2} \quad (3.1)$$

$$\text{direct tensile strength: } f_t' = g_t [w_c (f_c')]^{1/2} \quad (3.2)$$

where

w_c = unit weight of concrete (lb/ft^3)
f_c' = compressive strength of concrete (psi)
g_r = 0.60 to 1.00 (0.012 to 0.021 for w_c in kg/m^3 and f_c' in MPa)
g_t = 0.33 (0.0069)

Both the flexure and splitting tests result in a sudden failure of the test specimen, indicating the brittle nature of plain concrete in tension. However, if the deformation of the specimen is controlled in a test, a significant descending branch of the tensile stress-strain diagram can be developed beyond the strain corresponding to maximum tensile stress. Evans and Marathe[7] illustrated this behavior on specimens loaded in direct tension in a testing machine modified to control deformation. Fig. 3.1 shows tensile stress-strain curves that include unloading beyond the maximum tensile stress. More recent work by Petersson[8] shows that the descending branch of the curve is controlled primarily by localized deformatiou across individual cracks, indicating that there are large differences between the average strain (Fig. 3.1) and local strains.

3.3-Development of cracks

When a reinforced concrete member is subjected to tension, two types of cracks eventually form (Fig. 3.2). One type is the visible crack that shows at the surface of the concrete, while the other type does not progress to the concrete surface. Broms[9] called cracks of the first type primary cracks and those of the second type secondary cracks.

Each of the two types of cracks has a different geometry. The primary or external cracks are widest at the surface of the concrete and narrowest at the surface of reinforcing bars.[10-12] The difference in crack width between the concrete surface and the reinforcing bar is small at low tension levels (just after crack formation), and increases as the tension level increases; therefore, the crack width at the reinforcing bar increases more slowly than the width at the concrete surface with an increase in load. The deformations on the reinforcing bars tend to control the crack width by limiting the slip between the concrete and the steel.

The secondary, or internal, cracks increase in width with distance away from the reinforcement before narrowing and closing prior to reaching the concrete surface. More detail on internal crack formation is presented in Reference 13.

Because of the variability in tensile strength along the length of a tension member, cracks do not all form at the same stress level. Clark and Spiers[14] estimated that the first major crack forms at about 90 percent of the average concrete tensile strength and the last major crack at about 110 percent of the average tensile strength. Somayaji and Shah[15] used a bilinear stress-strain diagram for concrete in tension to model the formation of cracks along the member at increasing load levels. They assumed that the tensile strength beyond first cracking was a function of the strain gradient in the concrete along the length of the bar.

Induced tensile stresses caused by restrained concrete shrinkage affect the amount of cracking that is visible at a given tensile force. This has been made apparent by tensile tests conducted to compare the performance of Type I cement and Type K shrinkage-compensating) cement in concrete specimens.[16] Specimens placed under the same conditions of environment and loading had markedly different cracking behavior.

When specimens made with Type I cement had fully developed external cracks, the specimens made with Type K cement exhibited fewer and narrower external cracks. The Type K specimens exhibited first cracking at a higher load than the Type I specimens, and in some tests no visible cracking was evident in Type K specimens.

The compressive stress induced in the concrete by the restrained expansion of the Type K cement was apparently responsible for increasing the loads both at first cracking and at which cracking was fully developed. Thus, efforts to compensate for concrete shrinkage also appear to help reduce cracking.

3.4-Crack spacing

As a result of the formation of cracks in a tension member, a new stress pattern develops between the cracks. The formation of additional primary cracks continues as the stress increases until the crack spacing is approximately twice the cover thickness as measured to the center of the reinforcing bar."

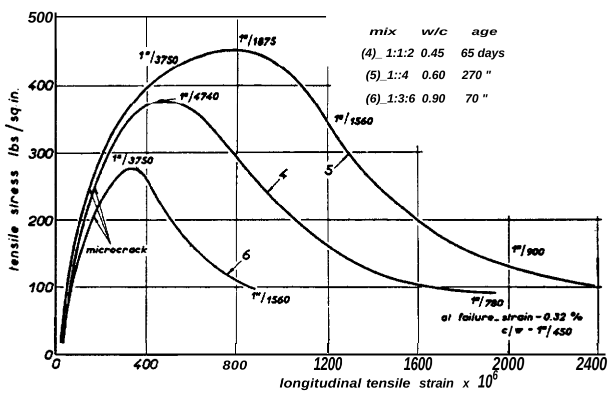


Fig. 3.1-Tensile stress-strain diagrams for concrete7 (includes unloading portion)

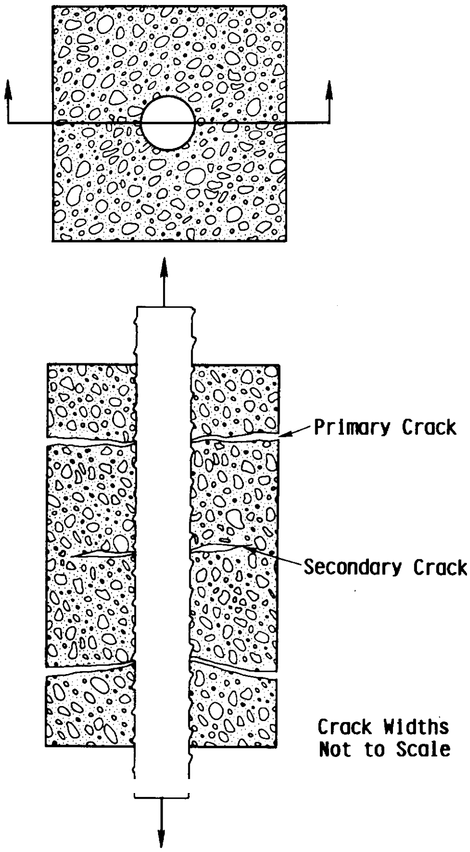


Fig. 3.2-Primary and secondary cracks in a reinforced concrete tension member

There is, of course, a considerable variation in the spacing of external cracks. The variability in the tensile strength of the concrete, the bond integrity of the bar, and the proximity of previous primary cracks, which tend to decrease the local tensile stress in the concrete, are the main cause of this variation in crack spacing. For the normal range of concrete covers, 1.25 to 3 in. (30 to 75 mm), the average crack spacing will not reach the limiting value of twice the cover until the reinforcement stress reaches 20 to 30 ksi (138 to 200 MPa).[11]

The expected value of the maximum crack spacing is about twice that of the average crack spacing.[11] That is, the maximum crack spacing is equal to about four times the concrete cover thickness. This range of crack spacing is more than 20 percent greater than observed for flexural members.

The number of visible cracks can be reduced at a given tensile force by simply increasing the concrete cover. With large cover, a larger percentage of the cracks will remain as internal cracks at a given level of tensile force. However, as will be discussed in Section 3.5, increased cover does result in wider visible cracks.

3.5-Crack width

The maximum crack width may be estimated by multiplying the maximum crack spacing (4 times concrete cover) at high steel stress by the average strain in the reinforcement. When tensile members with more than one reinforcing bar are considered," the actual concrete cover is not the most appropriate variable. Instead, an effective concrete cover t_e is used. t_e is defined as a function of the reinforcement spacing, as well as the concrete cover measured to the center of the reinforcement.[11] The greater the reinforcement spacing, the greater will be the crack width. This is reflected as an increased effective cover. Based on the work of Broms and Lutz,[11] the effective concrete cover is

$$t_e = d_c \sqrt{1 + \left(\frac{s}{4d_c}\right)^2} \tag{3.3}$$

in which d_c = distance from center of bar to extreme tension fiber, in., and s = bar spacing, in.

The variable t_e is similar to the variable $\sqrt[3]{d_c A}$ used in the Gergely-Lutz crack width expression for flexural members,[17] in which A = area of concrete symmetric with reinforcing steel divided by number of bars (in.^{2}). Using t_e, it is possible to express the maximum crack width in a form similar to the Gergely-Lutz expression.

Due to the larger variability in crack width in tension members, the maximum crack width in direct tension is expected to be larger than the maximum crack width in flexure at the same steel stress.

The larger crack width in tensile members may be due to the lack of crack restraint provided by the compression zone in flexural members. The stress gradient in a flexural member causes cracks to initiate at the most highly stressed location and to develop more gradually than in a tensile member that is uniformly stressed.

The expression for the maximum tensile crack width developed by Broms and Lutz" is

$$W_{max} = 4\,\epsilon_s t_e = 0.138 f_s t_e \times 10^{-3} \tag{3.4}$$

Using the definition of t_e given in Eq. (3.3), W_{max} may be expressed as

$$W_{max} = 0.138\, f_s d_c \sqrt{1 + \left(\frac{s}{4d_c}\right)^2} \times 10^{-3} \tag{3.5}$$

The parameter $\sqrt[3]{d_c A}$ for a single layer of reinforcement is $d_c \sqrt[3]{2s/d_c}$ (see Fig. 3.3), which is approximately equal to $1.35 d_c \sqrt{1 + (s/4d_c)^2}$ for S/d_c between 1 and 2. Thus, for tensile cracking

$$W_{max} = 0.10\, f_s \sqrt[3]{d_c A} \times 10^{-3} \tag{3.6}$$

Eq. (3.6) can be used to predict the probable maximum crack width in fully cracked tensile members. As with flexural members, there is a large variability in the maximum crack width. One should fully expect the maximum crack width to be 30 percent larger or smaller than

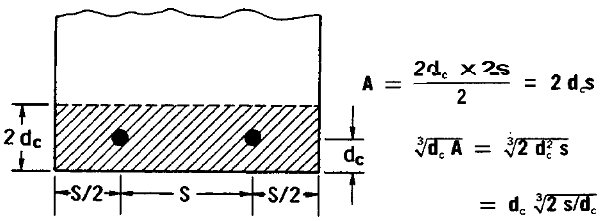


Fig. 3.3—$\sqrt[3]{d_c A}$ parameter in terms of bar spacing

the value obtained from Eq. (3.6).

The maximum flexural crack width expression[17]

$$W_{max} = 0.076\, \beta\, f_s \sqrt[3]{d_c A} \times 10^{-3} \tag{3.7}$$

in which β = ratio of distance between neutral axis and tension face to distance between neutral axis and centroid of reinforcing steel $\approx$ 1.20 in beams may be used tocompare the crack widths obtained in flexure and tension.

Using a value of β = 1.20, the coefficient 0.076β in Eq. (3.7) becomes 0.091 compared to a coefficient of 0.10 in Eq. (3.6) for tensile cracks. This indicates that for the same section and steel stress f_s the maximum tensile crack will be about 10 percent wider than the maximum flexural crack.

The flexural crack width expression in Eq. (3.7), with β = 1.2, is used in the following form in ACI 318

$$z = f_s \sqrt[3]{d_c A} \tag{3.8}$$

A maximum value of z = 175 kips/in. (30.6 MN/m) is permitted for interior exposure, corresponding to a limiting crack width of 0.016 in. (0.41 mm). ACI 318 limits the value of z to 145 kips/in. (25.4 MN/m) for exterior exposure, corresponding to a crack width of 0.013 in. (0.33 mm). To obtain similar crack widths for tensile members, the z-factors of 145 and 175 for flexural members should be multiplied by the ratio of coefficients in Eq. (3.7) and (3.6) (= 0.91). Using the same definition of z for both tensile and flexural members, this produces z-values of 132 and 160, respectively, for tensile members.

Rizkalla and Hwang[18] reported recently on tests in direct tension and presented an alternative procedure for computing crack widths and crack spacing based on expressions given by Beeby[19] and Leonhardt.[20]

CHAPTER 4-EFFECT OF CRACKING ON AXIAL STIFFNESS

4.1-Axial stiffness of one-dimensional members

When a symmetrical uncracked reinforced concrete member is loaded in tension. the tensile force is distributed between the reinforcing steel and concrete in proportion to their respective stiffnesses. Total load at strain ϵ is given by

$$P = P_c + P_s = (E_c A_c + n E_c A_s)\,\epsilon = E_c A_g\,(1 - p + n\,\rho)\,\epsilon = (EA)_{uc}\epsilon \tag{4.1}$$

in which E_c = modulus of elasticity of concrete. Loads carried by the concrete and reinforcing steel are, respectively

$$P_c = \left(\frac{1}{1 + n\rho}\right) P \tag{4.2}$$

and

$$P_s = \left(\frac{n\rho}{1 + n\rho}\right) P \tag{4.3}$$

Cracking occurs when the strain ϵ corresponds to the tensile strength of concrete. If the ascending branch of the tensile stress-strain curve is assumed to be linear $\epsilon = E_t = f_{t1}'/E_c$, in which f_{t1}' is the tensile stress causing the first crack. The total load at cracking P_{cr} is carried across the crack entirely by the reinforcement. If the applied force remains unchanged, steel stress after the crack occurs f_{scr} is given by

$$f_{scr} = \frac{P_{cr}}{A_s} = f_{t1}'\left(\frac{1}{\rho} - 1 + n\right) \tag{4.4}$$

The load carried across the crack by the reinforcement is gradually transferred by bond to the concrete on each side of the crack. As the applied load increases, additional cracks form at discrete intervals along the member as discussed in Chapter 3. The contribution of concrete between cracks to the net stiffness of a member is known as *tension stiffening.* The gradual reduction in stiffness

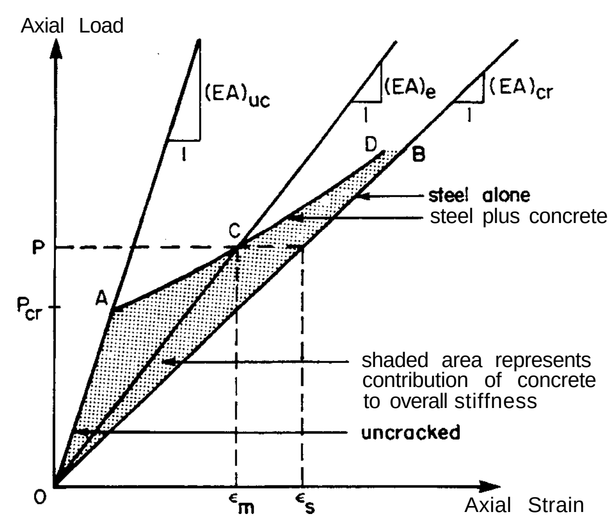


Fig. 4.1-Tensile load versus strain diagram

due to progressive cracking is referred to as *strain softening*.

The stiffening effect of the concrete between cracks can be illustrated by considering the relationship between the load and the average strain in both the uncracked and cracked states. A tensile load versus strain curve is shown in Fig. 4.1. In the range $P = 0$ to $P = P_{cr}$, the member is uncracked, and the response follows the line *OA*. The load-strain relationship [Eq. (4.1)] is given by

$$P = E_c A_g (1 - \rho + n\rho)\epsilon = (EA)_{uc}\epsilon \qquad (4.5)$$

If the contribution to stiffness provided by the concrete is ignored, the response follows the line OB, and the load-strain relationship is given by

$$P = E_s A_s \epsilon = n E_c \rho A_g \epsilon = (EA)_{cr}\epsilon \qquad (4.6)$$

For loads greater than P_{cr}, the actual response is intermediate between the uncracked and fully cracked limits, and response follows the line *AD*. At Point C on *AD*, where *P* is greater than P_{cr}, a relationship can be developed between the load *P* and average strain in the member ϵ_m

$$P = (EA)_e \epsilon_m \qquad (4.7)$$

The term $(EA)_e$ can be referred to as the effective axial cross-sectional stiffness of the member. This term can be written in terms of the actual area of steel A_s, and an effective modulus of elasticity E_{sm} of the steel bars

$$P = E_{sm} A_s \epsilon_m \qquad (4.8)$$

or

$$E_{sm} = \frac{f_s}{\epsilon_m} \qquad (4.9)$$

Several methods can be used to determine ϵ_m. For example, the CEB Model Code gives

$$\epsilon_m = \epsilon_s \left[1 - k\left(\frac{f_{scr}}{f_s}\right)^2\right] \qquad (4.10)$$

in which f_{scr} is given by Eq. (4.4), $f_s = P/A_s$, $\epsilon_s = f_s/E_s$, and k = 1.0 for first loading and 0.5 for repeated or sustained loading.

Combining Eq. (4.9) and (4.10)

$$E_{sm} = \frac{E_s}{\left[1 - k\left(\frac{f_{scr}}{f_s}\right)^2\right]} \qquad (4.11)$$

The CEB expression is based on tests of direct tension members conducted at the University of Stuttgart.[20] Other methods for determining ϵ_m are reviewed by Moosecker and Grasser.[21]

An alternative approach is to write the effective stiffness $(EA)_c$ in terms of the modulus of elasticity of the concrete and an effective (reduced) area of concrete, i.e.

$$P = E_c A_e \epsilon_m \qquad (4.12)$$

This approach is analogous to the effective moment of inertia concept for the evaluation of deflections developed by Branson[22] and incorporated in ACI 318.

Using the same form of the equation as used for the effective moment of inertia, the effective cross-sectional area for a member can be written as

$$A_e = A_g\left(\frac{P_{cr}}{P}\right)^3 + A_{cr}\left[1 - \left(\frac{P_{cr}}{P}\right)^3\right] \qquad (4.13)$$

where A_g = gross cross-sectional area and $A_{cr} = nA_s$.

The term A_g could be replaced by the transformed area A_e to include the contribution of reinforcing steel to the uncracked system [$A_t = A_c + nA_s = A_g + (n - 1) A_s$].

The load-strain relationships obtained using the CEB expression and the effective cross-sectional area [Eq. (4.13)] compare quite favorably, as shown in Fig. 4.2.

A third approach that has been used in finite element analysis of concrete structures involves a progressive reduction of the effective modulus of elasticity of concrete with increased cracking.

4.2-Finite element applications

Extensive research has been done in recent years on the application of finite elements to modeling the behavior of reinforced concrete and is summarized in a report of the ASCE Task Committee on Finite Element Analysis of Reinforced Concrete.[23] Two basic approaches, the discrete crack approach and the smeared crack approach, have been used to model cracking and tension stiffening.

In the discrete crack approach, originally used by Ngo and Scordelis,[24] individual cracks are modeled by using separate nodal points for concrete elements located at cracks, as shown for a flexural member in Fig. 4.3. This allows separation of elements at cracks. Effects of bond degradation on tension stiffening can be modeled by linear[24] or nonlinear [25] bond-slip linkage elements connecting concrete and steel elements.

The finite element method combined with nonlinear fracture mechanics was used by Gerstle, Ingraffea, and [26] to study the tension stiffening effect in tension members. The sequence of formation of primary and secondary cracks was studied using discrete crack modeling. A comparison of analyses with test results is shown in Fig. 4.4.

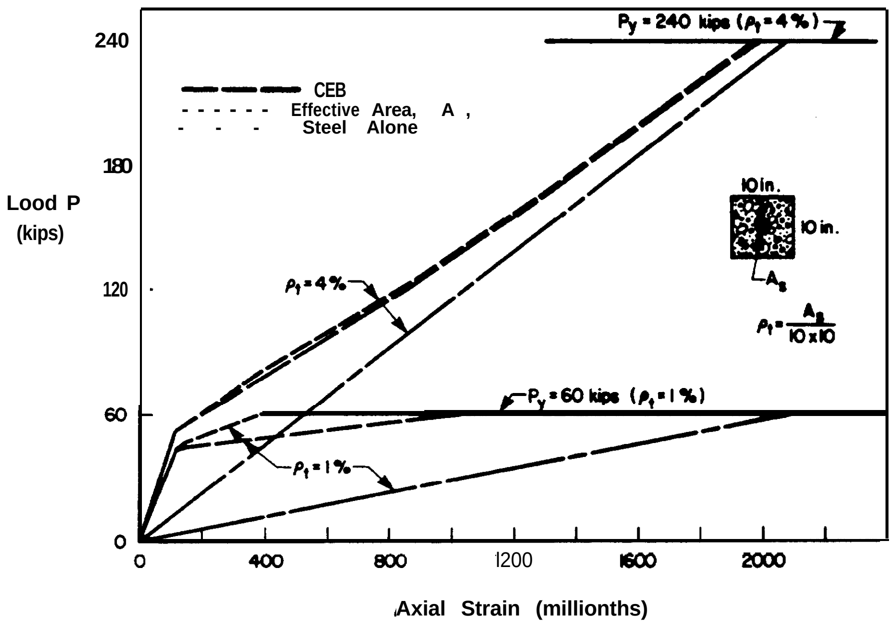


Fig. 4.2-Tensile load versus strain diagrams based on CEB and effective cross-sectional area expressions

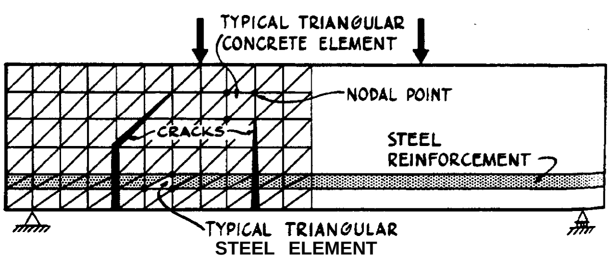


Fig. 4.3-Finite element modeling by the discrete crack approach[24]

In the smeared crack approach, tension stiffening is modeled either by retaining a decreasing concrete modulus of elasticity and leaving the steel modulus unchanged, or by first increasing and then gradually decreasing the steel modulus of elasticity and setting the concrete modulus to zero as cracking progresses. Scanlon and Murray’ introduced the concept of degrading concrete stiffness to model tension stiffening in two-way slabs. Variations of this approach have been used in finite element models by a number of researchers.[28-31]

Gilbert and Warner[31] used the smeared crack concept and a layered plate model to compare results using the degrading concrete stiffness approach and the increased steel stiffness approach. Various models compared by Gilbert and Warner are shown in Fig. 4.5. Satisfactory results were obtained using all of the models considered. However, the approach using a modified steel stiffness was found to be numerically the most efficient. More recent works[32] has shown that the energy consumed in fracture must be correctly modeled to obtain objective finite element results in general cases. While most of these models have been applied to flexural members, the same general approach can be used for members in direct tension.

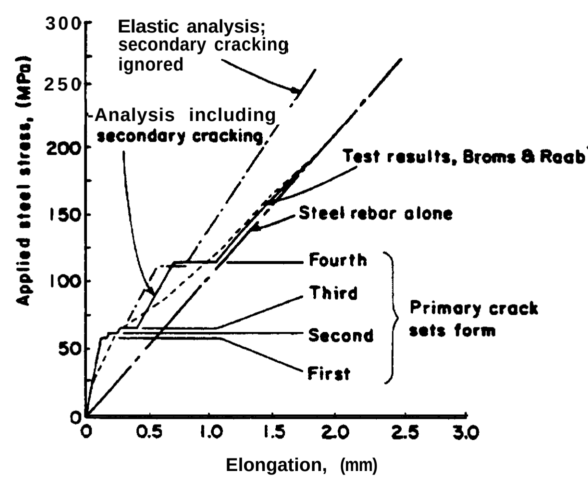
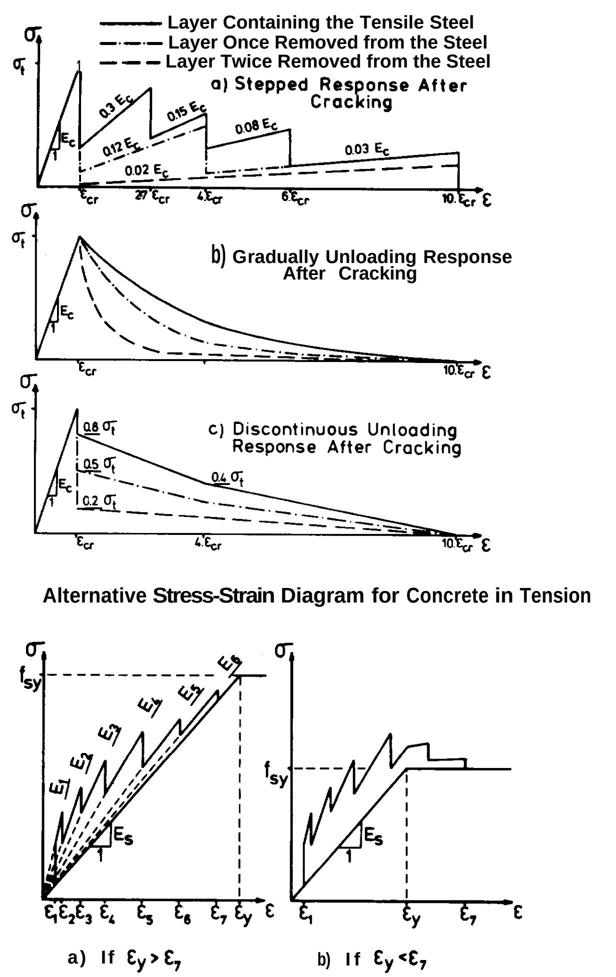


Fig. 4.4-Steel stress versus elongation curves for tension specimen based on nonlinear fracture mechanics approach[26]

4.3-Summary

Several methods have been proposed in the literature to estimate the axial stiffness of cracked reinforced concrete members. The CEB Model Code approach involves the modification of the effective steel modulus of elasticity and appears to be well-established in Europe. An alternative approach, suggested in this report, involves an expression for the effective cross-sectional area that is analogous to the well-known effective moment of inertia concept. Both of these approaches appear to be acceptable for the analysis of one-dimensional members.



Material Modelling Law:

ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
ε_{cr}	$1.5\varepsilon_{cr}$	$3.\varepsilon_{cr}$	$5.\varepsilon_{cr}$	$8.\varepsilon_{cr}$	$11.\varepsilon_{cr}$	$14.\varepsilon_{cr}$

E_1	E_2	E_3	E_4	E_5	E_6
$4.0E_s$	$2.7E_s$	$2.0E_s$	$1.6E_s$	$1.15E_s$	$1.05E_s$

Fig. 4.5-Tension stiffening models proposed for smeared crack finite element approach[31] *(σ_t = tensile strength of concrete, f_{sy} = yield strength of reinforcment, σ = stress, and ϵ = strain)*

For more complex systems, finite element analysis procedures have been used successfully to model the behavior of cracked reinforced concrete, using a variety of stiffness models.

CHAPTER 5-CONTROL OF CRACKING CAUSED BY DIRECT TENSION

5.1-Introduction

The three previous chapters emphasized predicting behavior when cracking due to direct tension occurs in reinforced concrete members. However, a major objective of design and construction of concrete structures should be to minimize and/or control cracking and its adverse effects. This chapter is intended to provide guidance to assist in achieving that objective.

The recommendations contained in ACI 224R apply where applicable. This chapter deals more specifically with members loaded in direct tension.

5.2-Control of cracking caused by applied loads

The main concern of crack control is to minimize maximum crack widths. In the past, tolerable crack widths have been related to exposure conditions (ACI 224R). However, at least in terms of protecting reinforcement from corrosion, the effect of surface crack width appears to be relatively less important than believed previously (ACI 224.1R). For severe exposures, it is preferable to provide a greater thickness of concrete cover even though this will lead to wider surface cracks. Tolerable crack widths may also be related to aesthetic or functional requirements. Based on experience using the *z*-factor for flexural cracking (ACI 318), a crack width of 0.016 in. (0.4 mm) may be acceptable for appearance in most cases. Functional requirements such as liquid storage (ACI 350R) may require narrower crack widths such as 0.008 in. (0.2 mm) for liquid-retaining structures.

Eq. (3.6) and (3.8) for members in direct tension may be used to select and arrange reinforcement to limit crack widths.

Since crack width is related to tensile stress in reinforcement, cracks attributed to live loads applied for short periods may not be as serious as cracks due to sustained load, since the cracks due to live load may be expected to close or at least decrease in width upon removal of the load. If acceptable crack control cannot be achieved by the use of mild reinforcement alone, prestressing can be used to reduce tensile stresses in a structure. Shrinkage-compensating concrete placed in accordance with ACI 223 can also be effective.

While measures can be taken at the design stage to control cracking, it is equally important to apply proper construction procedures to insure the intended performance of the structure. This requires avoiding overloading the structure during construction. Careful placement of reinforcement is also essential, including provision for properly designed lap splices (ACI 318).

5.3-Control of cracking caused by restraint of volume change

Cracking due to the restraint of volume change during the early life of a structure can be minimized by protecting new concrete for as long as practical from drying or temperature drop of the surface of the member that would cause tensile stress in the member greater than its early age strength.

Drying shrinkage at early ages can be controlled by using proper curing procedures. Shrinkage-compensating concrete can be very effective in limiting cracks due to drying shrinkage (ACI 223).

Temperature changes related to the heat of hydration can be minimized by placing concrete at lower than norma1 temperatures (precooling). For example, placing concrete at approximately 50 F (10 C) has significantly reduced cracking in concrete tunnel linings.[33] It should be noted that concrete placed at 50 F (10 C) tends to develop higher strength at later ages than concrete placed at higher temperatures.

Circumferential cracks in tunnel linings (as well as cast-in-place conduits and pipelines) can be greatly reduced in number and width if the tunnel is kept bulkheaded against air movement and shallow ponds of water are kept in the invert from the time concrete is placed until the tunnel goes into service (see Fig. 35 of Reference 34).

To minimize crack widths caused by restraint stresses, bonded "temperature" reinforcement should be provided. As a general rule, reinforcement controls the width and spacing of cracks most effectively when bar diameters are as small as possible, with correspondingly closer spacing for a given total area of steel. Fiber reinforced concrete may also have application in minimizing the width of cracks induced by restraint stresses (ACI 544.1R).

If tensile forces in a restrained concrete member will result in unacceptably wide cracks, the degree of restraint can be reduced by using joints where feasible or leaving empty pour strips that are subsequently filled with concrete after the adjacent members have gained strength and been allowed to dry. Flatwork will be restrained by the anchorage of the slab reinforcement to perimeter slabs or footings. When each slab is free to shrink from all sides towards its center, cracking is minimized. For slabs on ground, contraction joints and perimeter supports should be designed accordingly (ACI 302.1R-80). Frequent contraction joints or deep grooves must be provided if it is desired to prevent or hide restraint cracking in walls, slabs, and tunnel linings [ACI 224R-80 (Revised 1984), ACI 302.1R-80].

NOTATION

A = area of concrete symmetric with reinforcing steel divided by number of bars, in.'
A_e = effective cross-sectional area of concrete, in.'
A_g = gross area of section, in.^{2}
A_s = area of nonprestressed tension reinforcement, in.^{2}
d_c = distance from center of bar to extreme tension fiber, in.
E_c = modulus of elasticity of concrete, ksi
E_s = modulus of elasticity of reinforcement, ksi
E_{sm} = effective modulus of elasticity of steel to that of concrete
f_r = modulus of rupture of concrete, psi
f_s = stress in reinforcement, ksi
f_{scr} = steel stress after crack occurs, ksi
f_c' = compressive strength of concrete, psi
f_t' = tensile strength of concrete, psi
n = the ratio of modulus of elasticity of steel to that of concrete
P = axial load
P_c = axial load carried by concrete
P_{cr} = axial load at which cracking occurs
P_s = axial load carried by reinforcement
s = bar spacing, in.
t_e = effective concrete cover, in.
w_c = unit weight of concrete, lb/ft^3
w_{max} = most probable maximum crack width, in.
z = factor limiting distribution of reinforcement
β = ratio of distance between neutral axis and tension face to distance between neutral axis and centroid of reinforcing steel ≈ 1.20 in. beams
ϵ_m = average strain in member (unit elongation)
ϵ_s = tensile strain in reinforcing bar assuming no tension in concrete
ρ = reinforcing ratio = A_s/A_g

CONVERSION FACTORS-SI EQUIVALENTS

1 in. = 25.4 mm
1 lb (mass) = 0.4536 kg
1 lb (force) = 4.488 N
1 lb/in.^{2} = 6.895 kPa
1 kip = 444.8 N
1 kip/in.^{2} = 6.895 MPa

Eq. (3.5)

$$W_{max} = 0.02\, f_s d_c \sqrt{1 + \left(\frac{s}{4d_c}\right)^2} \times 10^{-3}$$

Eq. (3.6)

$$W_{max} = 0.0145 f_s \sqrt[3]{d_c A} \times 10^{-3}$$

Eq. (3.7)

$$W_{max} = 0.011 \beta f_s \sqrt[3]{d_c A} \times 10^{-3}$$

W_{max} in mm, f_s in MPa, d_c in mm, A in mm^2, and s in mm.

CHAPTER 6-REFERENCES

6.1-Recommended references

The documents of the various standards-producing organizations referred to in this report are listed below with their serial designation.

American Concrete Institute

209R Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures
223 Standard Practice for the Use of Shrinkage-

	Compensating Concrete
224R	Control of Cracking in Concrete Structures
224.1R	Causes, Evaluation, and Repair of Cracks in Concrete Structures
302.1R	Guide for Concrete Floor and Slab Construction
318	Building Code Requirements for Reinforced Concrete
350R	Environmental Engineering Concrete Structures
544.1R	State-of-the-Art Report on Fiber Reinforced Concrete

ASTM

C 78	Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)
C 293	Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Center-Point Loading)
C 496	Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens

Comité Euro-International du Béton/Fédération Internationale de Ia Précontrainte

CEB-FIP Model Code for Concrete Structures

These publications may be obtained from:

American Concrete Institute
PO Box 19150
Detroit, MI 48219

ASTM
1916 Race St.
Philadelphia, PA 19103

Comité Euro-International du Béton
EPFL Case Postale 88
CH 1015 Lausanne, Switzerland

6.2-Cited references

1. Scanlon, Andrew, and Murray, David W., "Practical Calculation of Two-Way Slab Deflections," *Concrete International: Design & Construction,* V. 4, No. 11, Nov. 1982, pp. 43-50.

2. Cole, Peter P.; Abel, John F.; and Billington, David P., "Buckling of Cooling-Tower Shells: State-of-the-Art," *Proceedings,* ASCE, V. 101, ST6, June 1975, pp. 1185-1203.

3. Tam, K.S.S., and Scanlon, A., "The Effects of Restrained Shrinkage on Concrete Slabs," *Structural Engineering Report No. 122*, Department of Civil Engineering, University of Alberta, Edmonton, Dec. 1984, 126 pp.

4. Neville, Adam M., *Hardened Concrete: Physical and Mechanical Aspects,* ACI Monograph No. 6, American Concrete Institute/Iowa State University Press, Detroit, 1971, 260 pp.

5. Wright, P.J.F., "Comments on Indirect Tensile Test on Concrete Cylinders," *Magazine of Concrete Research* (London), V. 7, No. 20, July 1955, pp. 87-96.

6. Price, Walter H., "Factors Influencing Concrete Strength," ACI JOURNAL, *Proceedings* V. 47, No. 6, Feb. 1951, pp. 417-432.

7. Evans, R.H., and Marathe, M.S., "Microcracking and Stress-Strain Curves for Concrete in Tension," *Materials and Structures, Research and Testing* (RILEM, Paris), V. 1, No. 1, Jan.-Feb. 1968, pp. 61-64.

8. Petersson, Per-Erik, "Crack Growth and Development of Fracture Zones in Plain Concrete and Similar Materials," *Report* No. TVBM-1006, Division of Building Materials, Lund Institute of Technology, 1981, 174 pp.

9. Broms, Bengt B., "Crack Width and Crack Spacing in Reinforced Concrete Members," ACI JOURNAL, *Proceedings* V. 62, No. 10, Oct. 1965, pp. 1237-1256.

10. Broms, Bengt B., "Stress Distribution in Reinforced Concrete Members With Tension Cracks," ACI JOURNAL, *Proceedings* V. 62, No. 9, Sept. 1965, pp. 1095-1108.

11. Broms, Bengt B., and Lutz, Leroy A., "Effects of Arrangement of Reinforcement on Crack Width and Spacing of Reinforced Concrete Members," ACI JOURNAL, *Proceedings* V. 62, No. 11, Nov. 1965, pp. 1395-1410.

12. Goto, Yukimasa, "Cracks Formed in Concrete Around Deformed Tension Bars," ACI JOURNAL, *Proceedings* V. 68, No. 4, Apr. 1971, pp. 244-251.

13. Goto, Y., and Otsuka, K., "Experimental Studies on Cracks Formed in Concrete Around Deformed Tension Bars," *Technology Reports of the Tohoku University,* V. 44, No. 1, June 1979, pp. 49-83.

14. Clark, L.A., and Spiers, D.M., "Tension Stiffening in Reinforced Concrete Beams and Slabs Under Short-Term Load," *Technical Report* No. 42.521, Cement and Concrete Association, Wexham Springs, 1978, 19 pp.

15. Somayaji, S., and Shah, S.P., "Bond Stress Versus Slip Relationship and Cracking Response of Tension Members," ACI JOURNAL, *Proceedings* V. 78, No. 3, May-June 1981, pp. 217-225.

16. Cusick, R.W., and Kesler, C.E., "Interim Report-Phase 3: Behavior of Shrinkage-Compensating Concretes Suitable for Use in Bridge Decks," T. & A.M. Report No. 409, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, July 1976, 41 pp.

17. Gergely, Peter, and Lutz, Leroy A., "Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete,* SP-20, American Concrete Institute, Detroit, 1968, pp.

87-117.

18. Rizkalla, S.H., and Hwang, L.S., "Crack Prediction for Members in Uniaxial Tension," ACI JOURNAL, *Proceedings* V. 81, No. 6, Nov.-Dec. 1984, pp. 572-579.

19. Beeby, A.W., "The Prediction of Crack Widths in Hardened Concrete," *Structural Engineer* (London), V. 57A, Jan. 1979, pp. 9-17.

20. Leonhardt, Fritz, "Crack Control in Concrete Structures," *IABSE Surveys* No. S-4/77, International Association for Bridge and Structural Engineering, Zurich, Aug. 1977, 26 pp.

21. Moosecker, W., and Grasser, E., "Evaluation of Tension Stiffening in Reinforced Concrete Linear Members," *Final Report*, IABSE Colloquium on Advanced Mechanics of Reinforced Concrete (Delft, 1981), International Association for Bridge and Structural Engineering, Zurich.

22. Branson, D.E., "Instantaneous and Time-Dependent Deflections of Simple and Continuous Reinforced Concrete Beams," *Research Report* No. 7, Alabama Highway Department, Montgomery, Aug. 1963, 94 pp.

23. *Finite Element Analysis of Reinforced Concrete*, American Society of Civil Engineers, New York, 1982, 545 pp.

24. Ngo, D., and Scordelis, A.C., "Finite Element Analysis of Reinforced Concrete Beams," ACI JOURNAL., *Proceedings* V. 64, No. 3, Mar. 1967, pp. 152-163.

25. Nilson, Arthur H., "Nonlinear Analysis of Reinforced Concrete by the Finite Element Method," ACI JOURNAL, *Proceedings* V. 65, No. 9, Sept. 1968, pp. 757-766.

26. Gerstle, Walter; Ingraffea, Anthony R.; and Gergely, Peter, ''Tension Stiffening: A Fracture Mechanics Approach," *Proceedings*, International Conference on Bond in Concrete (Paisely, June 1982), Applied Science Publishers, London, 1982, pp. 97-106.

27. Scanlon, A., and Murray, D.W., "An Analysis to Determine the Effects of Cracking in Reinforced Concrete Slabs," *Proceedings*, Specialty Conference on the Finite Element Method in Civil Engineering, EIC/McGill University, Montreal, 1972, pp. 841-867.

28. Lin, Cheng-Shung, and Scordelis, Alexander C., "Nonlinear Analysis of RC Shells of General Form," *Proceedings*, ASCE, V. 101, ST3, Mar. 1975, pp. 523-538.

29. Chitnuyanondh, L.; Rizkalla, S.; Murray, D.W.; and MacGregor, J.G., "An Effective Uniaxial Tensile Stress-Strain Relationship for Prestressed Concrete," *Structural Engineering Report* No. 74, University of Alberta, Edmonton, Feb. 1979, 91 pp.

30. Argyris, J.H.; Faust, G.; Szimmat, J.; Warnke, P., and William, K.J., "Recent Developments in the Finite Element Analysis of Prestressed Concrete Reactor Vessels," *Preprints*, 2nd International Conference on Structural Mechanics in Reactor Technology (Berlin, Sept. 1973), Commission of the European Communities, Luxembourg, V. 3, Paper H l/l, 20 pp. Also, *Nuclear Engineering and Design* (Amsterdam), V. 28, 1974.

31. Gilbert, R. Ian, and Warner, Robert F., ''Tension Stiffening in Reinforced Concrete Slabs," *Proceedings*, ASCE, V. 104, ST12, Dec. 1978, pp. 1885-1900.

32. Bazant, Zdenek, and Cedolin, Luigi, "Blunt Crack Band Propagation in Finite Element Analysis," *Proceedings*, ASCE, V. 105, EM2, Apr. 1979, pp. 297-315.

33. Tuthill, Lewis H., "Tunnel Lining With Pumped Concrete," ACI JOURNAL., *Proceedings* V. 68, No. 4, Apr. 1971, pp. 252-262.

34. *Concrete Manual*, 8th Edition, U.S. Bureau of Reclamation, Denver, 1975, 627 pp.