

Chapter 5

Association Analysis: Basic Concepts

Introduction to Data Mining, 2nd Edition

by

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Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
- Dimensionality (number of items) of the data set
- Size of database
- Average transaction width

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 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
 -

- Size of database
 -

- Average transaction width
 -

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Impact of Support Based Pruning

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk



Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 \\ 6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Minimum Support = 2

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 \\ 6 + 15 + 20 + 15 = 56$$

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- Dimensionality (number of items) of the data set
 - More space is needed to store support count of itemsets
 - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
- Average transaction width

<i>TID</i>	<i>Items</i>
1	Bread, Milk
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 - run time of algorithm increases with number of transactions
- Average transaction width
 - transaction width increases the max length of frequent itemsets
 - number of subsets in a transaction increases with its width, increasing computation time for support counting

Compact Representation of Frequent Itemsets

- Some frequent itemsets are redundant because their supersets are also frequent

Consider the following data set. Assume support threshold =5

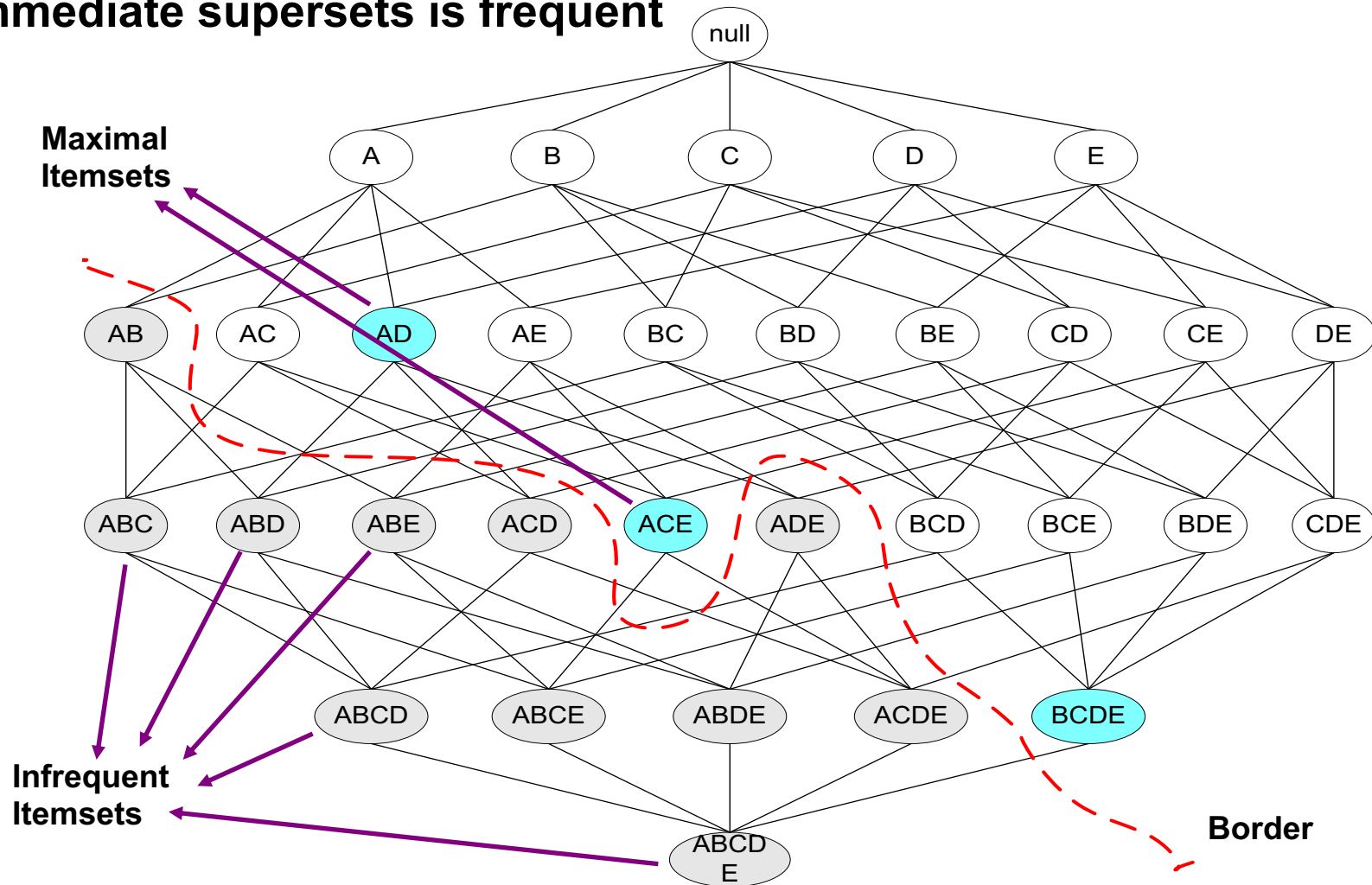
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

$$\text{Number of frequent itemsets} = 3 \times \sum_{k=1}^{10} \binom{10}{k}$$

- Need a compact representation

Maximal Frequent Itemset

An itemset is maximal frequent if it is frequent and none of its immediate supersets is frequent



What are the Maximal Frequent Itemsets in this Data?

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

Minimum support threshold = 5

(A1-A10)

(B1-B10)

(C1-C10)

An illustrative example

		Items									
		A	B	C	D	E	F	G	H	I	J
Transactions	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

Support threshold (by count) : 5

Frequent itemsets: ?

Maximal itemsets: ?

An illustrative example

Transactions	Items									
	A	B	C	D	E	F	G	H	I	J
	1									
	2									
	3									
	4									
	5									
	6									
	7									
	8									
	9									
	10									

Support threshold (by count) : 5

Frequent itemsets: {F}

Maximal itemsets: {F}

Support threshold (by count): 4

Frequent itemsets: ?

Maximal itemsets: ?

An illustrative example

		Items									
Transactions		A	B	C	D	E	F	G	H	I	J
	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

Support threshold (by count) : 5

Frequent itemsets: {F}

Maximal itemsets: {F}

Support threshold (by count): 4

Frequent itemsets: {E}, {F}, {E,F}, {J}

Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3

Frequent itemsets: ?

Maximal itemsets: ?

An illustrative example

		Items									
Transactions		A	B	C	D	E	F	G	H	I	J
	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

Support threshold (by count) : 5

Frequent itemsets: {F}

Maximal itemsets: {F}

Support threshold (by count): 4

Frequent itemsets: {E}, {F}, {E,F}, {J}

Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3

Frequent itemsets:

All subsets of {C,D,E,F} + {J}

Maximal itemsets:

{C,D,E,F}, {J}

Another illustrative example

Transactions	Items									
	A	B	C	D	E	F	G	H	I	J
	1									
	2									
	3									
	4									
	5									
	6									
	7									
	8									
	9									
	10									

Support threshold (by count) : 5

Maximal itemsets: {A}, {B}, {C}

Support threshold (by count): 4

Maximal itemsets: {A,B}, {A,C},{B,C}

Support threshold (by count): 3

Maximal itemsets: {A,B,C}

Closed Itemset

- An itemset X is closed if none of its immediate supersets has the same support as the itemset X .
- X is not closed if at least one of its immediate supersets has support count as X .

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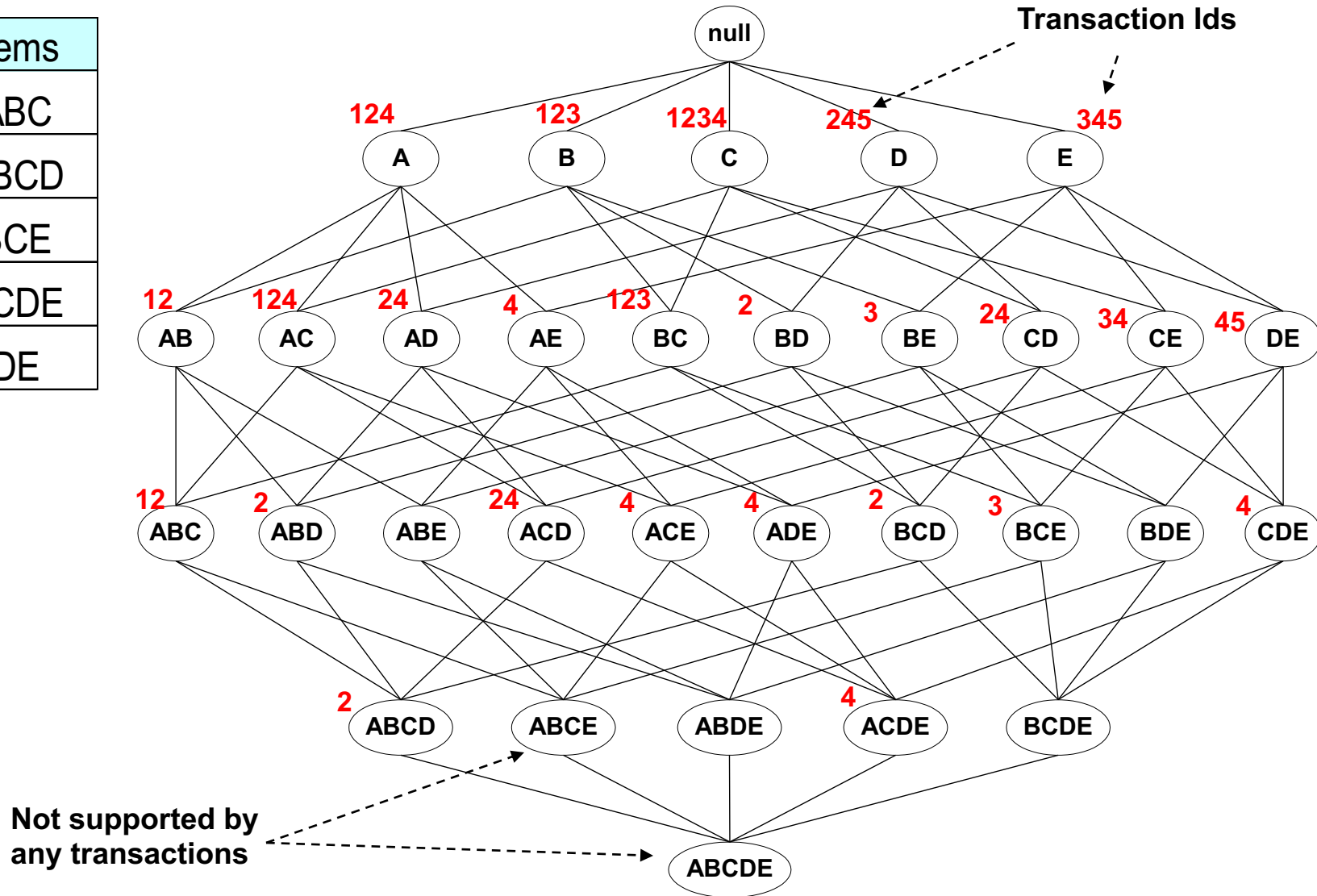
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	2
{A,B,C,D}	2

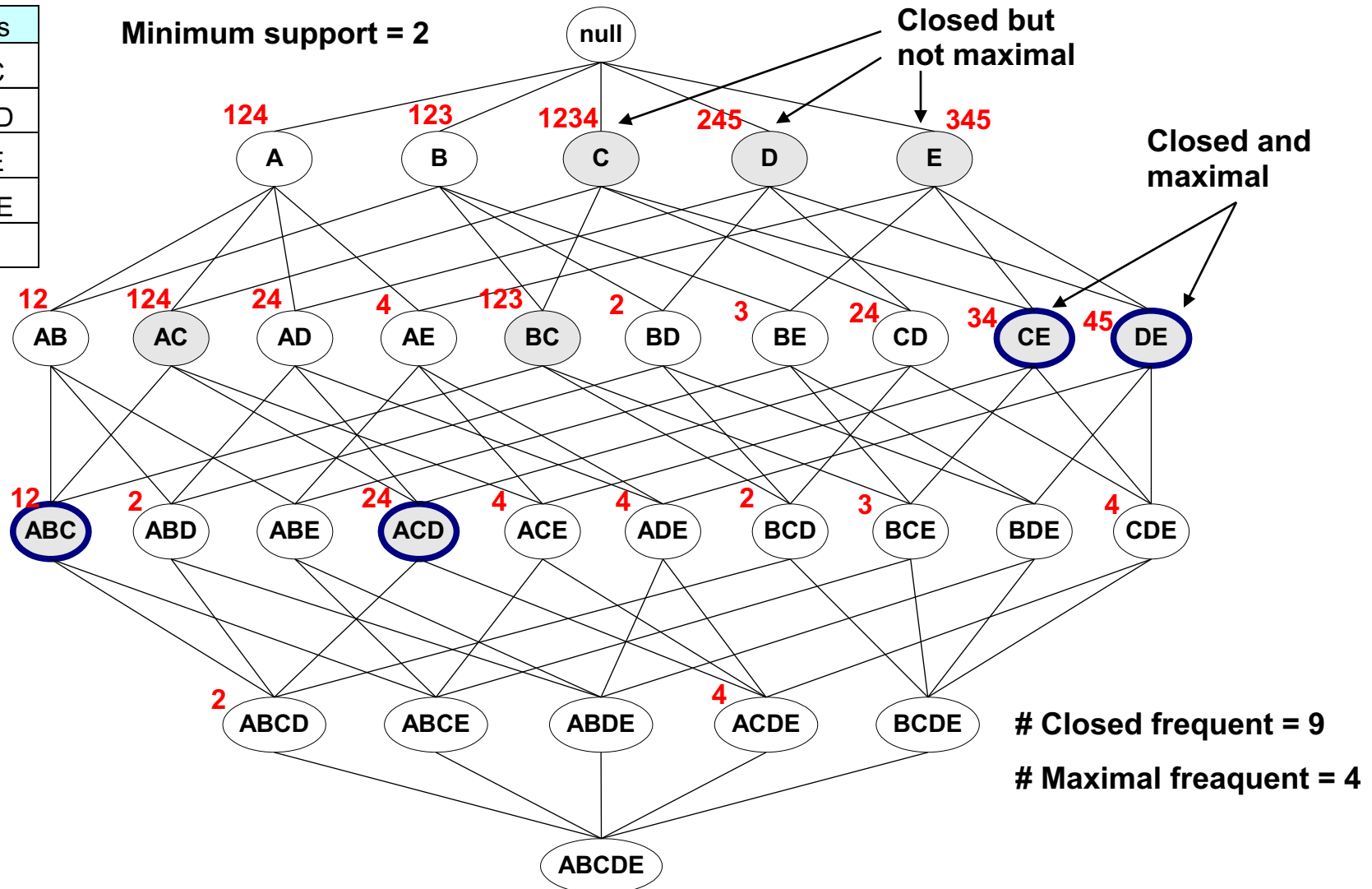
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal Frequent vs Closed Frequent Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



What are the Closed Itemsets in this Data?

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

(A1-A10)

(B1-B10)

(C1-C10)

Example 1

		Items									
Transactions		A	B	C	D	E	F	G	H	I	J
	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{C,D}	2	

Example 1

	Items									
	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Itemsets	Support (counts)	Closed itemsets
{C}	3	✓
{D}	2	
{C,D}	2	✓

Example 2

		Items									
Transactions		A	B	C	D	E	F	G	H	I	J
	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	2	

Example 2

Transactions	Items									
	A	B	C	D	E	F	G	H	I	J
	1									
	2									
	3									
	4									
	5									
	6									
	7									
	8									
	9									
	10									

Itemsets	Support (counts)	Closed itemsets
{C}	3	✓
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	2	✓

Example 3

Items

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Closed itemsets: {C,D,E,F}, {C,F}

Example 4

Items

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Closed itemsets: {C,D,E,F}, {C}, {F}

Maximal vs Closed Itemsets

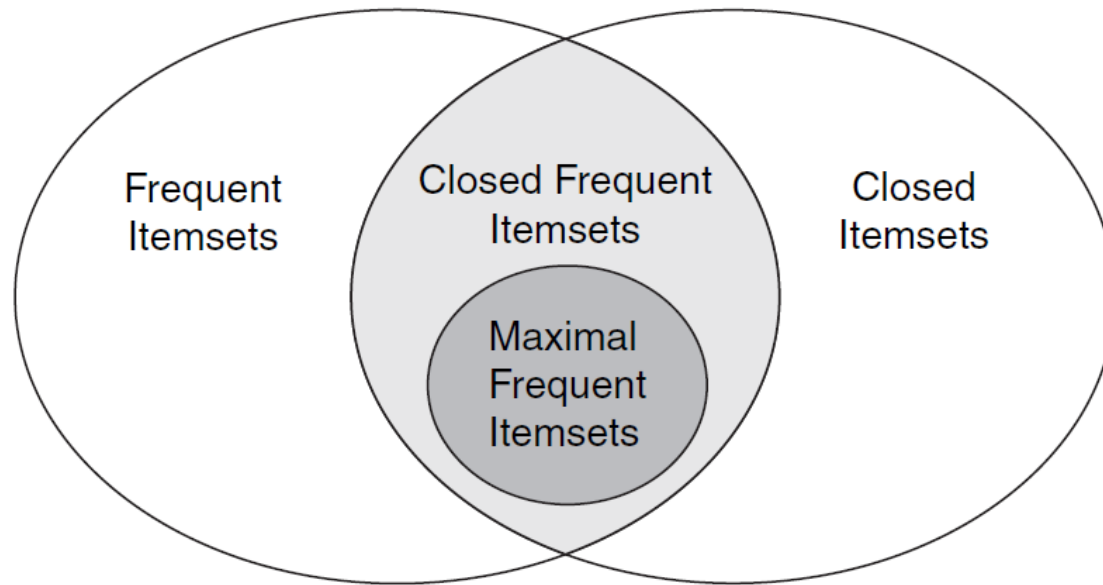
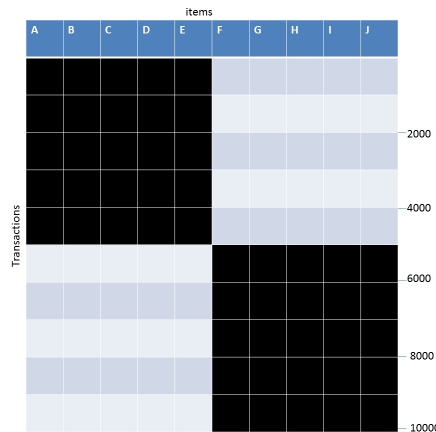


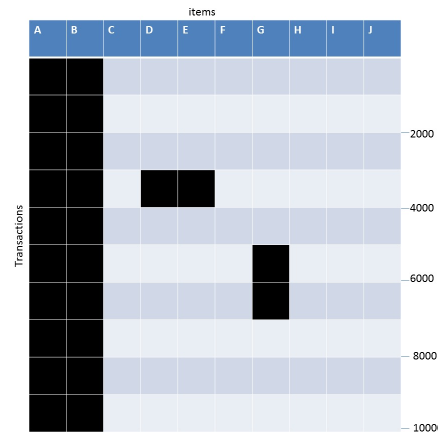
Figure 5.18. Relationships among frequent, closed, closed frequent, and maximal frequent itemsets.

Example question

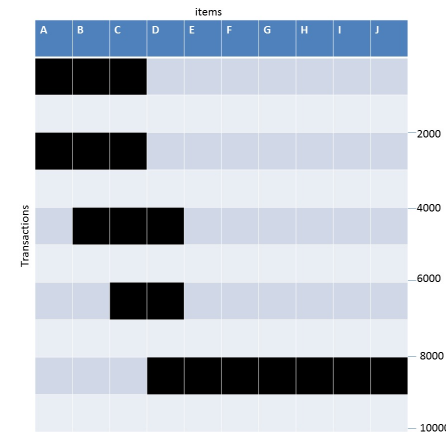
- Given the following transaction data sets (dark cells indicate presence of an item in a transaction) and a support threshold of 20%, answer the following questions



DataSet: A



Data Set: B



Data Set: C

- What is the number of frequent itemsets for each dataset? Which dataset will produce the most number of frequent itemsets?
- Which dataset will produce the longest frequent itemset?
- Which dataset will produce frequent itemsets with highest maximum support?
- Which dataset will produce frequent itemsets containing items with widely varying support levels (i.e., itemsets containing items with mixed support, ranging from 20% to more than 70%)?
- What is the number of maximal frequent itemsets for each dataset? Which dataset will produce the most number of maximal frequent itemsets?
- What is the number of closed frequent itemsets for each dataset? Which dataset will produce the most number of closed frequent itemsets?

Pattern Evaluation

- Association rule algorithms can produce large number of rules
- Interestingness measures can be used to prune/rank the patterns
 - In the original formulation, support & confidence are the only measures used

Computing Interestingness Measure

- Given $X \rightarrow Y$ or $\{X, Y\}$, information needed to compute interestingness can be obtained from a contingency table

Contingency table

	Y	\overline{Y}	
X	f_{11}	f_{10}	f_{1+}
\overline{X}	f_{01}	f_{00}	f_{0+}
	f_{+1}	f_{+0}	N

f_{11} : support of X and Y

f_{10} : support of X and \overline{Y}

f_{01} : support of \overline{X} and Y

f_{00} : support of \overline{X} and \overline{Y}

Used to define various measures

- ◆ support, confidence, Gini, entropy, etc.

Drawback of Confidence

Custo mers	Tea	Coffee	...
C1	0	1	...
C2	1	0	...
C3	1	1	...
C4	1	0	...
...			

	<i>Coffee</i>	\overline{Coffee}	
<i>Tea</i>	150	50	200
\overline{Tea}	650	150	800
	800	200	1000

Association Rule: Tea \rightarrow Coffee

Confidence $\cong P(\text{Coffee}|\text{Tea}) = 150/200 = 0.75$

Confidence > 50%, meaning people who drink tea are more likely to drink coffee than not drink coffee

So rule seems reasonable

Drawback of Confidence

	Coffee	<u>Coffee</u>	
Tea	150	50	200
<u>Tea</u>	650	150	800
	800	200	1000

Association Rule: Tea \rightarrow Coffee

$$\text{Confidence} = P(\text{Coffee}|\text{Tea}) = 150/200 = 0.75$$

but $P(\text{Coffee}) = 0.8$, which means knowing that a person drinks tea reduces the probability that the person drinks coffee!

$$\Rightarrow \text{Note that } P(\text{Coffee}|\overline{\text{Tea}}) = 650/800 = 0.8125$$

Drawback of Confidence

Custo mers	Tea	Honey	...
C1	0	1	...
C2	1	0	...
C3	1	1	...
C4	1	0	...
...			

	<i>Honey</i>	\overline{Honey}	
<i>Tea</i>	100	100	200
\overline{Tea}	20	780	800
	120	880	1000

Association Rule: Tea \rightarrow Honey

Confidence $\cong P(\text{Honey}|\text{Tea}) = 100/200 = 0.50$

Confidence = 50%, which may mean that drinking tea has little influence whether honey is used or not

So rule seems uninteresting

But $P(\text{Honey}) = 120/1000 = .12$ (hence tea drinkers are far more likely to have honey)

Measure for Association Rules

- So, what kind of rules do we really want?
 - Confidence($X \rightarrow Y$) should be sufficiently high
 - ◆ To ensure that people who buy X will more likely buy Y than not buy Y
 - Confidence($X \rightarrow Y$) > support(Y)
 - ◆ Otherwise, rule will be misleading because having item X actually reduces the chance of having item Y in the same transaction
 - ◆ Is there any measure that capture this constraint?
 - Answer: Yes. There are many of them.

Statistical Relationship between X and Y

- The criterion
 $\text{confidence}(X \rightarrow Y) = \text{support}(Y)$

is equivalent to:

- $P(Y|X) = P(Y)$
- $P(X,Y) = P(X) \times P(Y)$ (X and Y are independent)

If $P(X,Y) > P(X) \times P(Y)$: X & Y are positively correlated

If $P(X,Y) < P(X) \times P(Y)$: X & Y are negatively correlated

Measures that take into account statistical dependence

$$\textit{Lift} = \frac{P(Y | X)}{P(Y)}$$

$$\textit{Interest} = \frac{P(X, Y)}{P(X)P(Y)}$$

lift is used for rules while
interest is used for itemsets

$$PS = P(X, Y) - P(X)P(Y)$$

$$\phi - \textit{coefficient} = \frac{P(X, Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

Example: Lift/Interest

	Coffee	<u>Coffee</u>	
Tea	150	50	200
<u>Tea</u>	650	150	800
	800	200	1000

Association Rule: Tea \rightarrow Coffee

Confidence = $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.8$

\Rightarrow Interest = $0.15 / (0.2 \times 0.8) = 0.9375$ (< 1 , therefore is negatively associated)

So, is it enough to use confidence/Interest for pruning?

There are lots of measures proposed in the literature

Measure (Symbol)	Definition
Correlation (ϕ)	$\frac{N f_{11} - f_{1+} f_{+1}}{\sqrt{f_{1+} f_{+1} f_{0+} f_{+0}}}$
Odds ratio (α)	$(f_{11} f_{00}) / (f_{10} f_{01})$
Kappa (κ)	$\frac{N f_{11} + N f_{00} - f_{1+} f_{+1} - f_{0+} f_{+0}}{N^2 - f_{1+} f_{+1} - f_{0+} f_{+0}}$
Interest (I)	$(N f_{11}) / (f_{1+} f_{+1})$
Cosine (IS)	$(f_{11}) / (\sqrt{f_{1+} f_{+1}})$
Piatetsky-Shapiro (PS)	$\frac{f_{11}}{N} - \frac{f_{1+} f_{+1}}{N^2}$
Collective strength (S)	$\frac{f_{11} + f_{00}}{f_{1+} f_{+1} + f_{0+} f_{+0}} \times \frac{N - f_{1+} f_{+1} - f_{0+} f_{+0}}{N - f_{11} - f_{00}}$
Jaccard (ζ)	$f_{11} / (f_{1+} + f_{+1} - f_{11})$
All-confidence (h)	$\min \left[\frac{f_{11}}{f_{1+}}, \frac{f_{11}}{f_{+1}} \right]$