Class 22: On Computable Numbers

Schedule

Problem Set 9 is now due on **Wednesday, 23 November**.

Problem Set ω is now posted and due on **Sunday, 4 December** or **Tuesday, 6 December** (see problem set for details). It is not like the others, and counts as a "bonus" optional assignment.

Cantor's Continuum "Hypothesis"

Aleph-naught: $\aleph_0 = |\mathbb{N}|$ is the *smallest infinite cardinal number.*

Recall: ω is the *smallest infinite ordinal.* The first ordinal after $0, 1, 2, \cdots$.

$$2^{\aleph_0} = |pow(\mathbb{N})| = |[0,1]| = |\mathbb{R}| = |\{0,1\}^\omega| > |\mathbb{N}$$

What does it mean to say it is proven to not be possible to settle the question of whether $\aleph_1 = 2^{\aleph_0}$ with the ZFC axioms?

On Computable Numbers

> *The "computable" numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means.* (Alan Turing, *On Computable Numbers, with an Application to the Entscheidungsproblem*, 1936.)

Is τ computable (by Turing's definition)? ($\tau = \frac{\text{Circumference of circle}}{\text{radius of circle}}$)

What are the problems with using the State Machine to model computation:

$$M = (S, G \subseteq S \times S, q_0 \in S)$$

What was Turing attempting to model in defining what we know call a Turing Machine?

$$TM = (S, T \subseteq S \times \Gamma \rightarrow S \times \Gamma \times \mathit{dir}, q_0 \in S, q_{Accept} \subseteq S)$$

S is a finite set (the "in-the-head" processing states)
Γ is a finite set (symbols that can be writte on the tape)
dir = {**Left**, **Right**, **Halt**} is the direction to move on the tape.

What is the cardinality of the set of all Turing Machines?

Prove that there are numbers that cannot be output by any Turing Machine.

