Class 19: Uncountable Sets

Schedule

Problem Set 8 is due **Friday (4 November) at 6:29pm**.

Friday, 4 November at 11am (Rotunda Dome Room): **Steve Huffman** (BSCS 2005, co-founder and CEO of Reddit), *Computer Science Distinguished Alumni Speaker*.

Exam 2 will be in class on **Thursday, 10 November**. See Class 18 notes for details.

Countable and Uncountable Sets

Definition. A set S is *countably infinite* if and only if there exists a bijection between S and $\mathbb{N}$.

Definition. A set S is *uncountable*, if there exists no bijection between S and $\mathbb{N}$.

The **power set** of A (pow(A))is the set of all subsets of A:

$$B \in \text{pow}(A) \iff B \subseteq A.$$

For all **finite** sets S, $|pow(S)| = 2^{|S|}$.

For **all** sets S, $|pow(S)| > |S|$.

Prove $pow(\mathbb{N})$ is uncountable.

bitstrings = $\forall n \in \mathbb{N}.\{0, 1\}^n$.

Ordinal and Cardinal Numbers

ω is the *smallest infinite ordinal.* The first ordinal after $0, 1, 2, \cdots$.

What is the difference between an *ordinal* and *cardinal* number?

What should 2ω mean?

Is InfiniteBitStrings = $\{0, 1\}^\omega$ countable?

Prove the number of real numbers in the interval $[0, 1]$ is uncountable.

What set is bigger than $\mathbb{R}$?

Aleph-naught: $\aleph_0 = |\mathbb{N}|$ is the *smallest infinite cardinal number.*

How is ω different from $\aleph_0$?

$$2^{\aleph_0} = |pow(\mathbb{N})| = |[0, 1]| = |\mathbb{R}| = |\text{InfiniteBitStrings}|$$

What is necessary to conclude that it is not possible to settle the question of whether $\aleph_1 = 2^{\aleph_0}$ with the ZFC axioms?

