# Class 9: Cardinality

**Schedule**

This week you should finish reading MSC Chapter 4 (section 4.5) and Section 5.1. We will discuss Induction (Section 5.1) next class.

**Problem Set 4** is due **Friday at 6:29pm**.

## Relation Practice

The *inverse* of a relation $R : A \to B, G \subseteq A \times B$ is defined by reversing all the arrows:

$$R^{-1} : B \to A, G^{-1} \subseteq B \times A$$

$$(b, a) \in G^{-1} \iff \underline{\hspace{6cm}}$$

What does it mean if $R \equiv R^{-1}$?

## Set Cardinality

**Finite Cardinality.** If $A$ is a finite set, the *cardinality* of $A$, written $|A|$, is the number of elements in $A$. Does this definition require adding a new fundamental set operation?

**Alternate definition:** The *cardinality* of the set

$$N_k = \{n | n \in \mathbb{N} \land n < k\}$$

is $k$. If there is a *bijection* between two sets, they have the same cardinality.

If there is a *surjective relation* between $A$ and $B$ what do we know about their cardinalities?

If there is a *surjective function* between $A$ and $B$ what do we know about their cardinalities?

If there is a *total surjective function* between $A$ and $B$ what do we know about their cardinalities?

If there is a *total surjective injective function* between $A$ and $B$ what do we know about their cardinalities?

What is the cardinality of $A \cup B$?

## Power Sets

The **power set** of $A$ (pow($A$))is the set of all subsets of $A$:

$$B \in \text{pow}(A) \iff B \subseteq A.$$

Prove that the size of the power set of a set $S$ with $|S| = N$ is $2^N$ using the well-ordering principle. (See MCS 4.5.1 for an alternate proof.)

  