Class 4: Logical Formulas

Schedule

Problem Set 1 is due **Friday at 6:29pm**.

I will not be able to hold my normal office hours on Monday.

Next week, we will cover the rest of Chapter 3 (Satisfiability and Quantifiers).

Notes and Questions

Well-Ordering Principle Proof

Odd Summation. (Problem 2.12) Prove that for all $n > 0$, the sum of the first n odd numbers is n^2.

Notations

Mathematics and other domains often use many symbols to mean the same thing. Section 3.2 of the book gives some common notations, but there are others in common use.

English	Logic	C, Java, Rust	Python
P IMPLIES Q	$P \implies Q$ *or* $P \longrightarrow Q$	-	-
NOT(P)	$\neg P$ *or* $\overline{P}$	`!p`	`not p`
P AND Q	$P \wedge Q$	`p && q`	`p and q`
P OR Q	$P \vee Q$	`P || Q`	`p or q`
P XOR Q	$P \oplus Q$	`p ^ q` (bitwise) or `p != q`	`p ^ q`

For what values in Java or C are `p ^ q` and `p != q` both valid, but have different meanings?

Logical Formulas

P	NOT(P)	________
T	**F**	
F	**T**	

How many one-input Boolean operators are there? How many do we need to produce them all?

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	________	$P \oplus Q$
T	**T**	**T**	**T**		**T**	**F**
T	**F**		**T**		**F**	**T**
F	**T**		**T**		**F**	**T**
F	**F**		**F**		**T**	**F**

How many two-input Boolean operators are there?

De Morgan's Laws:

$$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q) \qquad \neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$$

How can these be written without the $\neg$ in front?

Prove that it is possible to make all two-input Boolean operators using just NOT and any *odd* two-input operator. (An operator is *odd*, if the number of outputs that are **True** are odd.)

Definition: valid. A logical formula is *valid* if there is no way to make it **false**. That is, no matter what truth values its variables have, it is always **true**. (Another name for this is a *tautology*.)

Definition: satisfiable. A logical formula is *satisfiable* if there is *some* way to make it **true**. That is, there is at least one assignment of truth value to its variables that makes the forumla true.

For each of the formulas below, determine if it is *valid* and if it is *satisfiable*.

1. $(P \vee \neg P)$
2. $(P \vee Q) \wedge (\neg P \vee Q)$
3. $((P \implies Q) \wedge (Q \implies P)) \vee (P \text{ XOR } Q)$

