Class 13: State Machines

Schedule

There is no problem set due this week!

Drop Date is today. If you have any concerns about whether you should stay in the class, please talk to me after class today.

Problem Set 6 (will be posted later this week) is due **21 October (Friday) at 6:29pm**.

Exam 1 will be returned during class today. Please confirm that the score on your exam matches what is recorded in collab, and that PS1-5 are all recorded correctly.

State Machines

A *state machine* is a binary relation (called a *transition relation*) on a set (both the domain and codomain are the same set). One state is designated as the *start state*.

$$M = (S, G : S \times S, q_0 \in S)$$

What does it mean if G is *total*?

What does it mean if G is not a *function*?

Parity Counter. Describe a state machine that determines if the number of steps is even.

Unbounded Counter. Describe a state machine that counts the number of steps.

How well do state machines model real computers? What kind of transition relation does the state machine modeling your computer have?

Invariant Principle

An *execution* of a state machine $M = (S, G \subseteq S \times S, q_0 \in S)$ is a (possibly infinite) sequence of states, $(x_0, x_1, \cdots, x_n)$ that:

1. $x_0 = q_0$ (it begins with the start state), and
2. $\forall i \in \{0, 1, \ldots, n-1\}. (x_i, x_{i+1}) \in G$ (if q and r are consecutive states in the sequence, then there is an edge $q \to r$ in G)

A state q is *reachable* if it appears in some execution. (That is, there is a sequence of transitions starting from q_0, following edges in G, that ends with q.)

$$M_1 = (S = \mathbb{N}, G = \{(x, y) | y = x^2\}, q_0 = 1)$$
$$M_2 = (S = \mathbb{N}, G = \{(x, y) | \exists k \in \mathbb{N}. y = kx\}, q_0 = 1)$$

Which states are *reachable* for M_1 and M_2?

A *preserved invariant* of a state machine $M = (S, G \subseteq S \times S, q_0 \in S)$ is a predicate, P, on states, such that whenever $P(q)$ is true of a state q, and $q \to r \in G$, then $P(r)$ is true.

Invariant Principle. If a *preserved invariant* of a state machine is true for the start state, it is true for all reachable states.

To show $P(q)$ for machine $M = (S, G \subseteq S \times S, q_0 \in S)$ all $q \in S$, show:

1. Base case: $P(\_\_\_\_)$
2. $\forall s \in S. \_\_\_\_ \implies \_\_\_\_$

Show the invariant principle holds using the principle of induction (hint: induction is on the *number of steps*).

