# Homework 1

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**Introduction to Signal and Image Processing**

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## 1 Fourier Transform

$$f_1(x) = \tfrac{1}{2}\delta(x) \rightarrow F_1(u) = \frac{1}{2}$$

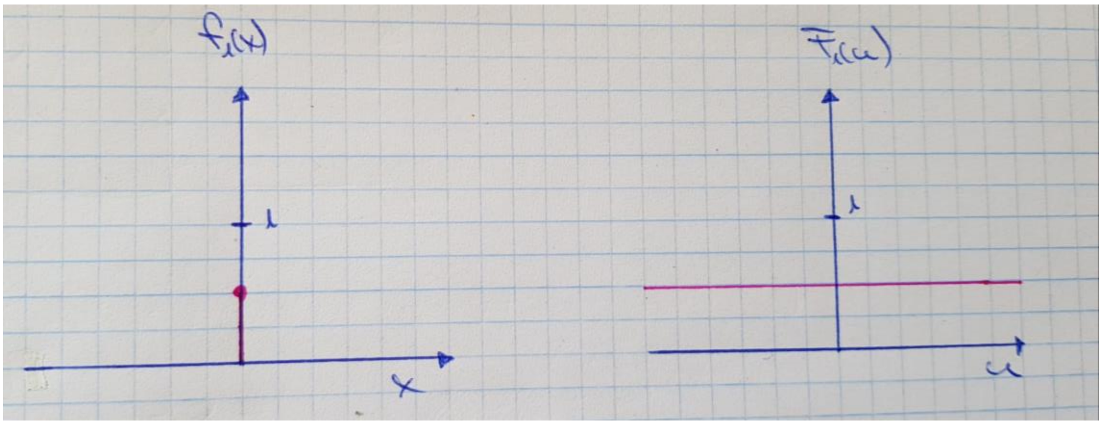


*Figure 1: Sketch of the solution with an impulse function in the time and the frequency domain*

$$f_2(x) = 2\cos(2\pi \mathrm{u}_a \mathrm{x}) + \cos(2\pi \mathrm{u}_b \mathrm{x}) \rightarrow \mathrm{F}_2(u) = \tfrac{1}{2}(2\delta(u-u_a) + 2\delta(u+u_a) + \delta(u-u_b) + \delta(u-u_b))$$

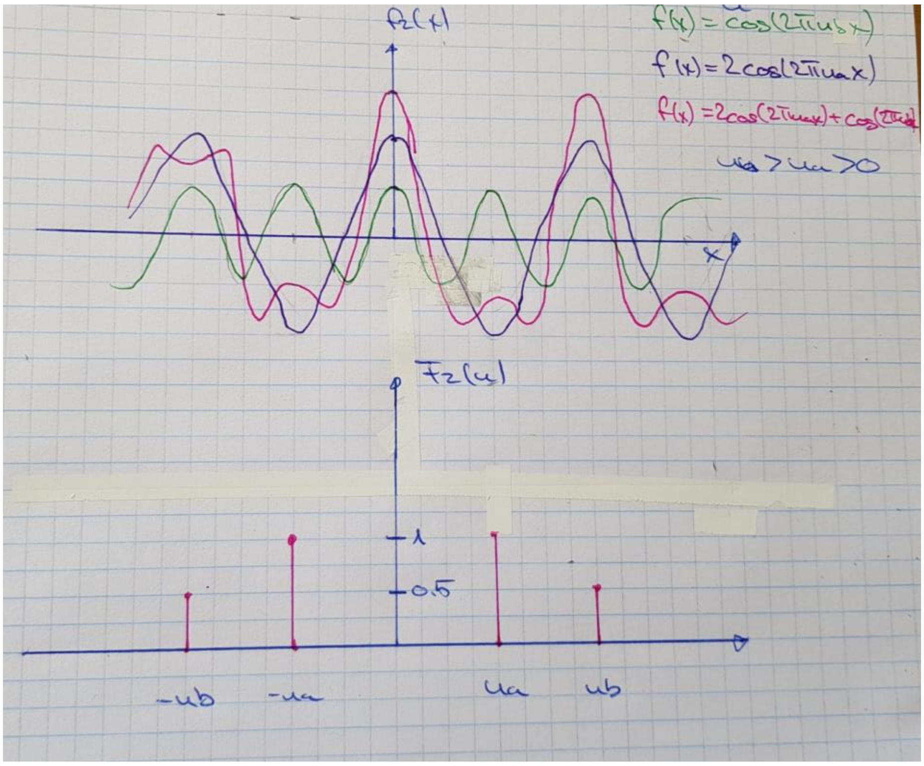


*Figure 2: Sketch of the solution with two overlaid cos functions in the time and the frequency domain.*

# 2 Lloyd-Max quantization

## 2.1

$$\varepsilon = \sum_{k=1}^{K} \int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz \tag{1}$$

The only step of the sum with $z_k$ :

$$\int_{z_{k-1}}^{z_k} (z - q_{k-1})^2 \, p(z) dz + \int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz$$

To get the minimum:

$$\frac{\partial}{\partial z_k} \left( \int_{z_{k-1}}^{z_k} (z - q_{k-1})^2 \, p(z) dz + \int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz \right) = 0$$

$$\frac{\partial}{\partial z_k} \left( \frac{\left(z_k - q_{k-1}\right)^3 p(z_k)}{3} - \frac{\left(z_{k-1} - q_{k-1}\right)^3 p(z_{k-1})}{3} + \frac{\left(z_{k+1} - q_k\right)^3 p(z_{k+1})}{3} - \frac{\left(z_k - q_k\right)^3 p(z_k)}{3} \right) = 0$$

$$\left(z_k - q_{k-1}\right)^2 p(z_k) - \left(z_k - q_k\right)^2 p(z_k) = 0$$

$${z_k}^2 - 2 z_k q_{k-1} + {q_{k-1}}^2 = {z_k}^2 - 2 z_k q_k + {q_k}^2$$

$$-2 z_k q_{k-1} + {q_{k-1}}^2 = -2 z_k q_k + q_k$$

$$2 z_k (q_k - q_{k-1}) = {q_k}^2 - {q_{k-1}}^2$$

$$2 z_k (q_k - q_{k-1}) = (q_k - q_{k-1})(q_k + q_{k-1})$$

$$z_k = \frac{(q_k + q_{k-1})}{2} \tag{2}$$

## 2.2

$$\varepsilon = \sum_{k=1}^{K} \int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz \tag{3}$$

The only step of the sum with $q_k$ :

$$\int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz$$

To get the minimum:

$$\frac{\delta}{\delta q_k} \int_{z_k}^{z_{k+1}} (z - q_k)^2 \, p(z) dz = 0$$

$$\int_{z_k}^{z_{k+1}} 2(z - q_k) \, p(z) dz = 0$$

$$\int_{z_k}^{z_{k+1}} 2z \cdot p(z) dz - \int_{z_k}^{z_{k+1}} 2q_k \cdot p(z) dz = 0$$

$$2\int_{z_k}^{z_{k+1}} z \cdot p(z) dz = 2q_k \int_{z_k}^{z_{k+1}} p(z) dz$$

$$q_k = \frac{\int_{z_k}^{z_{k+1}} z \cdot p(z) dz}{\int_{z_k}^{z_{k+1}} p(z) dz} \tag{4}$$

## 2.3

Random initialisation:

$$q_1 = 0.3, q_2 = 0.8$$
$$p(z) = 1$$

First iteration with random values:

$$z_2 = \frac{q_1 + q_2}{2} = \frac{0.3 + 0.8}{2} = 0.55$$

$$q_1 = \frac{\int_{z_1}^{z_2} z \cdot p(z)dz}{\int_{z_1}^{z_2} p(z)dz} = \frac{\int_0^{0.55} z \cdot 1dz}{\int_0^{0.55} 1dz} = \frac{\frac{0.55^2}{2}}{0.55} = 0.275$$

$$q_2 = \frac{\int_{z_2}^{z3} z \cdot p(z)dz}{\int_{z_2}^{z_3} p(z)dz} = \frac{\int_{0.55}^{1} z \cdot 1dz}{\int_{0.55}^{1} 1dz} = \frac{\frac{1^2 - 0.55^2}{2}}{1 - 0.55} = 0.775$$

Second iteration with the values from the first iteration:

$$z_2 = \frac{q_1 + q_2}{2} = \frac{0.275 + 0.775}{2} = 0.525$$

$$q_1 = \frac{\int_{z_1}^{z_2} z \cdot p(z)dz}{\int_{z_1}^{z_2} p(z)dz} = \frac{\int_0^{0.525} z \cdot 1dz}{\int_0^{0.525} 1dz} = \frac{\frac{0.525^2}{2}}{0.525} = 0.2625$$

$$q_2 = \frac{\int_{z_2}^{z3} z \cdot p(z)dz}{\int_{z_2}^{z_3} p(z)dz} = \frac{\int_{0.525}^{1} z \cdot 1dz}{\int_{0.525}^{1} 1dz} = \frac{\frac{1^2 - 0.525^2}{2}}{1 - 0.525} = 0.7626$$

$z_2$ will converge to 0.5. $q_1$ will converge to 0.25 and $q_2$ will converge to 0.75

# 3 Chamfer distance maps

To generate the chamfer distance maps the neighbouring pixels approach was used. The distance was calculated with the L-norm.

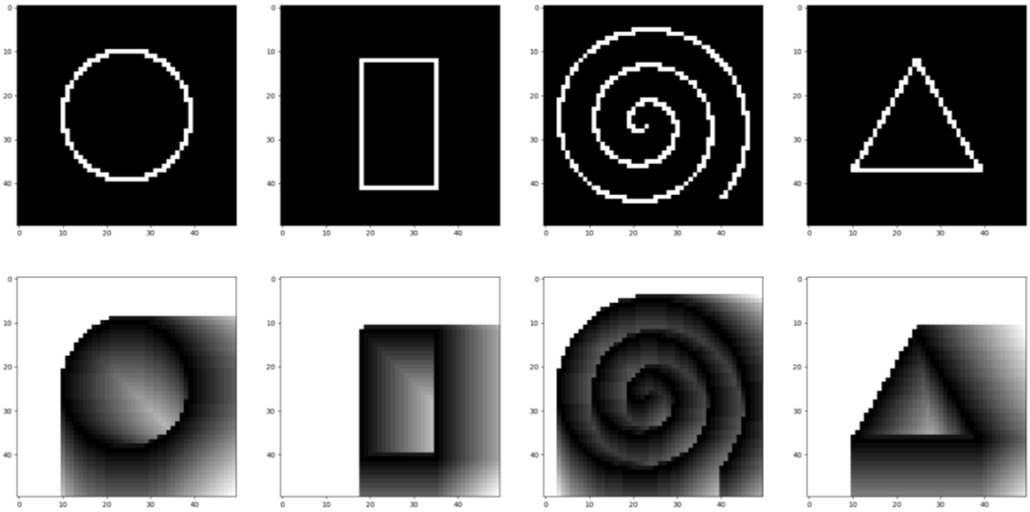

*Figure 3: Upper row shows the shapes and the lower row shows the distance map of the respective shape after the first pass.*

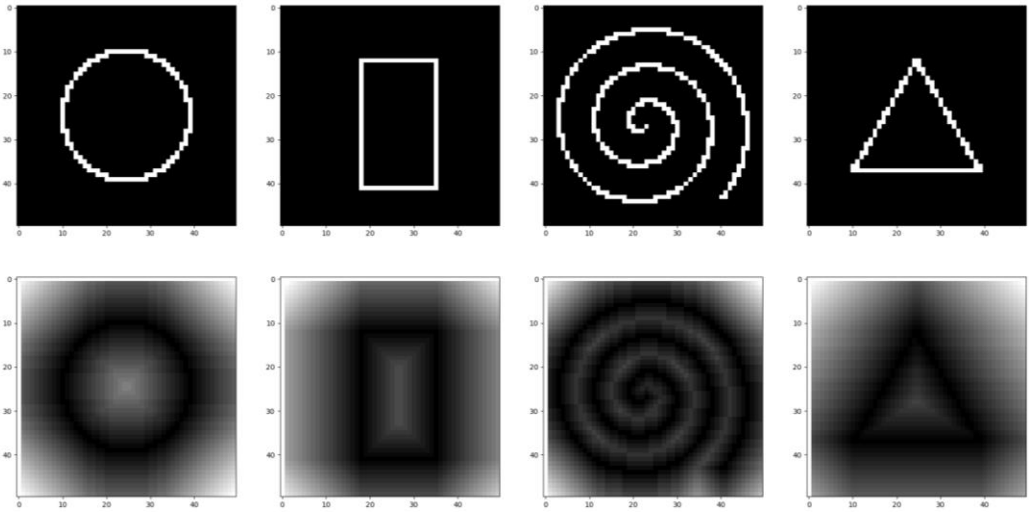

*Figure 4: Upper row shows the shapes and the lower row shows the final distance map of the respective shape*

# 4 Bilinear interpolation

The graphics in the following section explains what the parts of the code are doing.

## 4.1 Linear interpolation

The linear interpolation can be used to extract a new data point from a discrete sampled signal and to resample the signal with more or less values as before.

### 4.1.1

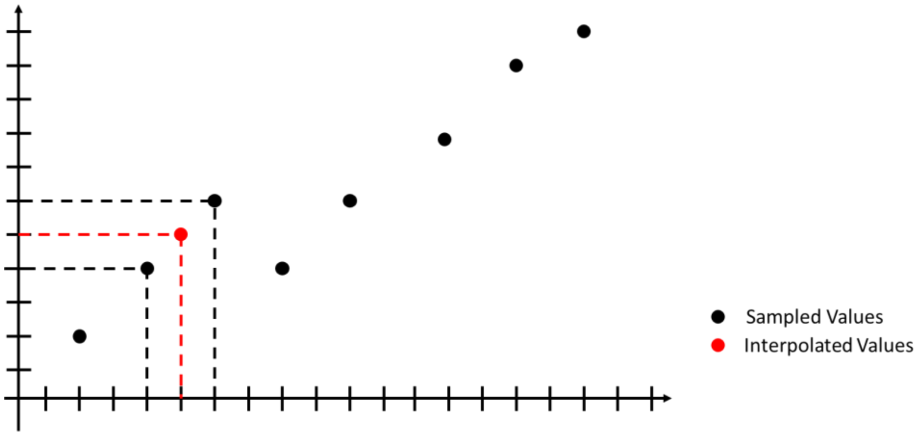


*Figure 5: Linear interpolation on a given x-value*

### 4.1.2

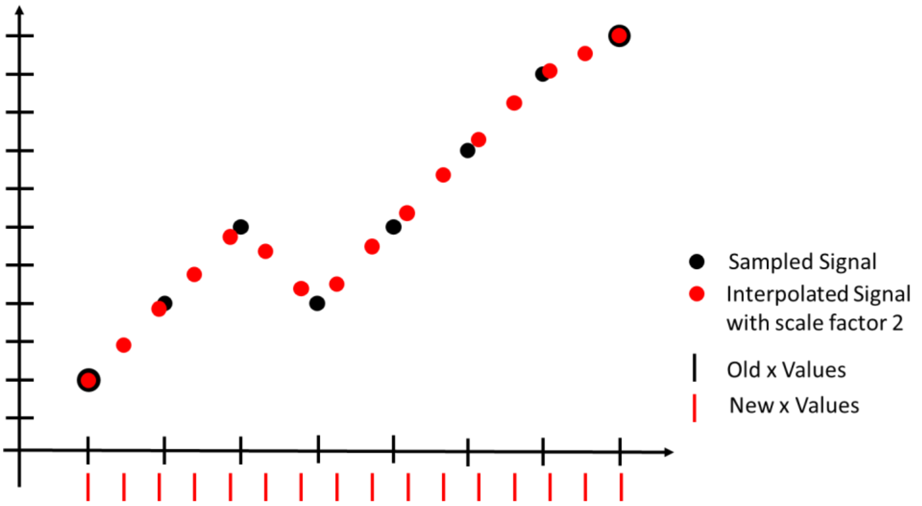


*Figure 6: Linear interpolation of  a signal with a given scale factor*

## 4.2 Bilinear interpolation

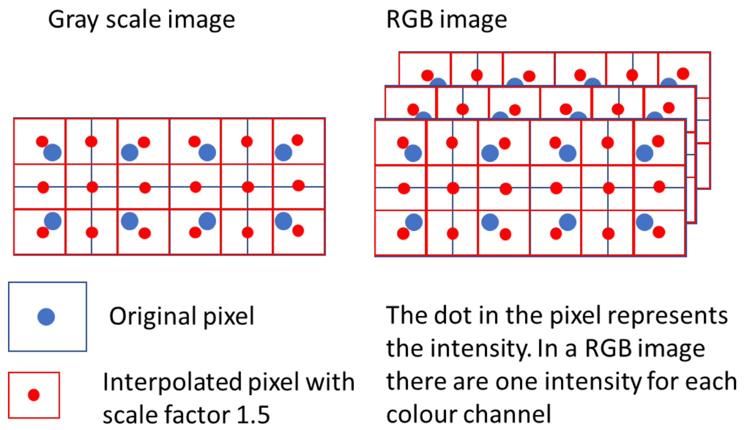


*Figure 7: Graphical illustration of a bilinear interpolation in a grayscale image and in a RGB image.*

### 4.2.1

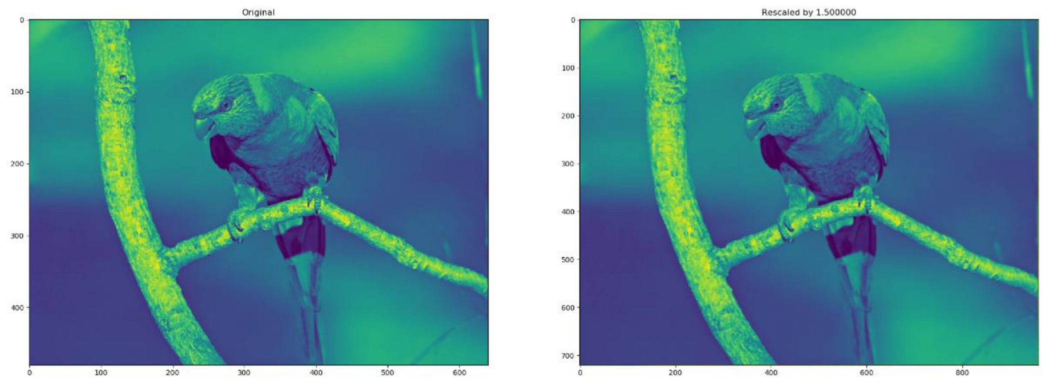


*Figure 8: Left shows the initial grayscale image and right shows the rescaled grayscale image by a scale factor of 1.5.*

### 4.2.2

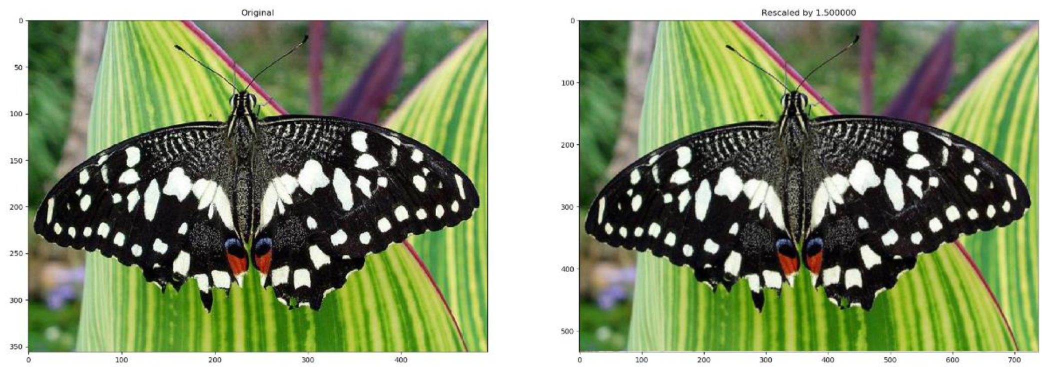


*Figure 9: Left shows the initial 3 channel RGB image and right shows the rescaled 3 channel RGB image by a scale factor of 1.5.*