Evolutionary game theory: A modified Ultimatum game model with algae as players

Stefanie Knebel

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1 What is the Ultimatum Game (UG)?

The Ultimatum Game (UG) is game theoretical model of fairness and is a focal point for studies in the evolution of social behaviour. In the UG, two players must decide on the division of resources. In contrast to the predictions of traditional game theory, participants appear to make irrational decisions; participants tend to divide the resources equally and would rather suffer a loss than to accept an unfair division. Here, we investigate a new UG theoretical model for the evolution of fairness, the Enhanced Ultimatum Game (EUG) where we introduce a cost associated with a demand.

The Ultimatum Game (UG) involves two players, the proposer and the responder, deciding on how to divide a resource between them. The proposer first offers an amount to the responder. If the responder accepts the offer then the resource is divided according to the proposal. If the offer is rejected then both players walk away with nothing.

In the Enhanced Ultimatum Game (EUG), the responder takes on an additional role as the demander. In this version, the responder first makes a demand for how to divide the resources and the proposer will then make an offer. The responder will then accept, with a cost, or reject.

The EUG models fairness as both players have the opportunity to be fair, split the resource evenly, or unfair, attempt to take a larger portion. Rejection of an unfair offer is interpreted as a form of punishment for deviating from fairness. A truly rational player would seek to maximize their payoff and should accept any offer which results in a positive payoff, fair or unfair, and to demand as much as possible. However, several models have shown that participants commonly make fair offers and reject unfair offers, leading to fairness as a potential evolutionary outcome.

This model is also applicable to

The EUG incorporates a demand and cost into the game and is played between two participants as follows:

- There are n dollars to be divided
- The responder demands an amount d such that $0 < d < n$
- The proposer then makes an offer p
- The responder has a minimum, M, that they are willing to accept such that $M \leq d$
- There is a cost to not giving a $d = M$. cost $= c(d - M)$, $c > 0$.
- If $p \leq M$ then the dollars are divided accordingly. If $P > M$ then both receive zero dollars.

2 How has the UG model been modified for this project? How is this model unique from literature?

The EUG model is a modified version of the model found within the literature, the UG model. Also, for this project, we are interested in the amount of Chlorophyll a within a lake and so, for the simulation, algae will act as the players in the EUG model. Also, several factors influence the evolution of chlorophyll in an algae population (ie. nutrients, sunlight, total phosphate etc). For instance, Total Phosphate (TP) found within a lake is thought to influence the quantity of a type of algae and sun exposure is associated with specific chlorophyll types. Therefore, for the purposes of this project, one may view TP and sunlight exposure as the resource to be divided in the EUG model. This is following an approach similar to the papers titles . This will allow us to make predictions about the group research question: Whether Chlorophyll a changes over time in a lake, given a particular environment.

3 Brief Description of simulation

The simulated data will be generated according to a variation on Agent-based modelling (ABM) that is commonly used in ecology, referred to as invidivual-based modelling (IBM) . In particular, we shall use the parameter of absorbance to simulate the evolution of an algae population in a lake according to individual behavior. The data obtained will be used to make predictions about how the enviroment will change according to varying levels of absorbance

4 In what program was the model coded?

I created my main code in python.

5 What are the parameters?

There are several possibilities (perhaps infinite) for parameter settings and so in the initial stages I was working out exactly what I wanted to analyse. Below is a list of the more interesting ways the simulation was altered:

- created a data set from various distributions (ie. poisson distribution)
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Only a few graphs are stored within the folder titled 'Plots with varying parameters' as it is not meant to be the point of interest.
For the graph displayed on the poster, the following parameters were used:

6 Pseudocode/Algorithm

Below is a outline/description of the python code:

{1} Import necessary packages (numpy and matplotlib.pyplot)

{2} Define variables (resource, cost, runs, popsize, generations, tsize, pmr, mdr, epoch)

{3} Create main loop.

i) Create characteristics of population: create an array for each algae within the population. Within each array describes the structure of the algae. The array contains 23 cells [3,4,2,6,2,14,15,13,2,6,7,8,6,4,3,3,7,8,9,9,7,18,0]. The first 20 cells correspond to proposal values that the alga will make, dependent on the demands. For example, if a second agla demands 3, then the first alga will propose the amount found in the 4th cell of the array. If the second alga demands 4, then the first alga will propose the amount found in the 5th cell of its array, and so on. The 21st cell is the alga's minaccept value, the 22nd is it's demand value, and the 23rd slot is it's fitness score. Suppose there is a population of n algae then n 23 celled arrays are created.

ii) Population interacts: Have each member of the population interact with all other members of the population twice, once as the demander and again as the proposer. Calculate and append fitness scores. Continue for g generations.

iii) Point mutation: Pull out a few members of the population, choose the two members with the highest fitness scores and have them produce two "offspring" that will replace two existing algae that have the lowest fitness scores.

iv) Stats: calculate average, minimum and maximum fitness scores across epochs

v) Plot: Create graph of maximum, minimum, and average fitness scores vs epoch

7 How were the parameters varied?

For the graph displayed on the poster, was run for values of

8 Any interesting findings?

It was found that

References

Michelutti, Neal, Jules M. Blais, Brian F. Cumming, Andrew M. Paterson, Kathleen Rühland, Alexander P. Wolfe, and John P. Smol. 2009. "Do Spectrally Inferred Determinations of Chlorophyll a Reflect Trends in Lake Trophic Status?" *Journal of Paleolimnology* 43 (2). Springer Nature: 205–17. doi:10.1007/s10933-009-9325-8.

Carlson, R.E. and J. Simpson. 1996. A Coordinator's Guide to Volunteer Lake Monitoring Methods. North American Lake Management Society. 96 pp.

For a complete list of references, see the "Literature" folder.