Evolutionary game theory: A modified Ultimatum game model with algae as players

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1 What is the Ultimatum Game (UG)?

The Ultimatum Game (UG) is game theoretical model of fairness and is a focal point for studies in the evolution of social behaviour. In the UG, two players must decide on the division of resources. In contrast to the predictions of traditional game theory, participants appear to make irrational decisions; participants tend to divide the resources equally and would rather suffer a loss than to accept an unfair division. Here, we investigate a new UG theoretical model for the evolution of fairness, the Enhanced Ultimatum Game (EUG) where we introduce a cost associated with a demand.

This model and other agent-based models are not limited to predicting human behaviour. Past studies have included such models to simulate behaviour of animals (Karsaia, Montanoa and Schmicklb,2016), plants (King, D. 1990), molecules (Bohl et al. (2014)) and more (Oswald & Schmikl, 2017; Zahadat and Schmickl (2016)). In these instances, what is of interest is the interaction of the members within a population and the changes that occur. It is my understanding that, if a particular population evolves via interactions of the members within its group, then the UG model may be applied.

For the purpose of a simple description, let's consider how the UG game would be applied to people. The Ultimatum Game (UG) involves two players, the proposer and the responder, deciding on how to divide a resource between them. The proposer first offers an amount to the responder. If the responder accepts the offer then the resource is divided according to the proposal. If the offer is rejected then both players walk away with nothing.

In the Enhanced Ultimatum Game (EUG), the responder takes on an additional role as the demander. In this version, the responder first makes a demand for how to divide the resources and the proposer will then make an offer. The responder will then accept, with a cost, or reject.

The EUG models fairness as both players have the opportunity to be fair, split the resource evenly, or unfair, attempt to take a larger portion. Rejection of an unfair offer is interpreted as a form of punishment for deviating from fairness. A truly rational player would seek to maximize their payoff and should accept any offer which results in a positive payoff, fair or unfair, and to demand as much as possible. However, several models have shown that participants commonly make fair offers and reject unfair offers, leading to fairness as a potential evolutionary outcome.

The EUG incorporates a demand and cost into the game and is played between two participants as follows:

- There are n dollars to be divided
- The responder demands an amount d such that $0 < d < n$
- The proposer then makes an offer p
- The responder has a minimum, M, that they are willing to accept such that $M \leq d$
- There is a cost to not giving a $d = M$. cost $= c(d - M)$, $c > 0$.
- If $p \leq M$ then the dollars are divided accordingly. If $P > M$ then both receive zero dollars

2 How has the UG model been modified for this project? How is this model unique from literature?

The EUG model is a modified version of the model found within the literature, the UG model. Also, for this project, we are interested in the amount of Chlorophyll a within a lake and so, for the simulation, algae will act as the players in the EUG model. Also, several factors influence the evolution of chlorophyll in an algae population (ie. nutrients, sunlight, total phosphate etc). For instance, Total Phosphate (TP) found within a lake is thought to influence the quantity of a type of algae and sun exposure is associated with specific chlorophyll types. Therefore, for the purposes of this project, one may view TP and sunlight exposure as the resource to be divided in the EUG model. This will allow us to make predictions about the group research question: Whether the quantity of Chlorophyll a changes over time in a lake, given a particular environment.

3 Brief description of simulation

The simulated data will be generated according to a variation on Agent-based modelling (ABM) that is commonly used in ecology, referred to as invidivual-based modelling (IBM) . In particular, we shall use the parameter of resource division to simulate the evolution of an algae population in a lake according to individual behavior. The data obtained will be used to make predictions about how an enviroment may change with respect to Chlorophyll a, given certain environmental conditions and an initial measure of Chlorophyll a.

4 In what program was the model coded?

I created my main code in python.

5 What are the parameters?

There are several possibilities (perhaps infinite) for parameter settings and so in the initial stages I was working out exactly what I wanted to analyse. Below is a list of the more interesting ways the simulation was altered:

- Created a data set from various distributions (ie. poisson distribution and varied lambda)
- Varied the cost parameter (ie. instead of n(D-MA) for some n>0, I tried n(D-2P+MA) for some n>0 etc).
- Introduced a resource probability parameter such that the resource available to any one alga would vary according to a normal distribution.
- Created an algae-type parameter (based on chlorophyll a vs. b) that was associated with proposal values.
- there ended up being many versions of this code!

Only a few graphs are stored within the folder titled 'Plots with varying parameters' as it is not meant to be the main point of interest for this project.

For the graph displayed on the poster, the following parameters were used:

- resource = 20 (Amount of resource to be divided)
- cost = 2 (Cost for demanding more than MA)

- runs = 1 (Number of interactions)
- popsize = 100 (population size/Number of algae)
- generations = 1000 (Number of generations)
- tsize = 4 (Tournament size)
- pmr=0.05 (Point mutation rate for proposal)
- mdr=0.5 (Point mutation rate for demand and MA)
- epoch=20

6 Algorithm/psuedocode

Below is a outline/description of the python code:

{1} Import necessary packages (numpy and matplotlib.pyplot)

{2} Define variables (resource, cost, runs, popsize, generations, tsize, pmr, mdr, epoch)

{3} Create main loop.

i) Create characteristics of population: create an array for each alga within the population. Within each array describes the structure of the alga. The array contains 23 cells [3,4,2,6,2,14,15,13,2,6,7,8,6,4,3,3,7,8,9,9,7,18,0]. The first 20 cells correspond to proposal values that the alga will make, dependent on the demands. For example, if a second agla demands 3, then the first alga will propose the amount found in the 4th cell of the array. If the second alga demands 4, then the first alga will propose the amount found in the 5th cell of its array, and so on. The 21st cell is the alga's minaccept value, the 22nd is it's demand value, and the 23rd slot is it's fitness score. Suppose there is a population of n algae then n 23 celled arrays are created.

ii) Population interacts: Have each member of the population interact with all other members of the population twice, once as the demander and again as the proposer. Calculate and append fitness scores. Continue for g generations.

iii) Point mutation: Pull out a few members of the population, choose the two members with the highest fitness scores and have them produce two "offspring" that will replace two existing algae that have the lowest fitness scores.

iv) Stats: calculate average, minimum and maximum fitness scores across epochs

v) Plot: Create graph of maximum, minimum, and average fitness scores vs epoch

7 How were the parameters varied?

For the graph displayed on the poster, all values found in the section of this paper titled "What are the parameters" were varied. However, I eventually fixed most values, as mentioned, except for the "runs", "generations" and "epochs" variables. I varied the parameters within the range that the memory of my computer would allow (approx. 0 - 100,000 units).

8 Any interesting findings?

It has proven difficult (or thought provoking?) to interpret a couple of the results. I found what presents as an expected Nash equilibrium such that a population will remain stable if members of the population accept anything while demanding as much as possible. This is seen in the plot (albeit labelled incorrectly as Absorbance) where the minaccept(green) is reducing over time but the demand (blue) is slightly higher. It can be seen that this trend continues when increasing the number of generations. However, if the parameters are altered, the findings match closer to what we would expect to be found with people and the division of resource. In this case, the strategy that appears to dominate the population is one that is offering a 50/50 split of the resource.This can be seen in the Proposalvsdemand plot where the amount proposed is half the resource.

The main finding: The simulation predicts that the algae population will tend towards a population with a higher “fitness” score. Given how “fitness” was defined for this model, we may expect the quantiy of chlorophyll a to grow substantially, as higher fitness implies more algae which implies more chlorophyll a. It's also interesing to note that the “fitness” score grows but appears to stabilize at a fixed value, suggesting an equilibrium is met. It's possible that the lake reaches a carrying capacity which could explain the fixed value. Likely what we're seeing is a particular type of algae (or some symbiotic relationship or a characteristic shared) dominating the population.

9 References

(For a complete list of references, see the “Literature” folder)

Carlson, R.E. and J. Simpson. 1996. A Coordinator's Guide to Volunteer Lake Monitoring Methods. North American Lake Management Society. 96 pp.

Bohl, Katrin, Sabine Hummert, Sarah Werner, David Basanta, Andreas Deutsch, Stefan Schuster, and Anja Schroeter. 2014. “Evolutionary Game Theory: Molecules as Players.” *Mol. BioSyst.* 10 (12). Royal Society of Chemistry (RSC): 3066–74. doi:10.1039/c3mb70601j.

Zahadat, Payam, and Thomas Schmickl. 2016. “Division of Labor in a Swarm of Autonomous Underwater Robots by Improved Partitioning Social Inhibition.” *Adaptive Behavior* 24 (2). SAGE Publications: 87–101. doi:10.1177/1059712316633028.