MAT 421: Introduction to Real Analysis I
Pranvere 2012, Provim 1

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Emri, Mbiemri:

Pergjigjuni 5 pyetje e meposhtme. Nuk i lejohet te perdore asgje pervec leter e bardhe dhe nje stilolaps. Maksimumi i pikeve te mundshme eshte 30.

1. A konvergjojne vargjet dhe seritet e meposhtme?:

   1. $\lim\limits_{n \to \infty} \frac{1}{n}$

   2. $\lim\limits_{n \to \infty} \frac{1}{(-1)^n}$

   3. $\lim\limits_{n \to \infty} \frac{n^2+1}{5n}$

   4. $\lim\limits_{n \to \infty} \frac{n^6}{n!}$

   5. $\lim\limits_{n \to \infty} \frac{2^n}{n!}$

   6. $\lim\limits_{n \to \infty} \frac{2^{2^n}}{n!}$

   7. $\sum_{n=1}^{\infty} \frac{1}{n}$

   8. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

   9. $\sum_{n=1}^{\infty} \frac{1}{5^n}$

   10. $\sum_{n=1}^{\infty} \frac{3^n}{7^n}$

   11. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

   12. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

   (12 pike, nje per cdo pergjigje te sakte)

2. Gjeni derivatin $f'(x)$ per funksionet e meposhtme:

   1. $f(x) = 4x^3 + 6x^2$

   2. $f(x) = (2x + 1)^{17}$

   3. $f(x) = \sin(x)^2$

   4. $f(x) = e^{2x}$

   5. $f(x) = \frac{2x}{x^2+1}$

   6. $f(x) = \frac{\sin(x)}{x^2+1}$

   (6 pike, nje per cdo pergjigje te sakte)

3. Gjeni te gjithe pikat e akumulimit te bashkesise $S := \{a^2 \mid a \in \mathbb{Q}\} \subset \mathbb{R}$.
   (4 pike)

4. Gjeni nje $a \in \mathbb{R}$ te tille qe funksioni

$$f : \ \mathbb{R} \to \mathbb{R}, \ x \mapsto \begin{cases} \sin(\frac{1}{x}) & \text{nese } x \neq 0, \\ a & \text{nese } x = 0 \end{cases}$$

eshte i vazhdueshem ne $x = 0$, apo tregoni qe nje $a \in \mathbb{R}$ te tille nuk egziston.
(4 pike)

5. Le te jete $S \subset \mathbb{R}$. Vertetoni apo gjeni kundershembuj:

   1. Nese S nuk permban nje interval $[a, b] \neq \emptyset$, S eshte e fundem apo e numerueshem.

   2. Nese S nuk eshte e ngjeshur ne asnje interval $[a, b] \neq \emptyset$, S eshte e fundem apo e numerueshem.

   (4 pike)