

# Classifying Finite Simple Groups with Respect to the Number of Orbits Under the Action of the Automorphism Group

– Supplementary Tables, Updated 2020-01-14 –

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The values  $\omega(G)$  in Table 1 have mostly been computed using **GAP** [4], all other data has been taken from the *Atlas of finite groups* [3]. Tables 2 and 4 have in parts been computed using **GAP**, and in parts using **MAGMA** [2] by Eamonn O’Brien in December 2019 – cf. Table 3. For Table 4, among various other information, also the bounds from [1] have been taken into account. The value  $\omega(\text{Ree}(8)) = 57$  has been determined independently by Frank Lübeck and Rob Wilson from the character table of the group and some insight on fusion of classes, and the value  $\omega(^2\text{E}_6(2)) = 98$  has been determined in a similar way by Frank Lübeck.

## 1 Orbit Numbers for Small Simple Groups

Table 1: Values  $\omega(G)$  for simple groups  $G$  (sorted by group order, the enumeration of groups  $G = \text{PSL}(2, q)$  was stopped at  $|G| = 10^6$ ).

$G$	$\omega(G)$	$ G $	Prime factorization of $ G $	$\text{Out}(G)$
$A_5 \cong \text{PSL}(2, 4)$				
$\cong \text{PSL}(2, 5)$	4	60	$2^2 \cdot 3 \cdot 5$	$C_2$
$\text{PSL}(3, 2) \cong \text{PSL}(2, 7)$	5	168	$2^3 \cdot 3 \cdot 7$	$C_2$
$A_6 \cong \text{PSL}(2, 9)$	5	360	$2^3 \cdot 3^2 \cdot 5$	$C_2^2$
$\text{PSL}(2, 8)$	5	504	$2^3 \cdot 3^2 \cdot 7$	$C_3$
$\text{PSL}(2, 11)$	7	660	$2^2 \cdot 3 \cdot 5 \cdot 11$	$C_2$
$\text{PSL}(2, 13)$	8	1092	$2^2 \cdot 3 \cdot 7 \cdot 13$	$C_2$
$\text{PSL}(2, 17)$	10	2448	$2^4 \cdot 3^2 \cdot 17$	$C_2$
$A_7$	8	2520	$2^3 \cdot 3^2 \cdot 5 \cdot 7$	$C_2$
$\text{PSL}(2, 19)$	11	3420	$2^2 \cdot 3^2 \cdot 5 \cdot 19$	$C_2$
$\text{PSL}(2, 16)$	7	4080	$2^4 \cdot 3 \cdot 5 \cdot 17$	$C_4$
$\text{PSL}(3, 3)$	9	5616	$2^4 \cdot 3^3 \cdot 13$	$C_2$
$\text{PSU}(3, 3) \cong G_2(2)'$	10	6048	$2^5 \cdot 3^3 \cdot 7$	$C_2$
$\text{PSL}(2, 23)$	13	6072	$2^3 \cdot 3 \cdot 11 \cdot 23$	$C_2$
$\text{PSL}(2, 25)$	10	7800	$2^3 \cdot 3 \cdot 5^2 \cdot 13$	$C_2^2$
$M_{11}$	10	7920	$2^4 \cdot 3^2 \cdot 5 \cdot 11$	1
<i>To be continued.</i>				

<i>Continued.</i>				
$G$	$\omega(G)$	$ G $	Prime factorization of $ G $	$\text{Out}(G)$
PSL(2, 27)	7	9828	$2^2 \cdot 3^3 \cdot 7 \cdot 13$	$C_6$
PSL(2, 29)	16	12180	$2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 29$	$C_2$
PSL(2, 31)	17	14880	$2^5 \cdot 3 \cdot 5 \cdot 31$	$C_2$
$A_8 \cong \text{PSL}(4, 2)$	12	20160	$2^6 \cdot 3^2 \cdot 5 \cdot 7$	$C_2$
PSL(3, 4)	6	20160	$2^6 \cdot 3^2 \cdot 5 \cdot 7$	$D_6$
PSL(2, 37)	20	25308	$2^2 \cdot 3^2 \cdot 19 \cdot 37$	$C_2$
$\text{PSU}(4, 2) \cong \text{O}(5, 3)$	15	25920	$2^6 \cdot 3^4 \cdot 5$	$C_2$
Sz(8)	7	29120	$2^6 \cdot 5 \cdot 7 \cdot 13$	$C_3$
PSL(2, 32)	9	32736	$2^5 \cdot 3 \cdot 11 \cdot 31$	$C_5$
PSL(2, 41)	22	34440	$2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 41$	$C_2$
PSL(2, 43)	23	39732	$2^2 \cdot 3 \cdot 7 \cdot 11 \cdot 43$	$C_2$
PSL(2, 47)	25	51888	$2^4 \cdot 3 \cdot 23 \cdot 47$	$C_2$
PSL(2, 49)	17	58800	$2^4 \cdot 3 \cdot 5^2 \cdot 7^2$	$C_2^2$
$\text{PSU}(3, 4)$	9	62400	$2^6 \cdot 3 \cdot 5^2 \cdot 13$	$C_4$
PSL(2, 53)	28	74412	$2^2 \cdot 3^3 \cdot 13 \cdot 53$	$C_2$
$M_{12}$	12	95040	$2^6 \cdot 3^3 \cdot 5 \cdot 11$	$C_2$
PSL(2, 59)	31	102660	$2^2 \cdot 3 \cdot 5 \cdot 29 \cdot 59$	$C_2$
PSL(2, 61)	32	113460	$2^2 \cdot 3 \cdot 5 \cdot 31 \cdot 61$	$C_2$
$\text{PSU}(3, 5)$	10	126000	$2^4 \cdot 3^2 \cdot 5^3 \cdot 7$	$S_3$
PSL(2, 67)	35	150348	$2^2 \cdot 3 \cdot 11 \cdot 17 \cdot 67$	$C_2$
$J_1$	15	175560	$2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$	1
PSL(2, 71)	37	178920	$2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 71$	$C_2$
$A_9$	16	181440	$2^6 \cdot 3^4 \cdot 5 \cdot 7$	$C_2$
PSL(2, 73)	38	194472	$2^3 \cdot 3^2 \cdot 37 \cdot 73$	$C_2$
PSL(2, 79)	41	246480	$2^4 \cdot 3 \cdot 5 \cdot 13 \cdot 79$	$C_2$
PSL(2, 64)	15	262080	$2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$	$C_6$
PSL(2, 81)	15	265680	$2^4 \cdot 3^4 \cdot 5 \cdot 41$	$C_2 \times C_4$
PSL(2, 83)	43	285852	$2^2 \cdot 3 \cdot 7 \cdot 41 \cdot 83$	$C_2$
PSL(2, 89)	46	352440	$2^3 \cdot 3^2 \cdot 5 \cdot 11 \cdot 89$	$C_2$
PSL(3, 5)	19	372000	$2^5 \cdot 3 \cdot 5^3 \cdot 31$	$C_2$
$M_{22}$	11	443520	$2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	$C_2$
PSL(2, 97)	50	456288	$2^5 \cdot 3 \cdot 7^2 \cdot 97$	$C_2$
PSL(2, 101)	52	515100	$2^2 \cdot 3 \cdot 5^2 \cdot 17 \cdot 101$	$C_2$
PSL(2, 103)	53	546312	$2^3 \cdot 3 \cdot 13 \cdot 17 \cdot 103$	$C_2$
$J_2$	16	604800	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7$	$C_2$
PSL(2, 107)	55	612468	$2^2 \cdot 3^3 \cdot 53 \cdot 107$	$C_2$
PSL(2, 109)	56	647460	$2^2 \cdot 3^3 \cdot 5 \cdot 11 \cdot 109$	$C_2$
PSL(2, 113)	58	721392	$2^4 \cdot 3 \cdot 7 \cdot 19 \cdot 113$	$C_2$
PSL(2, 121)	37	885720	$2^3 \cdot 3 \cdot 5 \cdot 11^2 \cdot 61$	$C_2^2$
PSL(2, 125)	24	976500	$2^2 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 31$	$C_6$
$\text{O}(5, 4)$	12	979200	$2^8 \cdot 3^2 \cdot 5^2 \cdot 17$	$C_4$
$\text{PSp}(6, 2)$	30	1451520	$2^9 \cdot 3^4 \cdot 5 \cdot 7$	1
$A_{10}$	22	1814400	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7$	$C_2$
PSL(3, 7)	16	1876896	$2^5 \cdot 3^2 \cdot 7^3 \cdot 19$	$S_3$
$\text{PSU}(4, 3)$	14	3265920	$2^7 \cdot 3^6 \cdot 5 \cdot 7$	$D_4$
$G_2(3)$	17	4245696	$2^6 \cdot 3^6 \cdot 7 \cdot 13$	$C_2$

*To be continued.*

<i>Continued.</i>				
$G$	$\omega(G)$	$ G $	Prime factorization of $ G $	$\text{Out}(G)$
$O(5, 5)$	27	4680000	$2^6 \cdot 3^2 \cdot 5^4 \cdot 13$	$C_2$
$PSU(3, 8)$	10	5515776	$2^9 \cdot 3^4 \cdot 7 \cdot 19$	$C_3 \times S_3$
$PSU(3, 7)$	34	5663616	$2^7 \cdot 3 \cdot 7^3 \cdot 43$	$C_2$
$PSL(4, 3)$	26	6065280	$2^7 \cdot 3^6 \cdot 5 \cdot 13$	$C_2^2$
$PSL(5, 2)$	20	9999360	$2^{10} \cdot 3^2 \cdot 5 \cdot 7 \cdot 31$	$C_2$
$M_{23}$	17	10200960	$2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	1
$PSU(5, 2)$	30	13685760	$2^{10} \cdot 3^5 \cdot 5 \cdot 11$	$C_2$
$PSL(3, 8)$	17	16482816	$2^9 \cdot 3^2 \cdot 7^2 \cdot 73$	$C_6$
${}^2F_4(2)'$ (Tits-G.)	17	17971200	$2^{11} \cdot 3^3 \cdot 5^2 \cdot 13$	$C_2$
$A_{11}$	29	19958400	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11$	$C_2$
$Sz(32)$	11	32537600	$2^{10} \cdot 5^2 \cdot 31 \cdot 41$	$C_5$
$PSL(3, 9)$	32	42456960	$2^7 \cdot 3^6 \cdot 5 \cdot 7 \cdot 13$	$C_2^2$
$PSU(3, 9)$	29	42573600	$2^5 \cdot 3^6 \cdot 5^2 \cdot 73$	$C_4$
$HS$	21	44352000	$2^9 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11$	$C_2$
$J_3$	17	50232960	$2^7 \cdot 3^5 \cdot 5 \cdot 17 \cdot 19$	$C_2$
$PSU(3, 11)$	30	70915680	$2^5 \cdot 3^2 \cdot 5 \cdot 11^3 \cdot 37$	$S_3$
$O(5, 7)$	43	138297600	$2^8 \cdot 3^2 \cdot 5^2 \cdot 7^4$	$C_2$
$O^+(8, 2)$	27	174182400	$2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$	$S_3$
$O^-(8, 2)$	33	197406720	$2^{12} \cdot 3^4 \cdot 5 \cdot 7 \cdot 17$	$C_2$
${}^3D_4(2)$	21	211341312	$2^{12} \cdot 3^4 \cdot 7^2 \cdot 13$	$C_3$
$PSL(3, 11)$	73	212427600	$2^4 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11^3 \cdot 19$	$C_2$
$A_{12}$	40	239500800	$2^9 \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 11$	$C_2$
$M_{24}$	26	244823040	$2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	1
$G_2(4)$	24	251596800	$2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$	$C_2$
$PSL(3, 13)$	39	270178272	$2^5 \cdot 3^2 \cdot 7 \cdot 13^3 \cdot 61$	$S_3$
$PSU(3, 13)$	100	811273008	$2^4 \cdot 3 \cdot 7^2 \cdot 13^3 \cdot 157$	$C_2$
$McL$	19	898128000	$2^7 \cdot 3^6 \cdot 5^3 \cdot 7 \cdot 11$	$C_2$
$PSL(4, 4)$	36	987033600	$2^{12} \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 17$	$C_2^2$
$PSU(4, 4)$	35	1018368000	$2^{12} \cdot 3^2 \cdot 5^3 \cdot 13 \cdot 17$	$C_4$
$O(5, 8)$	21	1056706560	$2^{12} \cdot 3^4 \cdot 5 \cdot 7^2 \cdot 13$	$C_6$
$PSL(3, 16)$	20	1425715200	$2^{12} \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 13 \cdot 17$	$C_4 \times S_3$
$O(5, 9)$	41	1721606400	$2^8 \cdot 3^8 \cdot 5^2 \cdot 41$	$C_2^2$
$PSU(3, 17)$	62	2317678272	$2^6 \cdot 3^4 \cdot 7 \cdot 13 \cdot 17^3$	$S_3$
$A_{13}$	52	3113510400	$2^9 \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	$C_2$
$He$	26	4030387200	$2^{10} \cdot 3^3 \cdot 5^2 \cdot 7^3 \cdot 17$	$C_2$
$PSU(3, 16)$	40	4279234560	$2^{12} \cdot 3 \cdot 5 \cdot 17^2 \cdot 241$	$C_8$
$PSp(6, 3)$	50	4585351680	$2^9 \cdot 3^9 \cdot 5 \cdot 7 \cdot 13$	$C_2$
$O(7, 3)$	52	4585351680	$2^9 \cdot 3^9 \cdot 5 \cdot 7 \cdot 13$	$C_2$
$PSL(3, 19)$	75	5644682640	$2^4 \cdot 3^4 \cdot 5 \cdot 19^3 \cdot 127$	$S_3$
$G_2(5)$	44	5859000000	$2^6 \cdot 3^3 \cdot 5^6 \cdot 7 \cdot 31$	1
$PSL(3, 17)$	163	6950204928	$2^9 \cdot 3^2 \cdot 17^3 \cdot 307$	$C_2$
$PSL(4, 5)$	34	7254000000	$2^7 \cdot 3^2 \cdot 5^6 \cdot 13 \cdot 31$	$D_4$
$PSU(6, 2)$	34	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	$S_3$

## 2 Simple Groups by Orbit Number

Table 2: Simple groups  $G$  for given  $\omega(G)$ ; if several groups are generically isomorphic, only one of them is mentioned. The table is complete for  $\omega(G) \leq 63$ .

$n$	Simple groups $G$ satisfying $\omega(G) = n$
4	$\text{PSL}(2, 4) \cong \text{PSL}(2, 5) \cong A_5$
5	$\text{PSL}(2, 7) \cong \text{PSL}(3, 2)$ , $\text{PSL}(2, 9) \cong A_6$ , $\text{PSL}(2, 8)$
6	$\text{PSL}(3, 4)$
7	$\text{PSL}(2, 11)$ , $\text{PSL}(2, 16)$ , $\text{PSL}(2, 27)$ , $\text{Sz}(8)$
8	$\text{PSL}(2, 13)$ , $A_7$
9	$\text{PSL}(3, 3)$ , $\text{PSL}(2, 32)$ , $\text{PSU}(3, 4)$
10	$\text{PSL}(2, 17)$ , $\text{PSU}(3, 3)$ , $\text{PSL}(2, 25)$ , $M_{11}$ , $\text{PSU}(3, 5)$ , $\text{PSU}(3, 8)$
11	$\text{PSL}(2, 19)$ , $M_{22}$ , $\text{Sz}(32)$
12	$\text{PSL}(4, 2) \cong A_8$ , $M_{12}$ , $O(5, 4)$
13	$\text{PSL}(2, 23)$
14	$\text{PSU}(4, 3)$
15	$\text{PSU}(4, 2) \cong O(5, 3)$ , $J_1$ , $\text{PSL}(2, 64)$ , $\text{PSL}(2, 81)$
16	$\text{PSL}(2, 29)$ , $A_9$ , $J_2$ , $\text{PSL}(3, 7)$
17	$\text{PSL}(2, 31)$ , $\text{PSL}(2, 49)$ , $G_2(3)$ , $M_{23}$ , $\text{PSL}(3, 8)$ , ${}^2F_4(2)'$ , $J_3$
18	
19	$\text{PSL}(3, 5)$ , $\text{Mcl}$ , $\text{Ree}(27)$
20	$\text{PSL}(2, 37)$ , $\text{PSL}(5, 2)$ , $\text{PSL}(3, 16)$
21	$\text{PSL}(2, 128)$ , $\text{HS}$ , ${}^3D_4(2)$ , $O(5, 8)$
22	$\text{PSL}(2, 41)$ , $A_{10}$
23	$\text{PSL}(2, 43)$ , $\text{Sz}(128)$
24	$\text{PSL}(2, 125)$ , $G_2(4)$
25	$\text{PSL}(2, 47)$ , $O'N$
26	$\text{PSL}(4, 3)$ , $M_{24}$ , $\text{He}$
27	$O(5, 5)$ , $\text{PSL}(2, 243)$ , $O^+(8, 2)$
28	$\text{PSL}(2, 53)$
29	$A_{11}$ , $\text{PSU}(3, 9)$
30	$O(7, 2) \cong \text{PSp}(6, 2)$ , $\text{PSU}(5, 2)$ , $\text{PSU}(3, 11)$
31	$\text{PSL}(2, 59)$
32	$\text{PSL}(2, 61)$ , $\text{PSL}(3, 9)$
33	$O^-(8, 2)$
34	$\text{PSU}(3, 7)$ , $\text{PSL}(4, 5)$ , $\text{PSU}(5, 4)$ , $\text{PSU}(6, 2)$
35	$\text{PSL}(2, 67)$ , $\text{PSU}(4, 4)$
36	$\text{PSL}(4, 4)$ , $\text{Ru}$
37	$\text{PSL}(2, 71)$ , $\text{PSL}(2, 121)$ , $\text{PSL}(2, 256)$ , $\text{Suz}$
38	$\text{PSL}(2, 73)$ , $O^+(8, 3)$
39	$\text{PSL}(3, 13)$
40	$A_{12}$ , $\text{PSU}(3, 16)$
41	$\text{PSL}(2, 79)$ , $O(5, 9)$
42	$\text{PSU}(3, 32)$ , $\text{Co}_3$
43	$\text{PSL}(2, 83)$ , $O(5, 7)$
<i>To be continued.</i>	

<i>Continued.</i>	
$n$	Simple groups $G$ satisfying $\omega(G) = n$
44	$G_2(5)$ , $\text{PSL}(6, 2)$ , HN
45	$O(5, 16)$
46	$\text{PSL}(2, 89)$
47	
48	Th
49	
50	$\text{PSL}(2, 97)$ , $\text{PSL}(2, 169)$ , $\text{PSp}(6, 3)$
51	
52	$\text{PSL}(2, 101)$ , $A_{13}$ , $O(7, 3)$
53	$\text{PSL}(2, 103)$ , Ly
54	
55	$\text{PSL}(2, 107)$
56	$\text{PSL}(2, 109)$ , ${}^3D_4(3)$
57	$\text{Ree}(8)$
58	$\text{PSL}(2, 113)$
59	$\text{Fi}_{22}$
60	$\text{Co}_2$
61	$\text{PSL}(2, 343)$ , $\text{PSL}(2, 512)$
62	$\text{PSU}(3, 17)$ , $F_4(2)$ , $J_4$
63	
64	$\text{PSU}(4, 5)$
65	$\text{PSL}(2, 127)$
66	
67	$\text{PSL}(2, 131)$
68	
69	$\text{PSL}(2, 729)$ , $A_{14}$
70	$\text{PSL}(2, 137)$
71	$\text{PSL}(2, 139)$
72	$\text{PSL}(3, 25)$ , $\text{PSL}(5, 3)$ , $G_2(7)$
73	$\text{PSL}(3, 11)$
74	
75	$\text{PSL}(3, 19)$ , $O(7, 4)$
76	$\text{PSL}(2, 149)$ , $\text{PSU}(4, 7)$
77	$\text{PSL}(2, 151)$ , $O^-(8, 3)$ , $\text{PSL}(7, 2)$
78	${}^3D_4(4)$
79	
80	$\text{PSL}(2, 157)$
81	$O(9, 2)$ , $\text{PSp}(8, 2)$
82	$\text{PSL}(2, 289)$
83	$\text{PSL}(2, 163)$
84	$O^+(10, 2)$ , $O^+(8, 4)$
85	$\text{PSL}(2, 167)$ , $\text{PSL}(4, 9)$
86	
87	$O(5, 11)$
88	$\text{PSL}(2, 173)$ , $\text{PSL}(2, 625)$
89	$\text{PSU}(5, 3)$
<i>To be continued.</i>	

<i>Continued.</i>	
$n$	Simple groups $G$ satisfying $\omega(G) = n$
90	$A_{15}$
91	$\text{PSL}(2, 179)$
92	$\text{PSL}(2, 181)$
93	$O^-(10, 2)$
94	
95	
96	
97	$\text{PSL}(2, 191), \text{Fi}'_{24}$
98	$\text{PSL}(2, 193), \text{Fi}_{23}, {}^2E_6(2)$
99	
100	$\text{PSL}(2, 197), \text{PSU}(3, 13)$

Table 3: Values  $\omega(G)$  computed by Eamonn O'Brien with MAGMA in December 2019.

$G$	$\omega(G)$
$O(5, 8)$	21
$\text{PSU}(6, 2)$	34
$\text{PSU}(5, 4)$	34
$\text{PSU}(4, 4)$	35
$O^+(8, 3)$	38
$\text{PSU}(3, 16)$	40
$O(5, 9)$	41
$\text{PSU}(3, 32)$	42
$O(5, 16)$	45
${}^3D_4(3)$	56
$\text{PSU}(3, 17)$	62
$F_4(2)$	62
$\text{PSU}(4, 5)$	64
$\text{PSL}(3, 25)$	72
$\text{PSL}(3, 11)$	73
$\text{PSL}(3, 19)$	75
$O(7, 4)$	75
$\text{PSU}(4, 7)$	76
$\text{PSL}(7, 2)$	77
$O^-(8, 3)$	77
${}^3D_4(4)$	78
$O^+(8, 4)$	84
$O^+(10, 2)$	84
$\text{PSL}(4, 9)$	85
$O(5, 11)$	87
$\text{PSU}(5, 3)$	89
$O^-(10, 2)$	93
$\text{PSU}(3, 13)$	100
$\text{PSU}(3, 23)$	106
$\text{PSL}(5, 4)$	110
<i>To be continued.</i>	

<i>Continued.</i>	
$n$	$\omega(G)$
$O(5, 13)$	115
$O^+(8, 5)$	116
$PSL(4, 8)$	119
$PSL(6, 3)$	122
$E_6(2)$	132
$PSp(6, 5)$	133
$O^-(8, 4)$	133
$O(7, 5)$	136
$PSL(4, 7)$	137
$PSU(4, 9)$	142
$O^-(10, 3)$	151
$O(5, 27)$	151
$PSU(6, 3)$	156
$PSU(3, 29)$	162
$PSL(6, 4)$	169
$O(5, 25)$	203
$PSU(4, 11)$	232
$PSU(9, 2)$	240
$O^+(10, 3)$	268
$O(7, 9)$	307
$PSU(3, 41)$	310

### 3 Remaining ‘Candidates’

Table 4: Bounds on orbit numbers for all remaining simple groups  $G$  which possibly satisfy  $\omega(G) \leq 100$ . We give the best lower bound computed so far.

$n$	Simple groups $G$ satisfying $\omega(G) \geq n$
64	$PSU(6, 5)$
77	$PSU(5, 9)$

## References

- [1] Alexander Bors, Michael Giudici, and Cheryl E. Praeger. *Documentation for the GAP code file OrbOrd.txt*, 2019. (<https://arxiv.org/abs/1910.12570>).
- [2] Wieb Bosma, John Cannon, and Catherine Playoust. The Magma algebra system. I. The user language. *J. Symbolic Comput.*, 24(3-4):235–265, 1997. Computational algebra and number theory (London, 1993).
- [3] John H. Conway, Robert T. Curtis, Simon P. Norton, Richard A. Parker, and Robert A. Wilson. *Atlas of finite groups*. Oxford University Press, 1985.
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