Prime Factorization - R vs. $R[x]$

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1 Definition Let R be an euclidean domain all of whose residue class rings are finite. We call a function $f : R \to R$ *analytic* if it satisfies the condition

$$\forall x, y \in R \quad f(x) \equiv f(x+y) \mod y.$$

We call the function f *reducible* if it can be written as (pointwise) product of two other analytic functions f_1, f_2 over R whose images do not entirely consist of units, and *irreducible* if not. We say that a product $f = \prod_{i=1}^{k} f_i$ of irreducible analytic functions f_i over R fulfills the *limited factoring condition* if and only if there is an $x \in R$ such that $f(x)$ has at most k prime factors. We say that f has *polynomial growth* if for all sequences $x_1, x_2, \ldots$ of elements of R satisfying $\lim_{n\to\infty} |R/x_nR| = \infty$ we have

$$\limsup_{n\to\infty} \frac{|R/f(x_n)R|}{|R/x_nR|!} \;=\; 0.$$

We call R a *limited factoring domain* if all analytic functions over R of polynomial growth fulfill the limited factoring condition.

2 Conjecture The ring $\mathbb{Z}$ is a limited factoring domain.

3 Question How to characterize limited factoring domains in general?