Computational Logic - Individual Homework	25. XI. 22
Resolution Proposition	3319 - FÎNTÎNĂ 913

1.3. $U_3 = (B \to A) \wedge (C \to A) \to (B \vee C \to A)$: Theorem $\Rightarrow$

U_3 is a theorem $\Leftrightarrow$ $CNF(\neg U_3) \vdash_{Res} \square$

$\downarrow$

U_3 is a theorem iff $\square$ (the empty clause) can be derived from $CNF(\neg U_3)$ (the normal form of the negated U_3) using the RESOLUTION ALGORHYTHM.

$$U_3 = (B \xrightarrow{\text{I}} A) \wedge (C \xrightarrow{\text{II}} A) \xrightarrow{\text{III}} (B \vee C \xrightarrow{\text{IV}} A)$$

I : $B \to A \equiv \neg B \vee A$

II : $C \to A \equiv \neg C \vee A$

III : $\neg((\neg B \vee A) \wedge (\neg C \vee A)) \vee (B \vee C \to A)$

IV : $\neg(B \vee C) \vee A$

After the normalization of U_3, we get:

$$U_3 = \neg((\neg B \vee A) \wedge (\neg C \vee A)) \vee (\neg(B \vee C) \vee A) \Rightarrow$$

$$\neg U_3 = \underbrace{(\neg B \vee A)}_{C_1} \wedge \underbrace{(\neg C \vee A)}_{C_2} \wedge \underbrace{(B \vee C)}_{C_3} \wedge \underbrace{\neg A}_{C_4}$$

The resolution algorhythm is based on the fact that $\neg q$ and q are clashing clauses upon the literal q and resolve upon it to derive $\square$ (empty cl.)

Therefore: The set S of clauses $\{C_1, C_2, C_3, C_4\}$ is inconsistent iff $S \vdash_{Res} \square \Leftrightarrow \vdash U_3$ iff $CNF(\neg U_3) \vdash_{Res} \square$

$$S = \{C_1 = \neg B \vee A,\ C_2 = \neg C \vee A,\ C_3 = B \vee C,\ C_4 = \neg A\}$$

$$S \ \vdash^{?}_{Res} \square$$

$C_1 = \neg B \vee A$

$C_2 = \neg C \vee A$

$C_3 = B \vee C$

$C_4 = \neg A$

General Resolution

$C_5 = Res_A(C_1, C_4) = \neg B \quad (Res_A(\neg B \vee \underline{\underline{A}}, \neg \underline{\underline{A}})$

$C_6 = Res_A(C_2, C_4) = \neg C \quad (Res_A(\neg C \vee \underline{\underline{A}}, \neg \underline{\underline{A}})$

$C_7 = Res_B(C_3, C_5) = C \quad (Res_B(\underline{\underline{B}} \vee C, \neg \underline{\underline{B}})$

$C_8 = Res_C(C_6, C_7) = \square \quad (Res_C(C, \neg C)$

Conclusion: $C_8\ (Res_C(C_6, C_7)) = \square \Rightarrow$

$Res_C(C, \neg C) = \square \Rightarrow$

U_3 - theorem

Upon the final clashing literal (C and $\neg C$), we denote that U_3 is indeed a theorem, because we were able to prove the inconsistency of the negated clause $\neg U_3$.