Symbolic integration

Integration is also a standard function in sympy, so we can find for example the integral

$$y(t) = \int_{-10}^{t} x(\lambda) d\lambda$$

for $x(t) = e^{-t/10}\cos(t)$:

In [25]:

```
xt = sp.exp(-t/10)*sp.cos(t);  # x(t)
lamb = sp.symbols('lamb');
xl = xt.subs(t,lamb);  # x(lamb)

yt = sp.integrate(xl, (lamb, -10, t));  # indefinite integral
yt
```

Out[25]:

$$\frac{100e}{101}\sin(10) + \frac{10e}{101}\cos(10) + \frac{100}{101}e^{-\frac{t}{10}}\sin(t) - \frac{10}{101}e^{-\frac{t}{10}}\cos(t)$$

Tasks

These tasks involve writing code, or modifying existing code, to meet the objectives described.

1. Define the expression $y(t) = v_0 t - \frac{1}{2}gt^2$ for some symbolic values of v_0 and g using sympy. You should recognise this as the "altitude" of a particle moving under the influence of gravity, given that the initial velocity at time $t = 0$ is v_0. Make a plot of the particle height in meters for $v_0 = 22.5m/s$ given $g = 9.8m/s^2$, over the range $t = 0$ to $t = 5s$.

2. Use symbolic math and the `roots` method to find an expression for the zeros of the expression $y(t)$ above for the same set of conditions. Substitute to find the nonzero numerical value of t for which your plot in the previous task crosses the x-axis.

3. Use symbolic differentiation to find the vertical velocity of the particle in the previous task as a function of time, given the same conditions. Make a plot of this velocity over the same time range.

4. Suppose the acceleration of a particle is given by $a(t) = 0.2 + \cos(t)$ for positive time. Use symbolic methods to find and plot the velocity $v(t)$ of the particle over the range $t = 0$ to $t = 5$ given the initial condition $v(0) = -0.3$. Then find and plot the position $s(t)$ of the particle over the same time period, given the additional auxiliary condition $s(0) = 0.1$.

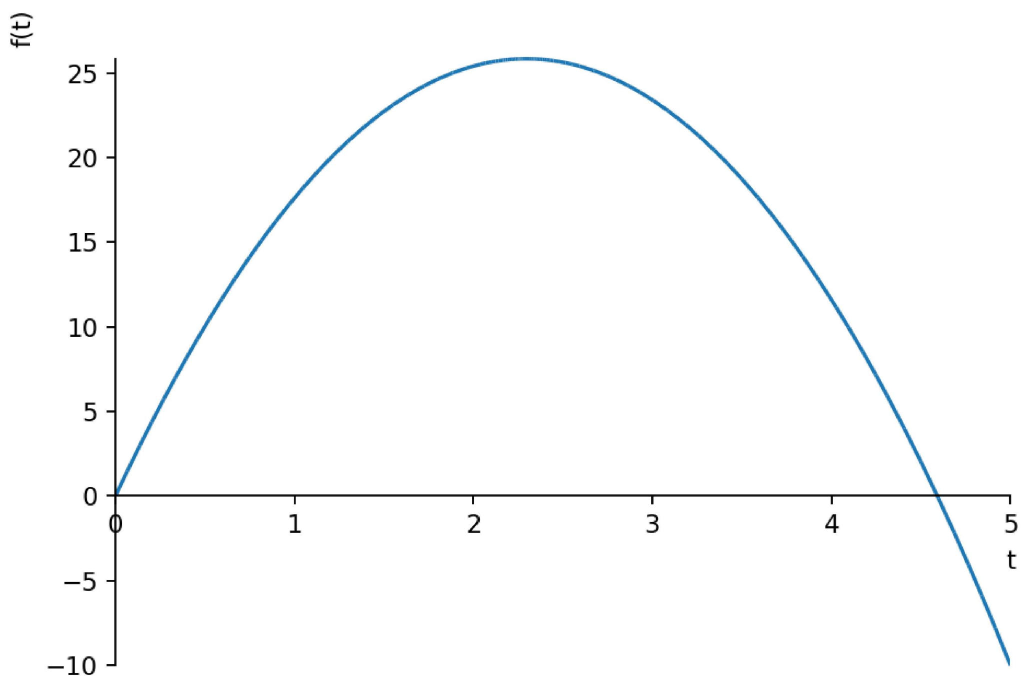
In [26]:

```
#imports
from IPython.display import display
import matplotlib.pyplot as plt
import numpy as np
import sympy as sp
%matplotlib notebook
```

In [27]:

```
v0, t, g = sp.symbols('v0 t g');
function = (v0*t - 0.5*g*t**2);
function = function.subs([(v0, 22.5), (g, 9.8)]);

display(function);

sp.plot(function, (t,0,5));
```

$$-4.9t^2 + 22.5t$$



In [29]:

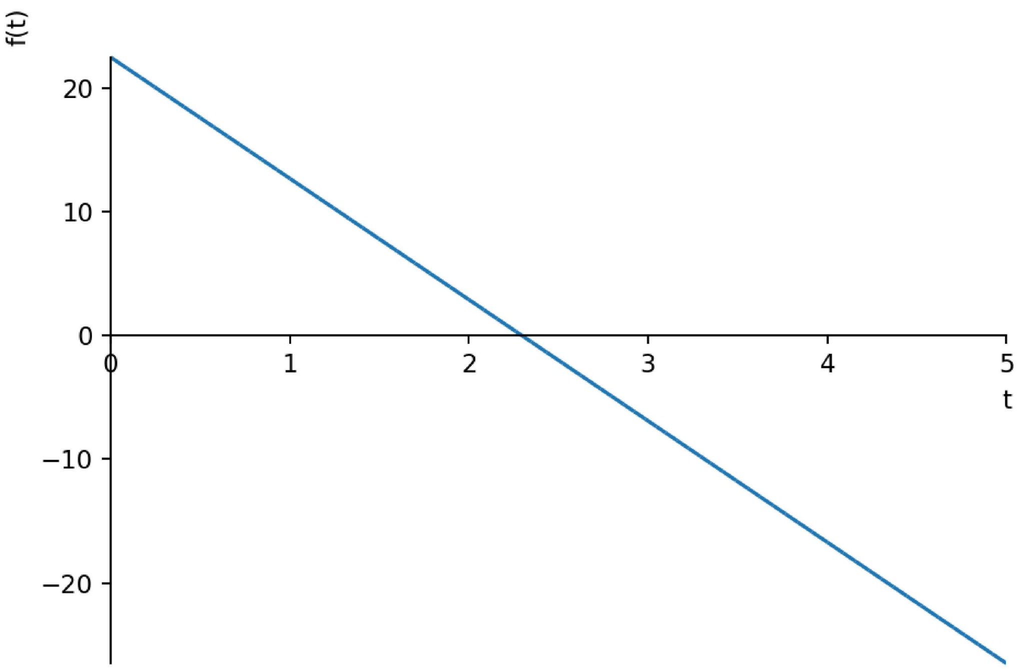
```
roots = list(sp.roots(function, t).keys())
print(roots)
```

```
[4.59183673469388, 0]
```

In [32]:

```
fdt = sp.diff(function, t);
display(fdt)
sp.plot(fdt, (t,0,5));
```

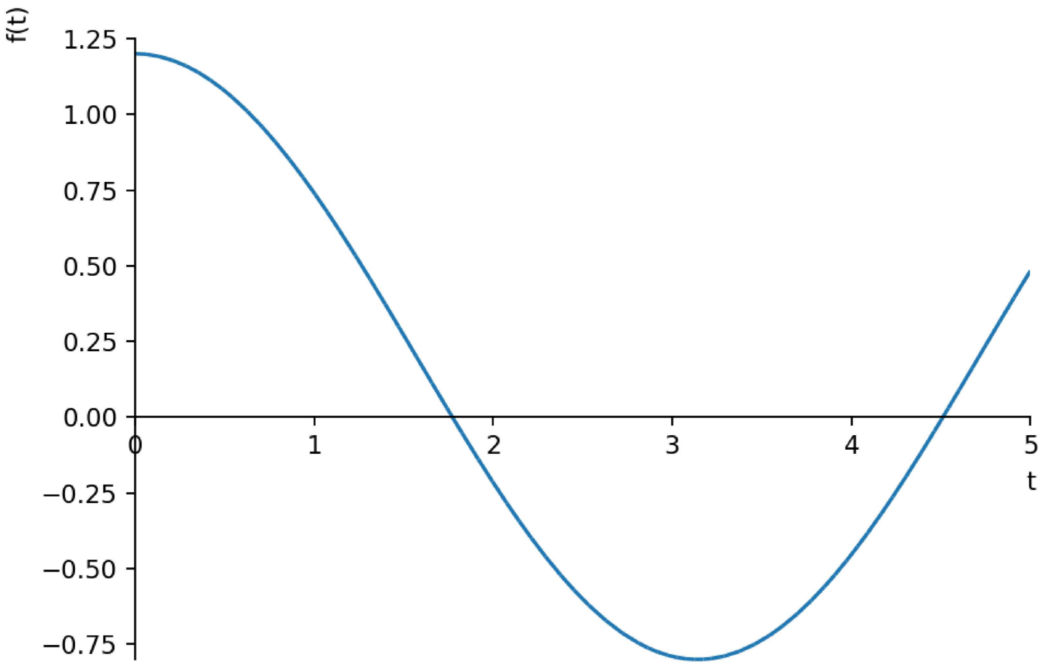
$-9.8t + 22.5$



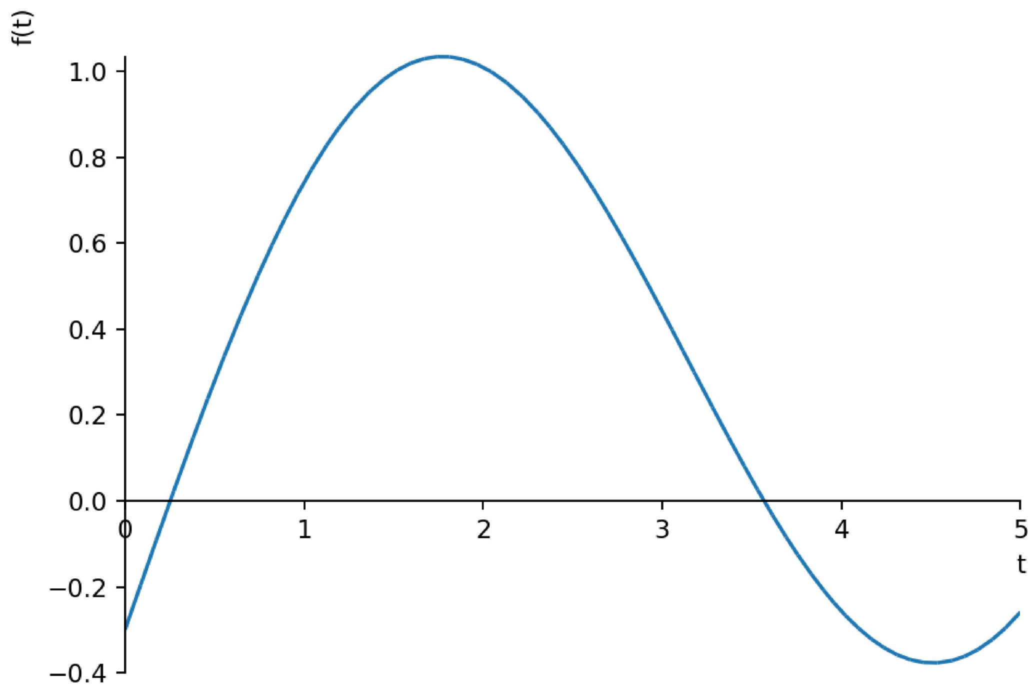
In [39]:

```
t = sp.symbols('t');
acceleration = 0.2 + sp.cos(t);
display(acceleration);
sp.plot(acceleration,(t,0,5));

velocity = sp.integrate(acceleration,(t));
velocity = velocity - 0.3;
display(velocity);
sp.plot(velocity,(t,0,5));

distance = sp.integrate(velocity, (t));
distance = distance + 0.1;
display(distance);
sp.plot(distance,(t,0,5));
```

$\cos(t) + 0.2$



$0.2t + \sin(t) - 0.3$



$0.1t^2 - 0.3t - \cos(t) + 0.1$