In [1]:

```
import numpy as np
import math
import pandas as pd
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec
from scipy.stats import uniform
from scipy.stats import beta
from scipy.stats import gamma
from scipy.stats import binom


mpl.rcParams['pdf.fonttype'] = 42
mpl.rcParams['ps.fonttype'] = 42
fig_dpi      = 300
fig_typeface = 'Helvetica'
fig_family   = 'monospace'
fig_style    = 'normal'
```

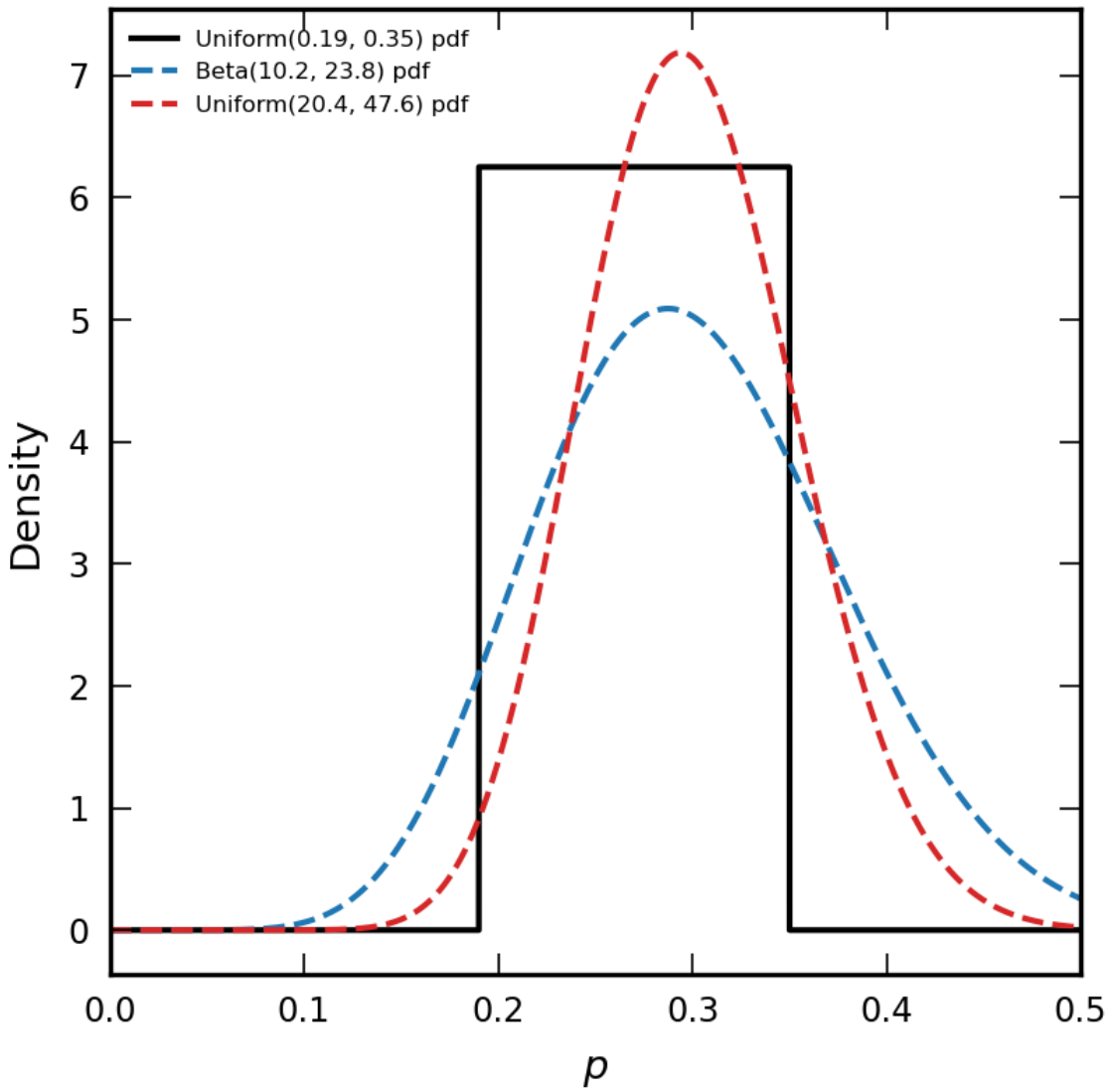
3.a

In [2]:

```
x = np.linspace(0, 0.5, int(1e6))
p_uniform = uniform.pdf(x, loc=0.19, scale=0.35-0.19)
p_beta1 = beta.pdf(x, 10.2, 23.8)
p_beta2 = beta.pdf(x, 20.4, 47.6)
```

In [3]:

```
f, ax = plt.subplots(1, 1, figsize=(3, 3), facecolor='white', dpi=300, gridspec_kw={
ax.plot(x, p_uniform, color = "black",ls = '-', lw=1, label='Uniform(0.19, 0.35) pdf
ax.plot(x, p_beta1, color = "tab:blue",ls = '--', lw=1, label='Beta(10.2, 23.8) pdf'
ax.plot(x, p_beta2, color = "tab:red",ls = '--', lw=1, label='Uniform(20.4, 47.6) pd

ax.tick_params(axis='both', which='both', labelsize='xx-small', right=True, top=True
ax.set_xlabel(r"$p$", size='x-small')
ax.set_ylabel("Density", size='x-small')
ax.set_xlim([0, 0.5])
ax.legend(loc = 2 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
plt.show()
# f.savefig("./HW4_3a.jpg", bbox_inches = "tight")
```



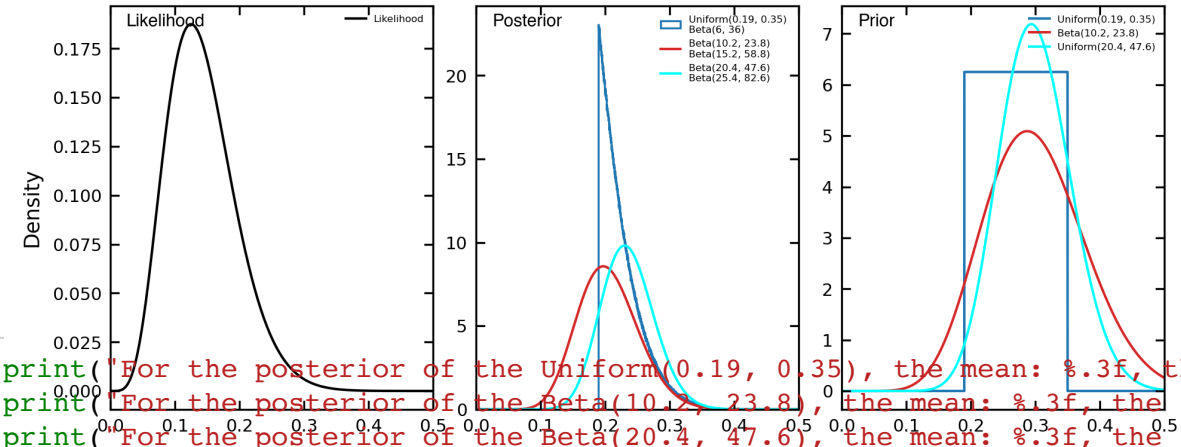
In [5]:

```
sample1 = np.random.beta(6, 36, int(1e8))
sample1 = sample1[sample1 >=0.19]
sample1 = sample1[sample1 <=0.35]
sample1 = np.random.choice(sample1, int(1e7))
sample2 = np.random.beta(15.2, 58.8, int(1e7))
sample3 = np.random.beta(25.4, 82.6, int(1e7))
sample1_mean = np.mean(sample1)
sample1_std = np.std(sample1)
sample1_prob = len(sample1[sample1>0.2])/int(1e7)
lb1 = np.percentile(sample1[sample1<sample1_mean], 5)
ub1 = np.percentile(sample1[sample1>=sample1_mean], 95)

sample2_mean = np.mean(sample2)
sample2_std = np.std(sample2)
sample2_prob = len(sample2[sample2>0.2])/int(1e7)
lb2 = np.percentile(sample2[sample2<sample2_mean], 5)
ub2 = np.percentile(sample2[sample2>=sample2_mean], 95)

sample3_mean = np.mean(sample3)
sample3_std = np.std(sample3)
sample3_prob = len(sample3[sample3>0.2])/int(1e7)
lb3 = np.percentile(sample3[sample3<sample3_mean], 5)
ub3 = np.percentile(sample3[sample3>=sample3_mean], 95)
```

In [12]:

```
f,ax = plt.subplots(figsize=(6, 2.5), dpi=300)
gs1 = gridspec.GridSpec(1, 1)
gs1.update(left=0.01, right=0.31, bottom=0.05, top=0.95, hspace=0.1, wspace=0.)
ax1 = plt.subplot(gs1[0])


gs2 = gridspec.GridSpec(1, 1)
gs2.update(left=0.35, right=0.65, bottom=0.05, top=0.95, hspace=0.1, wspace=0.)
ax2 = plt.subplot(gs2[0])


gs3 = gridspec.GridSpec(1, 1)
gs3.update(left=0.69, right=0.99, bottom=0.05, top=0.95, hspace=0.1, wspace=0.)
ax3 = plt.subplot(gs3[0])

# Likelihood
ax1.plot(x, binom.pmf(5, 40, x), color = 'black',ls = "-", lw=1, label='Likelihood',
# Posterior
ax2.hist(sample1, density= 1, bins=1000, histtype='step',lw= 0.8, color='tab:blue',
# ax2.plot(x, beta.pdf(x, 6, 36), color = 'tab:blue', ls = "-",lw=1, label='Uniform(
ax2.plot(x, beta.pdf(x, 15.2, 58.8), color = 'tab:red', ls = "-",lw=1, label='Beta(1
ax2.plot(x, beta.pdf(x, 25.4, 82.6), color = 'cyan', ls = "-",lw=1, label='Beta(20.4
# Prior
ax3.plot(x, p_uniform, color = "tab:blue",ls = '-', lw=1, label='Uniform(0.19, 0.35)
ax3.plot(x, p_beta1, color = "tab:red",ls = '-', lw=1, label='Beta(10.2, 23.8)', zor
ax3.plot(x, p_beta2, color = "cyan",ls = '-', lw=1, label='Uniform(20.4, 47.6)', zor

ax1.text(0.05, 0.95, "Likelihood", size=7, weight = 'bold',style=fig_style, family=f
ax2.text(0.05, 0.95, "Posterior", size=7, weight = 'bold',style=fig_style, family=fi
ax3.text(0.05, 0.95, "Prior", size=7, weight = 'bold',style=fig_style, family=fig_fa

ax1.set_xlim(0,0.5)
ax2.set_xlim(0,0.5)
ax3.set_xlim(0,0.5)
ax1.legend(loc = 1 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
ax2.legend(loc = 1 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
ax3.legend(loc = 1 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
ax1.tick_params(axis='both', which='both',labelleft = True, labelsize='x-small', rig
ax2.tick_params(axis='both', which='both',labelleft = True, labelsize='x-small', rig
ax3.tick_params(axis='both', which='both',labelleft = True, labelsize='x-small', rig

ax1.set_xlabel('p', size='small'); ax2.set_xlabel('p', size='small');  ax3.set_xlabe
ax1.set_ylabel('Density', size = "small")
# f.savefig("./HW4_3b.jpg", bbox_inches = "tight")
plt.show()
```

```
/var/folders/dy/y_4bw3nj3nl7cw3b482fcf_c0000gn/T/ipykernel_56870/26030
88003.py:4: MatplotlibDeprecationWarning: Auto-removal of overlapping
axes is deprecated since 3.6 and will be removed two minor releases la
ter; explicitly call ax.remove() as needed.
  ax1 = plt.subplot(gs1[0])
```




```
print("For the posterior of the Uniform(0.19, 0.35), the mean: %.3f, the std: %.3f,
print("For the posterior of the Beta(10.2, 23.8), the mean: %.3f, the std: %.3f, Pr(
print("For the posterior of the Beta(20.4, 47.6), the mean: %.3f, the std: %.3f, Pr(
print("95 percent HPDs for p are respectively: [%.3f, %.3f], [%.3f, %.3f], [%.3f, %.
```

```
For the posterior of the Uniform(0.19, 0.35), the mean: 0.226, the st
d: 0.031, Pr(p>0.2) = 0.789
For the posterior of the Beta(10.2, 23.8), the mean: 0.205, the std:
0.047, Pr(p>0.2) = 0.523
For the posterior of the Beta(20.4, 47.6), the mean: 0.235, the std:
0.041, Pr(p>0.2) = 0.803
95 percent HPDs for p are respectively: [0.191, 0.311], [0.123, 0.30
5], [0.161, 0.320]
```

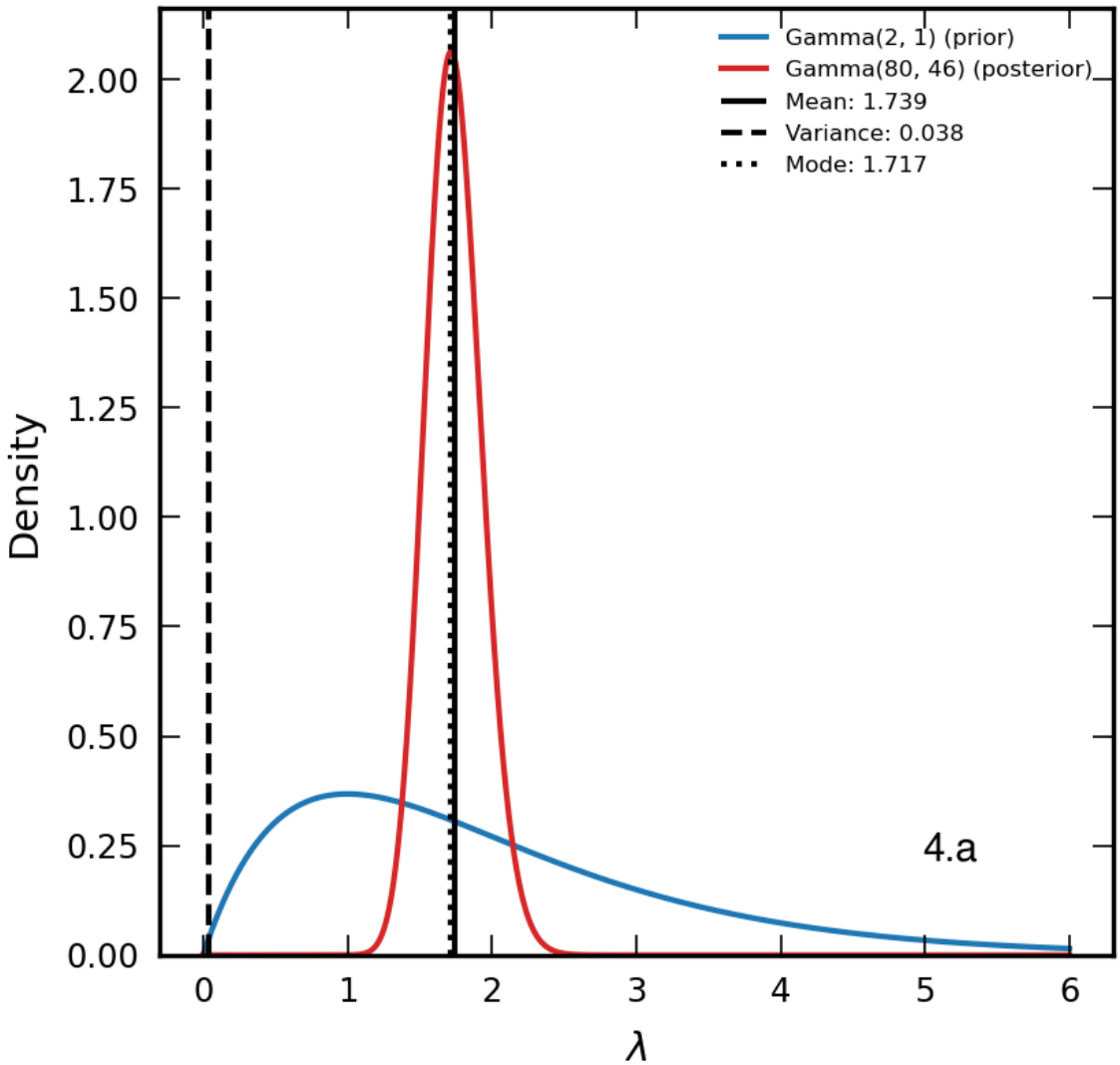
4.a

In [89]:

```
x_ = np.linspace(0,6, int(1e7))
p_gamma_prior = gamma.pdf(x_, 2, loc=0, scale=1)
p_gamma_posterior = gamma.pdf(x_, 80, loc=0, scale=1/46)
pos_mode = x_[gamma.pdf(x_, 80, loc=0, scale=1/46)==np.max(p_gamma_posterior)]
pos_mean,pos_var = gamma.stats(80, loc=0, scale=1/46, moments="mv")
```

In [100]:

```
f, ax1 = plt.subplots(1, 1, figsize=(3, 3), facecolor='white', dpi=300, gridspec_kw=
ax1.plot(x_, p_gamma_prior, color = "tab:blue",ls = '-', lw=1, label='Gamma(2, 1) (p
ax1.plot(x_, p_gamma_posterior, color = "tab:red",ls = '-', lw=1, label='Gamma(80, 4
ax1.vlines(pos_mean, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles='-',
ax1.vlines(pos_var, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles='--',
ax1.vlines(pos_mode, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles=':',


ax1.tick_params(axis='both', which='both', labelsize='xx-small', right=True, top=Tru
ax1.set_xlabel(r"$\lambda$", size='x-small')
ax1.set_ylabel("Density", size='x-small')
# ax1.set_xlim([0, 6])
ax1.set_ylim([0, np.max(p_gamma_posterior)+0.1])
ax1.legend(loc = 1 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
ax1.text(0.8, 0.1, "4.a", size=7, weight = 'bold',style=fig_style, family=fig_family

plt.show()
# f.savefig("./HW4_4a.jpg", bbox_inches = "tight")
```

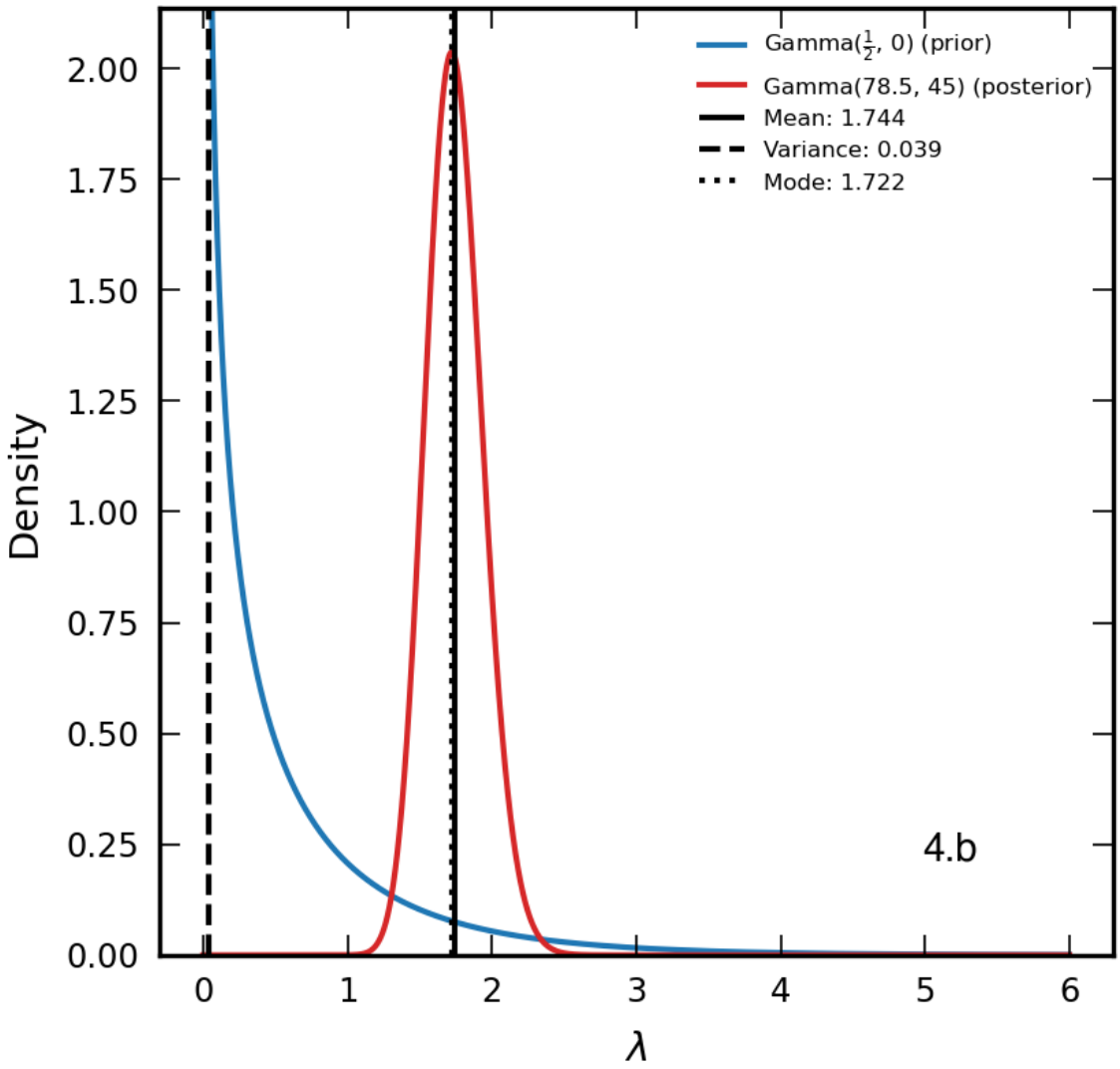


In [105]:

```
x_ = np.linspace(0,6, int(1e7))
p_gamma_prior = gamma.pdf(x_, 1/2, loc=0, scale=1)
p_gamma_posterior = gamma.pdf(x_, 78.5, loc=0, scale=1/45)
pos_mode = x_[gamma.pdf(x_, 78.5, loc=0, scale=1/45)==np.max(p_gamma_posterior)]
pos_mean,pos_var = gamma.stats(78.5, loc=0, scale=1/45, moments="mv")
```

In [106]:

```
f, ax1 = plt.subplots(1, 1, figsize=(3, 3), facecolor='white', dpi=300, gridspec_kw=
ax1.plot(x_, p_gamma_prior, color = "tab:blue",ls = '-', lw=1, label=r'Gamma($\frac{
ax1.plot(x_, p_gamma_posterior, color = "tab:red",ls = '-', lw=1, label='Gamma(78.5,
ax1.vlines(pos_mean, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles='-',
ax1.vlines(pos_var, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles='--',
ax1.vlines(pos_mode, 0, np.max(p_gamma_posterior)+1, colors='black', linestyles=':',


ax1.tick_params(axis='both', which='both', labelsize='xx-small', right=True, top=Tru
ax1.set_xlabel(r"$\lambda$", size='x-small')
ax1.set_ylabel("Density", size='x-small')
# ax1.set_xlim([0, 6])
ax1.set_ylim([0, np.max(p_gamma_posterior)+0.1])
ax1.legend(loc = 1 ,fontsize = 4,markerscale = 3,ncol = 1,scatterpoints= 1,frameon =
ax1.text(0.8, 0.1, "4.b", size=7, weight = 'bold',style=fig_style, family=fig_family

plt.show()
# f.savefig("./HW4_4b.jpg", bbox_inches = "tight")
```



In [ ]:

```
Initial = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8]
marks = ["o","^", "v", "<", ">", "*"]
figure, ax1 = plt.subplots(1, 1, figsize=(3, 3), facecolor='white', dpi=300, gridspe
for theta in Initial:
    j = 0
    i = 0
    convergence = []
    convergence.append(theta)
    while(i<10):
        f = math.factorial(197)/(math.factorial(125)*math.factorial(18)*math.factori
        logf = np.log(f)
        logf_prime = -38/(1-theta) + 125/(2+theta) + 34/theta
        theta = theta - logf/(logf_prime)
        convergence.append(theta)
        i = i + 1
    ax.plot(np.arange(11), convergence, marks[j], ms = 1)
    j = j + 1
```