HW3

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Homework 3

Question:

Suppose we want to estimate S=E$[X^2]$ when X has the density that is proportional to $q(x) = e^{-\frac{|x|^3}{3}}$.
1. Use rejection sampling to estimate E$[X^2]$. Describe your algorithm first. Attach your R code and output. Be sure to count and report your acceptance ratio.

Solution:
1. I first need to define an envelop function with a particular density function. I want to sample y from standard normal distribution. Then I need to find an α s.t.

$$\frac{1}{\alpha\sqrt{2\pi}}e^{-\frac{x^2}{2}} \geq e^{-\frac{|x|^3}{3}}$$

Take the algorithm:

$$\log \alpha\sqrt{2\pi} \leq \frac{|x|^3}{3} - \frac{x^2}{2}$$

, and $RHS \geq -\frac{1}{6}$, I take $\alpha = \frac{e^{-\frac{1}{6}}}{\sqrt{2\pi}}$
Then I get the envelop function

$$e(y) = \frac{g(y)}{\alpha} = e^{-\frac{x^2}{2}+\frac{1}{6}}$$

```
deci <- function(){
  y_i <- rnorm(1)
  q_y <- exp(-abs(y_i)^3/3)
  e_y <- exp(-y_i^2/2 + 1/6)
  u <- runif(1)
  if(u<=q_y/e_y){
    return(y_i)
  }else{return(NULL)}
}
```

Then I want to sample 1000, 10000 and 100000 y from f(x)

```
sSizes <- c(10000, 100000, 1000000)

for(sSize in sSizes){
  i = 0
  j = 0
  y <- c()
  while(i<=sSize){
    j = j + 1
    y_i <- deci()
```

```
    if(is.null(y_i)){
      next
    }else{
      y[i] <- y_i
      i = i+1
    }
  }
  estimation = sum(y^2)/sSize
  print(sprintf("After obtaining %d numbers from f(x), E[X^2] is estimated to be %.3f", sSize, estimatio
  print(sprintf("The acceptance ratio is: %.2f", sSize/j))
}
```

```
## [1] "After obtaining 10000 numbers from f(x), E[X^2] is estimated to be 0.779"
## [1] "The acceptance ratio is: 0.87"
## [1] "After obtaining 100000 numbers from f(x), E[X^2] is estimated to be 0.776"
## [1] "The acceptance ratio is: 0.87"
## [1] "After obtaining 1000000 numbers from f(x), E[X^2] is estimated to be 0.775"
## [1] "The acceptance ratio is: 0.87"
```

After using MC to estimate the integral of q(x), I find $f(x) = \frac{q(x)}{2.5758}$, and the acceptance rate is $\frac{\alpha}{c} = 0.869841$, which approximates with the accepatance above.