HW2

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Question 1

1. Use Simulation to approximate the following integrals. Describe your algorithm first. Attach your code and output.

- $$\int_1^3 \frac{x}{(1+x^2)^2}\,dx$$
- $$\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$$
- $$\int_{-1}^{1}\int_{-1}^{1} |x-y|\,dx\,dy$$

Solutions

Because we use

$$\frac{1}{n}\sum_{i=1}^{n} h(x)$$

to estimate E[h(X)].We should define h(X) and f(X) for each integral. (1) For the first integral The former intgral can be writtern as

$$\int_1^3 \frac{1}{2} * 2\frac{x}{(1+x^2)^2}\,dx$$

$$X \sim Uniform(1,3),\ f(x) = \frac{1}{2},\ 1 <= x <= 3$$

$$h(x) = \frac{2x}{(1+x^2)^2}$$

```
n_list <- c(1000, 10000, 100000,1000000)
for(n in n_list){
  # Sampling
  x <- runif(n,1,3)

  s <- 2*x/(1+x^2)^2
  outcome <- 1/n*sum(s)
  print(sprintf("The integral is %.4f with the sample size of %d", outcome, n))
}
```

```
## [1] "The integral is 0.1999 with the sample size of 1000"
## [1] "The integral is 0.1976 with the sample size of 10000"
## [1] "The integral is 0.1998 with the sample size of 100000"
## [1] "The integral is 0.2001 with the sample size of 1000000"
```

(2) For the second integral The former intgral can be writtern as

$$\int_0^\infty \frac{1}{2\sqrt{x}} e^{-x}\, dx$$

$$X \sim Exp(1),\ f(x) = e^{-x},\ x > 0$$

$$h(x) = \frac{1}{2\sqrt{x}}$$

```
n_list <- c(1000, 10000, 100000,1000000)
for(n in n_list){
  # Sampling
  x <- rexp(n,1)

  s <- 1/(2*sqrt(x))
  outcome <- 1/n*sum(s)
  print(sprintf("The integral is %.4f with the sample size of %d", outcome, n))
}
```

```
## [1] "The integral is 0.8959 with the sample size of 1000"
## [1] "The integral is 0.8913 with the sample size of 10000"
## [1] "The integral is 0.8914 with the sample size of 100000"
## [1] "The integral is 0.8870 with the sample size of 1000000"
```

(3) For the third integral The former intgral can be writtern as

$$\int_{-1}^{1} \frac{1}{2} \int_{-1}^{1} \frac{1}{2} 4|x-y|\, dx\, dy$$

$$X \sim Uniform(-1,1),\ Y \sim Uniform(-1,1)$$

$$f(x,y) = \int_{-1}^{1}\int_{-1}^{1} \frac{1}{4}\, dx\, dy,\ x > 0$$

$$h(x,y) = 4|x-y|$$

```
n_list <- c(1000, 10000, 100000,1000000)
for(n in n_list){
  # Sampling
  x <- runif(n,-1,1)
  y <- runif(n,-1,1)

  s <- 4*abs(x-y)
  outcome <- 1/n*sum(s)
  print(sprintf("The integral is %.4f with the sample size of %d", outcome, n))
}
```

```
## [1] "The integral is 2.7443 with the sample size of 1000"
## [1] "The integral is 2.6518 with the sample size of 10000"
## [1] "The integral is 2.6634 with the sample size of 100000"
## [1] "The integral is 2.6674 with the sample size of 1000000"
```

Question 2

Consider the model given by

$$X \sim lognormal(0,1) \quad and\ logY = 9 + 3logX + e\,, e \sim N(0,1)$$

is independt of X. Use simulation to estimate E[Y/X].
Let Z be the standard normal distribution, then f(x,z) = f(x)f(z) because of the independence of x and z.

$$h(x,z) = \frac{Y}{X} = e^{9+z}x^2$$

$$E[Y/X] = \iint_{X,Z} h(x,z)f(x,z)\, dxdy = \int_X x^2 f(x)dx \int_Z e^{9+z} f(z)dz$$

```r
n_list <- c(1000, 10000, 100000,1000000,10000000)

for(n in n_list){
  # Sampling
  x <- rlnorm(n,0,1)
  z <- rnorm(n,0,1)
  s1 <- sum(x^2)/n
  s2 <- sum(exp(9+z))/n
  outcome <- s1*s2
  print(sprintf("The integral is %.4f with the sample size of %d", outcome, n))
}
```

```
## [1] "The integral is 97330.5124 with the sample size of 1000"
## [1] "The integral is 97778.6645 with the sample size of 10000"
## [1] "The integral is 96550.9683 with the sample size of 100000"
## [1] "The integral is 97928.9744 with the sample size of 1000000"
## [1] "The integral is 98745.8364 with the sample size of 10000000"
```