Laborator 11-12.12.2023

Ex 1

1) d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y(x) = 0$; $y(0) = 1$; $\frac{dy}{dx}(0) = 3$

Ec. caracteristică asociată

$r^2 + 4r + 4 = 0$

$\Delta = 0 \Rightarrow r_1 = r_2 = -\frac{4}{2} = -2$

Soluția generală a ec. omogene

$y(x) = C_1 \cancel{(x)} \cdot e^{rx} + C_2 \cdot x \cdot e^{rx} = C_1 e^{-2x} + \cancel{2x}\, x\, C_2 e^{-2x}$

$y(0)=1$

$C_1=1$

$\frac{dy}{dx}(0)=3.$

$\frac{dy}{dx}=\cancel{\frac{dy}{dx}}-2\underset{1}{C_1}\cdot e^{-2x}+C_2\cdot e^{-2x}-2x\cdot C_2\cdot e^{-2x}$

$-2e^{-2x}+C_2\cdot e^{-2x}-2x\,C_2\cdot e^{-2x}$

$\frac{dy}{dx}(0)=-2\cdot 1+C_2=3$

$-2+C_2=3$

$\boxed{C_2=5}$

Solutia pb cu conditii initiale

$y(x)=e^{-2x}\cancel{-2x\cdot 5}\cdot -5x\cdot e^{-2x}$

e) $\frac{d^2y}{dx^2}+\frac{2dy}{dx}+5y(x)=0$

Ec. caracteristică

$y^2+2y+5=0$

$\Delta=4-5\cdot 4=-16=16i^2$

$a=-1$

$b=2i$

$y_1=\frac{-2-4i}{2}=\frac{+2(-1-2i)}{2}=-1-2i$

$y_2=-1+2i$

$y(x)=e^{-1\cdot x}(c_1\cos 2x+c_2\cdot \sin 2x)$

3) a)

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} - 3y(x) = 5e^{-3x} \quad (1)$$

I. Soluția generală a ec. omogene

$r^2 + 2r - 3 = 0$

$\Delta = 4 + 12 = 16$

$r_1 = \frac{-2-4}{2} = -3$

$r_2 = \frac{-2+4}{2} = 1$

$y_{omogen} = c_1 e^{-3x} + c_2 e^{x}$

II. Soluția particulară a ec. neomogene (Metoda coeficienților nedeterminați)

$f(x) = 5e^{-3x}$

$\cancel{C(x)} = C x^m \cdot e^{rx} \Rightarrow C = 5$

$r = -3 \Rightarrow \Delta = 1$

$x^m = 1 \Rightarrow m = 0.$

$y_p(x) = \cancel{x}$

$x \cdot A_0 \cdot e^{-3x}$

$\frac{dy_p}{dx} = A_0 e^{-3x} - 3 \cdot A_0 e^{-3x}$

$\frac{d^2y_p}{dx^2} = -3A_0 e^{-2x} - 3A_0 e^{-3x} + 9xA_0 e^{-3x}$

Înlocuim în (1)

$-3A_0 e^{-3x} - 3A_0 e^{-3x} + 9xA_0 e^{-3x} + 2A_0 e^{-3x} - 6xA_0 e^{-3x} - 3xA_0 e^{-3x} = 5e^{-3x}$

$-4A_0 \cancel{e^{-3x}} = 5\cancel{e^{-3x}}$

$-4A_0 = 5$

$A_0 = \frac{-5}{4}$

$y_p = \frac{-5}{4} \cdot x \cdot e^{-3x}$

$y_p = x^2(A_1 x + A_0) e^{3x}$

$\frac{dy_p}{dx} =$

III soluția generală a ec. neomogene:

$y = C_1 e^{-3x} + C_2 e^{x} - \frac{5}{4} x e^{-3x}$.

$\frac{dy_p}{dx} = \left(x^2(A_1x + A_0) \cdot e^{3x}\right)' = 2x(A_1x + A_0) \cdot e^{3x} + x^2(A_1$

$\frac{dy}{}$

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y(x) = 0$

$r^2 - 2r = 2 = 0$

$\Delta = -4 = 4i \,;\, \sqrt{\Delta} = 2i$

$r_1, r_2 = 1 \pm i$

$a = 1$

$b = 1$

$y_{omogen} = c_1 e^{x} \cdot \cos x_1 + c_2 \cdot e^{x} \sin x$

$f(x) = 5 e^{2x} \cos x$

$C x^{mx} e^{a \cdot x} \cos bx \Rightarrow C = 5$

$x^{m} = 1 \Rightarrow m = 0$

$e^{ax} = e^{2x} \Rightarrow a = 2$

$b = 1$ $\Rightarrow s = 0.$

$y_p(x) = x^{0} \cdot e^{2x} \cdot A_0 \cdot \cos bx + x^{0} e^{2x} \cdot B \cdot \sin bx$

$= e^{2x} A_0 \cdot \cos bx + e^{2x} B_0 \sin bx$

$\frac{dy_p}{dx} = \cancel{2e^{2x} A_0}$

$= 2e^{2x} \cdot A_0 \cos bx + e^{2x} \cdot A \cdot (-\sin bx) + 2e^{2x} B_0 \sin x \cdot$

$+ e^{2x} b \cos bx \cdot B_0$

$= \cancel{2}$

$\frac{d^2 y_p}{dx^2} = 4e^{2x} A \cos x - 2e^{2x} A \sin x - 2 e^{2x} A \sin x - e^{2x} A \cdot \cos x$

$+ 4e^{2x}$

$= 2e^{2x} B \cos x + 2e^{2x} B \cos x - e^{2x} B \cdot \sin x =$

$= + 3e^{2x} A \cdot \cos x - 4e^{2x} A \sin x + 3e^{2x} B \sin x + 4 e^{2x}$

$+x \cdot b_1 \sin x + b_1 \cos x$

Laborator 4

$$\frac{d^4 y}{dx^4} + \frac{2 d^2 y}{dx^2} + y(x) = 0$$

Ec. caract.:

$r^4 + 2r^2 + 1 = 0$

$\cancel{r^4 - 1 + 2r^2 + 2 = 0}$

$(r^2+1)^2 = 0 \Rightarrow r^2 + 1 = 0 \Rightarrow r^2 = -1 \Leftrightarrow$

$r_1 = i \Rightarrow m_1 = 2$; $r \cdot a = 0$ $\Rightarrow$ polinom de grad $m_1 - 1$

$r_2 = -i \Rightarrow m_2 = 2$

$\Rightarrow$ grad $= 1$.

Soluția omogenă $y(x) = (A_0 x + A_1) \cdot e^{0 \cdot x} \cos x + (B x + B_1) e^{0x} \sin x$

$y(x) = (A x + A_1) \cos x + (B x + B) \sin x$

b) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y(t) = 0$; $y(0) = 0$; $\frac{dy}{dx}(0) = 1$

$\frac{d^2y}{dx^2}(0) = 2$.

Ec. caracteristică

$r^3 - 2r^2 - r + 2 = 0$

$r^2(r-2) - (r-2) = 0$

$(r-2)(r^2-1) = 0$

$r - 2 = 0 \Rightarrow r_1 = 2$; $m_1 = 1$

$r^2 - 1 = 0 \Rightarrow r_2 = 1$; $m_2 = 1$

$r_3 = -1$; $m_3 = 1$

Sol. ec. omogene:

$y(x) = C_1 \cdot e^{2x} + C_2 \cdot e^{x} + C_3 \cdot e^{-x}$

$y(0) = 0 \Leftrightarrow C_1 \cdot e^0 + C_2 \cdot e^0 + C_3 \cdot e^0 = 0$

$C_1 + C_2 + C_3 = 0$ (1).

~~$\frac{dy}{dx}(0) = 1 \Leftrightarrow$~~

$\frac{dy}{dx} = C_1 \cdot 2 \cdot e^{2x} + C_2 \cdot e^{x} - C_3 \cdot e^{-x}$

$\frac{dy}{dx}(0) = 1 \Leftrightarrow 2C_1 + C_2 - C_3 = 1$ (2)

$\frac{d^2y}{dx^2} = 4C_1 \cdot e^{2x} + C_2 \cdot e^{x} + C_3 \cdot e^{-x}$

$\frac{d^2y}{dx^2}(0) = 2 \Leftrightarrow 4C_1 + C_2 + C_3 = 2$ (3).

$$\begin{cases} c_1 + c_2 + c_3 = 0 \\ 2c_1 + c_2 - c_3 = 1 \\ 4c_1 + c_2 + c_3 = 2 \end{cases} \iff \Bigg\{$$

2)b)

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y(x) = e^{-2x}$$

I. Solutia generală a ec. omogene.

$r^3 + 3r^2 - 4 = 0$

$r^3 - 1 + 3r^2 - 3 = 0$

$(r-1)(r^2+r+1) + 3(r-1)(r+1) = 0$

$(r-1)(r^2+r+1+3r)=0$

• $r = ~~1 = 0 \Rightarrow~~ r_1 = 1$

• $~~r^2 + r + 4 = 0 \Rightarrow \Delta = 1 - 16 = -15 = 15i^2 \Rightarrow r_2 = \sqrt{15}i~~$

$~~r_3 = \sqrt{15}i~~$

$r - 1 = 0 \Rightarrow r_1 = 1$

$r^2 + 4r + 4 = 0 \Rightarrow r_{2,3} = ~~-2~~ 2$

$(r-1)(r^2 + r + 1 + 3r + 3) = 0$

$r^2 + 4r + 4 = 0$

$\Delta = 4 -$

$(r+2)(r^2 + 2r - 2) = 0 \Rightarrow r_{2,3} = -2 \Rightarrow m = 2$

$r_{2,3} = 1$

	r^3	r^2	r^1	r
	1	3	0	-4
1	1	4	4	0

$(r-1)(r^2+4r+4)=0.$

Sol gen a ec omogene

$y_{omogen} = (A_1x + A_0)e^{-2x} + A \cdot e^{x}$

2 Solutie gen part a ec neomogene (MCN)

$y_{part} =$

$f(x) = e^{-2x}$

$c \cdot x^m \cdot e^{rx} = e^{-2x} \Rightarrow C = 1$

$m = 0$

$r = -2 \Rightarrow \Delta = 2$

$y_{part}(x) = x^{\Delta} \cdot A \cdot e^{rx} = x^2 A \cdot e^{-2x}$

$$\frac{dy_p}{dx_p} = 2xAe^{-2x} - 2x^2Ae^{-2x}$$

$$\frac{d^2y_p}{dx_p^2} = 2Ae^{-2x} - 4xe^{-2x}A - 4xAe^{-2x} + 4x^2Ae^{-2x}$$

$$= +2Ae^{-2x} - 8xe^{-2x}A + 4x^2Ae^{-2x}$$

$$\frac{d^3y_p}{dx_p^3} = -4Ae^{-2x} - 8Ae^{-2x} + 16xAe^{-2x} + 8x^2Ae^{-2x} + 8x^2Ae^{-2x}$$

$-12Ae^{-2x}+24xAe^{-2x}-8x^2A\cdot e^{-2x}$

Înlocuim în ec de mai sus

$-12Ae^{-2x}+\cancel{24xAe^{-2x}}-8x^2Ae^{-2x}+3(2Ae^{-2x}-\cancel{8xe^{-2x}A}+4x^2Ae^{-2x})$

$-4x^2Ae^{-2x}=e^{-2x} \mid : e^{-2x}$

$\cancel{-12}-6A\ \cancel{12Ax^2}=1 \Rightarrow A=\frac{-1}{6}$

sol part: $y_p(x)=\frac{-1}{6}x^2\cdot e^{-2x}$

III Sol gen a ec neomogene:

$y(x)=y_{omogen}(x)+y_p(x)$

d) $\frac{d^3y}{dx^3}-\frac{dy}{dx}=\sin 3x$ (1)

I Soluția ec gen ec ~~caract~~ omogenă

$r^3-r=0$

$r(r^2-1)=0 \Rightarrow r_1=0 \quad m_1=1$

$r_{2,3}=\pm 1, \quad m_2=1$

$y_{omogen}(x)=c_1+c_2e^{-x}+c_3e^{x} \quad m_3=1.$

II. Soluția part a ec. neomogene

$y_{part}=$

$f(x)=\sin 3x \qquad c\cdot x^m\cdot e^{ax}\cdot\sin bx=\sin 3x$

$c=1 \quad ; a=0 \qquad z=a+bi$

$m=0 \qquad b=3. \qquad =3i \Rightarrow$

$$y_p(x) = x^0 \cdot A_0 \cdot e^0 \cdot \sin 3x + x^0 \cdot A_1 \cdot e^0 \cos 3x$$

$$A_0 \sin 3x + A_1 \cdot \cos 3x = \cancel{\sin 3x}$$

$$\frac{dy_p}{dx} = \cancel{A \sin 3} \cdot A_0 \cdot 3\cos 3x - 3A_1 \sin 3x$$

$$\frac{d^2y_p}{dx^2} = -9A_0 \sin 3x - 9A_1 \cos 3x$$

$$\frac{d^3y_p}{dx^3} = -27A_0 \overset{\cos 3x}{\cancel{\sin 3x}} + 27A_1 \sin 3x.$$

Inlocuim in (1)

$$-27A_0 \cos 3x + 27A_1 \sin 3x - 3A_0 \cos 3x + 3A_1 \sin 3x = \sin 3x.$$

$$-30A_0 \cos 3x + 30A_1 \sin 3x = \sin 3x. \mid :$$

$$-30A_0 \cos 3x + 30A_1 \sin 3x - \sin 3x = 0.$$

$$-30A_0 \cos 3x + \sin 3x(30A_1 - 1) = 0.$$

$$\begin{cases} 30A_1 - 1 = 0 \Rightarrow A_1 = \frac{1}{30} \\ -30A_0 = 0 \Rightarrow A_0 = 0. \end{cases}$$

$$y_{part} = \frac{-1}{30} \sin 3x.$$

III Sol gen a ec neomogene

$$y(x) = y_{omogen} + y_{part}$$