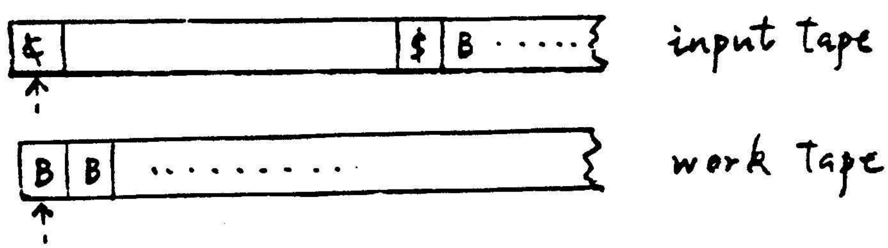
## Resource Bounded TM's

△ **A TM has unlimited resources: time and space.**
**In practice, we need to restrict them.**

△ **TMs with restricted resources are the topic of complexity theory. Here, we briefly introduce the basic concepts of $P$, $NP$, etc. .**

△ **We use the off-line DTMs and NTMs which have been introduced before. An off-line TM (DTM or NTM) has**
— **a read-only input tape;**
— **a read-write work tape; (storage)**



**Note:**

1. **There are two end-markers at the two ends of the input word. The read-only head never moves out of the two ends.**
2. **Both heads can make three kinds of moves, i.e.,** $L, \ R, \ \lambda$.
3. **The work tape is initially blank.**

$\triangle$ **Given an off-line DTM** $M$**, if each accepted word of length n causes** $M$ **to visit at most** $S(n)$ **distinct cells on the work tape, then** $M$ **is said to be an** $S(n)$ **space-bounded DTM.**

**Note:**

**(1) We only consider accepted words.**

**(2)** $S(n)$ **is a function from** $N$ **to** $N$**.**

**(3) We only count the cells used on the work tape.**

$\triangle$ **Given an off-line DTM $M$, if each accepted word of length $n$ causes $M$ to compute at most $T(n)$ steps before $M$ accepts, then $M$ is said to be a $T(n)$ time-bounded DTM.**

$\triangle$ **Given an off-line NTM $M$, if each accepting configuration sequence visits at most $S(n)$ distinct cells on the work tape, then $M$ is said to be an $S(n)$ space-bounded NTM.**

$\triangle$ **Similarly, we define $T(n)$ time-bounded NTMs.**

$\triangle$ **A language $L$ is of**

**(1) deterministic–space complexity $S(n)$, if $L = L(M)$ and $M$ is $S(n)$ space-bounded off-line DTM.**

**(2) deterministic-time complexity $T(n)$.**

**(3) nondeterministic-space complexity $S(n)$.**

**(4) nondeterministic-time complexity $T(n)$.**

**Attention has focused on <u>polynomial functions</u>, since these appear to be the the most practical.**

**<u>Example:</u>**

| $n$ | 1 | 10 | 20 | ... |
|---|---|---|---|---|
| $100n^2$ | 100 | 10000 | 40000 | ... |
| $2^n$ | 2 | 1024 | 1048576 | ... |

**We define the following**
**four families of languages:**

**(1)** $\mathcal{L}_{DPSPACE} = \{L \mid L$ **has det.-space complexity** $S(n),$ **for some polynomial** $S(n)\}$

**(2)** $\mathcal{L}_{DPTIME} = \{L \mid L$ **has deterministic-time complexity** $T(n),$ **for some polynomial** $T(n)\}$

**(3)** $\mathcal{L}_{NPSPACE} = \{L \mid L$ **has nondet.-space complexity** $S(n),$ **for some polynomial** $S(n)\}$

**(4)** $\mathcal{L}_{NPTIME} = \{L \mid L$ **has nondet.-time complexity** $T(n),$ **for some polynomial** $T(n)\}$

△ **We know that**

$$\mathcal{L}_{DPSPACE} \subseteq \mathcal{L}_{NPSPACE}$$

**and**

$$\mathcal{L}_{DPTIME} \subseteq \mathcal{L}_{NPTIME}$$

**Why?**

$\triangle$ **We also know that**

$$\mathcal{L}_{DPTIME} \subseteq \mathcal{L}_{DPSPACE}$$

**and**

$$\mathcal{L}_{NPTIME} \subseteq \mathcal{L}_{NPSPACE}$$

**Why?**

$\triangle$ **It has been proven that**

$$\mathcal{L}_{DPSPACE} = \mathcal{L}_{NPSPACE}$$

**and that**

$$\mathcal{L}_{NPSPACE} \subset \mathcal{L}_{REC}$$

$\triangle$ **So, we have**

$$\underbrace{\mathcal{L}_{DPTIME}}_{P} \subseteq \underbrace{\mathcal{L}_{NPTIME}}_{NP} \subseteq \overbrace{\mathcal{L}_{DPSPACE}}^{\mathcal{L}_{NPSPACE}} \subset \mathcal{L}_{REC}$$

**Corresponding to these four families of languages, we have four <u>classes</u> of problems:**

**(1) PSPACE : the class of all problems that can be solved in deterministic polynomial space.**
**(2) P : the class of all problems that can be solved in deterministic polynomial time.**
**(3) NPSPACE : . . . in nondeterministic polynomial space**
**(4) NP: . . . in nondeterministic polynomial time.**

$$P \subseteq NP \subseteq PSPACE = NPSPACE$$

**problems $\Leftrightarrow$ decision problems $\Leftrightarrow$ languages**

$$P? =?NP$$

$\triangle$ **We define a relation $\prec$ on problems (languages).**

**For $L_1, L_2$ in $\mathcal{L}_{REC}$,**

$$L_1 \prec L_2$$

**If $L_1$ requires no more time than $L_2$ to accept.**

$\triangle$ $\alpha$ **is transitive and reflexive.**
**It is a pre-order.**

$\triangle$ $L$ **is said to be NP-hard if $L' \ \alpha \ L$ for all $L'$ in $\mathcal{L}_{NPTIME}$.**
$\triangle$ $L$ **is said to be NP-complete if it is also in $\mathcal{L}_{NPTIME}$.**

**A clear concept of NP-completness was given by Steven Cook. He showed that satisfiability problem is NP-complete.**

**SATISFIABILITY PROBLEM (SAT)**
**INSTANCE : A set $U$ of variables and a collection $C$ of clauses over $U$.**
**QUESTION : Is there a satisfying truth assignment for $C$?**

**SAT is NP-complete.**