Exercises for class #9

Write your answer on latex and prepare a pdf file. The file should be clearly written and organized. Make sure the figures are readable. You should think of that file as a short version of a paper. The appendix of that file should include your code (it can be in MATLAB, C, Fortran, Python, Julia, or anything you want). Be prepared to explain your code in class. Email me that pdf file before the deadline.

Capital income taxation

Agents live forever. They supply one unit of labor inelastically. They receive the wage w. They save in physical capital $k \geq 0$ (no borrowing) that they rent every period to the firms at the rate r.

A household productivity z takes one of these 11 values:

$$Z = [0.26061824, 0.3356399, 0.43225732, 0.55668708, 0.71693522, 0.92331246, \\ 1.18909753, 1.53139158, 1.97221853, 2.53994208, 3.27109075]$$

and the transition matrix is

$$\begin{aligned}\Pi_z = [&0.776, 0.199, 0.023, 0.002, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000 \\ &0.020, 0.781, 0.180, 0.018, 0.001, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000 \\ &0.001, 0.040, 0.785, 0.160, 0.014, 0.001, 0.000, 0.000, 0.000, 0.000, 0.000 \\ &0.000, 0.002, 0.060, 0.787, 0.140, 0.011, 0.000, 0.000, 0.000, 0.000, 0.000 \\ &0.000, 0.000, 0.003, 0.080, 0.789, 0.120, 0.008, 0.000, 0.000, 0.000, 0.000 \\ &0.000, 0.000, 0.000, 0.005, 0.100, 0.789, 0.100, 0.005, 0.000, 0.000, 0.000 \\ &0.000, 0.000, 0.000, 0.000, 0.008, 0.120, 0.789, 0.080, 0.003, 0.000, 0.000 \\ &0.000, 0.000, 0.000, 0.000, 0.000, 0.011, 0.140, 0.787, 0.060, 0.002, 0.000 \\ &0.000, 0.000, 0.000, 0.000, 0.000, 0.001, 0.014, 0.160, 0.785, 0.040, 0.001 \\ &0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.001, 0.018, 0.180, 0.781, 0.020 \\ &0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.002, 0.023, 0.199, 0.776].\end{aligned}$$

Other parameters:

- Preference are CRRA with coefficient of relative risk aversion = 2.0.
- Discount factor β = 0.964.
- Production function is $Y = K^{\alpha}$ with $\alpha = 0.3$.
- The depreciation $\delta = 0.1$.

The firm solves the following problem:

$$Profits = \max_{K,L}(1-\tau)\left[K^{\alpha}L^{1-\alpha} - wL - \delta K\right] - rK$$

and from the FOCs and setting $L = 1$ we have

$$r(K) = (1-\tau)\left[\alpha K^{(\alpha-1)} - \delta\right],$$
$$w(K) = (1-\alpha)K^{\alpha}.$$

1. Steady state analysis (40 points)

a. (10 points)

Set $\tau = 0.05$. Assume $r = 0.029$. Make a grid for k with at least 250 points, starting from 0 and up to $\bar{k} = 100$. Solve for the policy function $k'(k, z)$. You can use your favorite method here. Do a plot of k' as a function of k showing 3 lines, one for each value of $z = \{0.43225732, 0.92331246, 1.97221853\}$.

b. (10 points)

Using the method I discussed in class to iterate on the probability density function, compute the distribution over individuals' capital k and productivity z_i. Let's refer to the distribution as $m_i(k)$. Plot the histogram for k. Compute total capital supply of the economy as

$$K^S = \sum_{i=1}^{11}\int_0^{\bar{k}} k m_i(k) dk.$$

Notice that this is the value associated with $r = 0.029$, which we will refer to as $K^S(0.029)$.

Compute the difference $\text{diff}(0.029) = r(K^S(0.029)) - 0.029$ where 0.029 was the assumed value of r. Note that they are equal, you found the steady state value of r and K.

c. (10 points)

Write a function that takes r as an input and return $\text{diff}(r)$. Then use you prefer root finding algorithm to find $\text{diff}(r^*) = 0$. Report the following aggregate variables: C, Y, K, r, w.

d. (10 points)

Set $\tau = 0.0$. Recompute the steady state equilibrium of the model. Then report the percent change in following aggregate variables because of the decline in taxes by 5 percentage points: C, Y, K, r, w.

2. An economic boom

Imagine the economy is in the steady state. The government wants to generate an economic boom in the economy so it is thinking about announcing that it will reduce taxes from 5% to 0% for a number of periods. It is considering announcing for either 4 or 8 years. You are the economist in charge of evaluating the idea. Your job is to figure out what would be the change in the aggregate variables C, Y, K, r, w during the next 50 years under the two alternative plans.

You will use the SSJ method that we discussed in the last class. The first step is to solve the steady state with taxes at 5%. That is exactly what you did in the previous questions.

a. DAG (10 points)

The second step in the SSJ method is to construct the DAG representation for this economy. Draw the DAG representation with the asset market clearing condition ($H = K^S - K$) making explicit the role of taxes. Where do they enter and why?

b. Simple Jacobians (10 points)

The next step is computing the Jacobians. In this question, you just need to compute and report the Jacobians for the representative firm block.

c. Fake News Algorithm (20 points)

Compute and report the Jacobians for the households block. Given that the DAG is for the asset market clearing condition, what are the Jacobians you need to compute?

Notice that the periods for which the aggregate variables would change go up to 50 but the shocks go up to 8 periods.

In this question I am asking you to use **numerical differentiation**. The centered difference formula is

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}.$$

d. GE (10 points)

Now you want to forward accumulate Jacobians along the DAG to obtain the Jacobians of H, and then solve for the change in dK for a given $d\tau$. Notice that here you are solving for the dK that clear the market every period. Show the formula that you used. Plot the percent deviation in K for the next 50 years following the announcement under the two alternative plans.

e. IRS (10 points)

Use Jacobians to back out the impulse response functions for the aggregate variables C, Y, r, w for the next 50 years following the announcement under the two alternative plans. Show the formulas that you used. Plot the percent deviation from steady state for the variable of interest for the next 50 years following the announcement under the two alternative plans. Interpret the results.