Numerical Mathematics Lab session 6

Stefan Evanghelides (s2895323) and Frank te Nijenhuis (S2462575)

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1 Preparation

1.1 The θ-method

2.

$e_1 = |u_1 - y_1| = |(y_1 - u_1^*) + (u_1^* - u_1)| = |h \cdot \tau(h) + (u_1^* - u_1)| \rightarrow$ It should be $\tau_1(h)$ but it's maximum, so it is also $\tau(h)$

$$e_1 \leq |h \cdot \tau(h)| + |(u_1^* - u_1)| \leq h \cdot |\tau(h)| + (1 + hL) \cdot |e_0| = h \cdot |\tau(h)|$$

$|u_1 - y_1| = |y_0 + \frac{h}{2}(f(t_0, y_0) + f(t_1, u_1)) - y_1| = |y_0 - y_1 + \frac{h}{2}(f(t_0, y_0) + f(t_1, u_1))| \leq |y_0 - y_1| + \frac{hL}{2}|y_0 - u_1| \leq |y_0 - y_1| +$

$$|f(t, y_0) - f(t, y_1)| \leq L|y0 - y1| \Rightarrow |y_0 - y_1| \geq \frac{|f(t, y_0) - f(t, y_1)|}{L}$$

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1.2 The n-body method

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2 Discussion

2.1 The θ-method

1. The θ-method with $\theta = 0$ gives $u_{n+1} = u_n + h(0 \cdot f_{n+1} + (1-0)f_n) = u_n + h \cdot f_n$ which exactly defines the forward Euler method. This method is consistent and converges with order 1, and it is absolutely stable (the effective step size of the solution is less than 1) in the region of the complex plane defined by $h\lambda \in \mathbb{C}$ s.t. $|1 + h\lambda| < 1$. Similarly, $\theta = 1$ gives $u_{n+1} = u_n + h \cdot f_{n+1}$, which defines the backward Euler method. This is another first order method, which is actually absolutely stable on the entire negative complex plane except for the circle of radius 1 centered at 1 in the complex plane. Finally, $\theta = \frac{1}{2}$ gives us $u_{n+1} = u_n + \frac{h}{2}(f_{n+1} + f_n)$, the Crank-Nicolson method. This method is second order consistent and is absolutely stable in the left plane.
2.
3. Depending on our choice of λ, we will land somewhere in the complex plane with $h\lambda$. If we take $\Re(\lambda) < 0$, we can use any of the methods depending on the specific value of $h\lambda$ and the computational cost.

2.2 The n-body method

2.2.1 *Two body problem*

1. The trapezoidal method has the same properties as the Crank-Nicolson method mentioned earlier. It is absolutely stable whenever $\Re(\lambda) < 0$, assuming of course that $h > 0$. It is second order accurate. Heun's method is also second order accurate. It has a smaller stability region, shaped as an ellipse (RK2).
2.Netwons method is exact, taking slightly longer time to compute. Broyden methond is an approximation of the Newton method, which almost as accurate as Newton's method, but slightly faster to compute.3

2.2.2 *The solar system*

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