Príklad z prednášky - 4 x 4 sústava

Riešte sústavu rovníc iteračnými metódami. Sústava nie je diagonálne dominantná v zadanom tvare.

$$
\begin{array}{rrrrl}
-1\,x_1 & +4\,x_2 & & -1\,x_4 & = 2 \\
+4\,x_1 & -1\,x_2 & -1\,x_3 & & = 1 \\
-1\,x_1 & & +4\,x_3 & -1\,x_4 & = 0 \\
 & -1\,x_2 & -1\,x_3 & +4\,x_4 & = 1
\end{array}
$$

Ukážeme si najskôr ako je možné túto sústavu riešiť neprogramátorskymi technikami - na hulváta - len definujem matice a spustím jeden cyklus. Potom si ukážeme aj programátorsky slušné riešenie.

Ak maticu ponecháme v originálnom tvare (nie je diagonálne dominantná) tak iteračný proces nebude fungovať.

In[216]:=
```
A = {{-1, 4, 0, -1}, {4, -1, -1, 0}, {-1, 0, 4, -1}, {0, -1, -1, 4}};
A // MatrixForm
```

Out[217]//MatrixForm=

$$
\begin{pmatrix} -1 & 4 & 0 & -1 \\ 4 & -1 & -1 & 0 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix}
$$

```
b = {2, 1, 0, 1};
b // MatrixForm
```

$$
\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}
$$

In[210]:=
```
LinearSolve[A, b] // N
```

Out[210]= {0.5, 0.75, 0.25, 0.5}

Definujeme si matice H a upravený vektor pravých strán tak, ako bolo odporúčané na prednáške. Z prvej rovnice si vyjadríme x_1 , z druhej rovnice si vyjadríme x_2 atď...

$$
\begin{array}{rrrrl}
-1\,x_1 & +4\,x_2 & & -1\,x_4 & = 2 \\
+4\,x_1 & -1\,x_2 & -1\,x_3 & & = 1 \\
-1\,x_1 & & +4\,x_3 & -1\,x_4 & = 0 \\
 & -1\,x_2 & -1\,x_3 & +4\,x_4 & = 1
\end{array}
$$

Vymeníme prvú a druhú rovnicu, aby sme zabezpečili diagonálnu dominantnosť matice A

$$
\begin{array}{rrrrl}
+4\,x_1 & -1\,x_2 & -1\,x_3 & & = 1 \\
-1\,x_1 & +4\,x_2 & & -1\,x_4 & = 2 \\
-1\,x_1 & & +4\,x_3 & -1\,x_4 & = 0 \\
 & -1\,x_2 & -1\,x_3 & +4\,x_4 & = 1
\end{array}
$$

Vyjadríme si jednotlivé premenné

$$
\begin{array}{rrl}
+4\,x_1 & & = 1 \quad +1\,x_2 + 1\,x_3 \\
 & +4\,x_2 & = 2 \quad +1\,x_1 \quad\quad\quad +1\,x_4
\end{array}
$$

$$
\begin{array}{llllll}
 & +4\,x_3 & & = 0 & +1\,x_1 & +1\,x_4 \\
 & & +4\,x_4 & = 1 & +1\,x_2\ +1\,x_3 &
\end{array}
$$

$$
\begin{array}{ll}
+x_1 & = 1/4 \times (1 \quad +1\,x_2 + 1\,x_3 \quad) \\
\quad +x_2 & = 1/4 \times (2 \ +1\,x_1 \quad +1\,x_4\ ) \\
\qquad +x_3 & = 1/4 \times (0 \ +1\,x_1 \quad +1\,x_4\ ) \\
\qquad\quad +x_4 & = 1/4 \times (1 \quad +1\,x_2 \ +1\,x_3 \quad)
\end{array}
$$

In[223]:=
```
H = 1 / 4 {{0, 1, 1, 0}, {1, 0, 0, 1}, {1, 0, 0, 1}, {0, 1, 1, 0}}
g = 1 / 4 {1, 2, 0, 1}
```

Out[223]= $\left\{\left\{0, \frac{1}{4}, \frac{1}{4}, 0\right\}, \left\{\frac{1}{4}, 0, 0, \frac{1}{4}\right\}, \left\{\frac{1}{4}, 0, 0, \frac{1}{4}\right\}, \left\{0, \frac{1}{4}, \frac{1}{4}, 0\right\}\right\}$

Out[224]= $\left\{\frac{1}{4}, \frac{1}{2}, 0, \frac{1}{4}\right\}$

In[228]:=
```
Clear[x]
x[0] = {0, 0, 0, 0}
Do[
 x[k + 1] = H .x[k] + g // N;
 Print[k + 1, ". iteracia: x=", x[k + 1]], {k, 0, 15}]
```

Out[229]= {0, 0, 0, 0}

```
1. iteracia: x={0.25, 0.5, 0., 0.25}
2. iteracia: x={0.375, 0.625, 0.125, 0.375}
3. iteracia: x={0.4375, 0.6875, 0.1875, 0.4375}
4. iteracia: x={0.46875, 0.71875, 0.21875, 0.46875}
5. iteracia: x={0.484375, 0.734375, 0.234375, 0.484375}
6. iteracia: x={0.492188, 0.742188, 0.242188, 0.492188}
7. iteracia: x={0.496094, 0.746094, 0.246094, 0.496094}
8. iteracia: x={0.498047, 0.748047, 0.248047, 0.498047}
9. iteracia: x={0.499023, 0.749023, 0.249023, 0.499023}
10. iteracia: x={0.499512, 0.749512, 0.249512, 0.499512}
11. iteracia: x={0.499756, 0.749756, 0.249756, 0.499756}
12. iteracia: x={0.499878, 0.749878, 0.249878, 0.499878}
13. iteracia: x={0.499939, 0.749939, 0.249939, 0.499939}
14. iteracia: x={0.499969, 0.749969, 0.249969, 0.499969}
15. iteracia: x={0.499985, 0.749985, 0.249985, 0.499985}
16. iteracia: x={0.499992, 0.749992, 0.249992, 0.499992}
```

Len pripomeňme, že presné riešenie bolo

In[•]:=
```
LinearSolve[A, b] // N
```

Out[•]= {0.5, 0.75, 0.25, 0.5}

Vráťme sa opäť k tomu istému príkladu a položme si otázku, či je naozaj tá diagonálna dominantnoť taká dôležitá? Ak by sme nevymenili prvú a druhú

$$
\begin{array}{l}
-1\,x_1 + 4\,x_2 \qquad\quad - 1\,x_4 = 2 \\
+4\,x_1 - 1\,x_2 - 1\,x_3 \qquad\quad = 1 \\
-1\,x_1 \qquad\quad + 4\,x_3 - 1\,x_4 = 0 \\
\qquad -1\,x_2 - 1\,x_3 + 4\,x_4 = 1
\end{array}
$$

$$
\begin{array}{lllll}
-1\,x_1 & = 2 & -4\,x_2 & & +1\,x_4 \\
-1\,x_2 & = 1 & -4\,x_1 & +1\,x_3 & \\
+4\,x_3 & = 0 & +1\,x_1 & & +1\,x_4 \\
+4\,x_4 & = 1 & +1\,x_2 & +1\,x_3 &
\end{array}
$$

$$
\begin{array}{ll}
+1\,x_1 & = -1 \times (2 - 4\,x_2 + 1\,x_4) \\
+1\,x_2 & = -1 \times (1 - 4\,x_1 + 1\,x_3) \\
+1\,x_3 & = 1/4 \times (0 + 1\,x_1 + 1\,x_4) \\
+1\,x_4 & = 1/4 \times (1 + 1\,x_2 + 1\,x_3)
\end{array}
$$

In[236]:=
```
H = {{0, +4, 0, -1}, {+4, 0, -1, 0}, {1 / 4, 0, 0, 1 / 4}, {0, 1 / 4, 1 / 4, 0}}
g = {-2, -1, 0, 1 / 4}
```

Out[236]= $\left\{\{0, 4, 0, -1\}, \{4, 0, -1, 0\}, \left\{\frac{1}{4}, 0, 0, \frac{1}{4}\right\}, \left\{0, \frac{1}{4}, \frac{1}{4}, 0\right\}\right\}$

Out[237]= $\left\{-2, -1, 0, \frac{1}{4}\right\}$

In[238]:=
```
Clear[x]
x[0] = {0, 0, 0, 0}
Do[
 x[k + 1] = H .x[k] + g // N;
 Print[k + 1, ". iteracia: x=", x[k + 1]], {k, 0, 15}]
```

Out[239]= {0, 0, 0, 0}

```
1. iteracia: x={-2., -1., 0., 0.25}
2. iteracia: x={-6.25, -9., -0.4375, 0.}
3. iteracia: x={-38., -25.5625, -1.5625, -2.10938}
4. iteracia: x={-102.141, -151.438, -10.0273, -6.53125}
5. iteracia: x={-601.219, -399.535, -27.168, -40.1162}
6. iteracia: x={-1560.02, -2378.71, -160.334, -106.426}
7. iteracia: x={-9410.4, -6080.76, -416.613, -634.51}
8. iteracia: x={-23690.5, -37226., -2511.23, -1624.09}
9. iteracia: x={-147282., -92252., -6328.66, -9934.06}
10. iteracia: x={-359076., -582800., -39304., -24644.9}
```

11. iteracia: x= $\{-2.30656\times10^{6}, -1.397\times10^{6}, -95930.2, -155526.\}$

12. iteracia: x= $\{-5.43248\times10^{6}, -9.1303\times10^{6}, -615521., -373232.\}$

13. iteracia: x= $\{-3.6148\times10^{7}, -2.11144\times10^{7}, -1.45143\times10^{6}, -2.43645\times10^{6}\}$

14. iteracia: x= $\{-8.20211\times10^{7}, -1.4314\times10^{8}, -9.6461\times10^{6}, -5.64145\times10^{6}\}$

15. iteracia: x= $\{-5.6692\times10^{8}, -3.18438\times10^{8}, -2.19156\times10^{7}, -3.81966\times10^{7}\}$

16. iteracia: x= $\{-1.23556\times10^{9}, -2.24577\times10^{9}, -1.51279\times10^{8}, -8.50885\times10^{7}\}$

Ak chceme použiť vzorce pomocou rozdelenia na trojuholníkové a diagonálnu matice a nechceme to programovať - možnosť je aj ich len priamo nadefinovať
poznámka - nemôže sa matica volať N (ako sme mali v teorii, lebo N je prikaz v Mathematice)

```
Diag = DiagonalMatrix[{4, 4, 4, 4}]; Diag // MatrixForm
M = {{0, 0, 0, 0}, {-1, 0, 0, 0}, {-1, 0, 0, 0}, {0, -1, -1, 0}};
M // MatrixForm
NH = {{0, -1, -1, 0}, {0, 0, 0, -1}, {0, 0, 0, -1}, {0, 0, 0, 0}};
NH // MatrixForm
A == M + Diag + NH
```

$$\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

True

Príklad 1 - 3 x 3 sústava

Riešte systém rovníc Jacobiho/Gauss-Seidlovou metódou s presnosťou 10^{-1}.

$$x_1 + x_2 - 3\,x_3 = -4$$
$$2\,x_1 + 5\,x_2 + x_3 = 5$$
$$4\,x_1 - x_2 + 2\,x_3 = -12$$

Presné riešenie

```
A = {{1, 1, -3}, {2, 5, 1}, {4, -1, 2}};
b = {-4, 5, -12};
n = Length[A];
LinearSolve[A, b] // N
```

```
{-3., 2., 1.}
```

Najskôr upravíme maticu *A* na tvar ostro diagonálnej, alebo symetrickej a pozitívne definitnej matice.

```
A = {{4, -1, 2}, {2, 5, 1}, {1, 1, -3}};
b = {-12, 5, -4};
```

Vytvoríme matice potrebné pre dosadenie do schémy

```
Clear[Upom, U, Lpom, L, Diag]
```

```
Upom[i_, j_ /; i < j] := A[[i, j]]
Upom[i_, j_ /; i ≥ j] := 0
U = Table[Upom[i, j], {i, 1, n}, {j, 1, n}];
U // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Lpom[i_, j_ /; i > j] := A[[i, j]]
Lpom[i_, j_ /; i ≤ j] := 0
L = Table[Lpom[i, j], {i, 1, n}, {j, 1, n}];
L // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

```
Diag = DiagonalMatrix[Table[A[[i, i]], {i, 1, n}]];
Diag // MatrixForm
```

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

Jacobiho metóda

Iteračná schéma pre túto metódu má tvar $x^{(k+1)} = -D^{-1}.(L + U)\,.\,x^{(k)} + D^{-1}.b$. To znamená, že je potrebné zvoliť $H = -D^{-1}.(L + U)$ a $g = D^{-1}.b$

```
H = -Inverse[Diag].(L + U);
H // MatrixForm
g = Inverse[Diag].b;
g // MatrixForm
```

$$\begin{pmatrix} 0 & \frac{1}{4} & -\frac{1}{2} \\ -\frac{2}{5} & 0 & -\frac{1}{5} \\ \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 1 \\ \frac{4}{3} \end{pmatrix}$$

```
Clear[x]
x[0] = {0, 0, 0};
pocetiteracii = 20;
Do[
 x[k + 1] = H .x[k] + g // N, {k, 0, pocetiteracii}]
```

```
TableForm[Table[NumberForm[x[k][[i]], {10, 4}], {k, 1, pocetiteracii}, {i, 1, n}],
 TableHeadings → {Automatic},
    TableSpacing → {1, 5}, TableAlignments → Right]
```

1	-3.0000	1.0000	1.3333
2	-3.4167	1.9333	0.6667
3	-2.8500	2.2333	0.8389
4	-2.8611	1.9722	1.1278
5	-3.0708	1.9189	1.0370
6	-3.0388	2.0209	0.9494
7	-2.9694	2.0256	0.9940
8	-2.9906	1.9890	1.0187
9	-3.0121	1.9925	0.9995
10	-3.0016	2.0050	0.9935
11	-2.9955	2.0019	1.0011
12	-3.0001	1.9980	1.0022
13	-3.0016	1.9996	0.9993
14	-2.9998	2.0008	0.9993
15	-2.9995	2.0000	1.0003
16	-3.0002	1.9997	1.0002
17	-3.0002	2.0000	0.9999
18	-2.9999	2.0001	1.0000
19	-3.0000	2.0000	1.0001
20	-3.0000	2.0000	1.0000

```
TableForm[Table[{NumberForm[x[k], {10, 4}], Max[Abs[x[k] - x[k - 1]]]},
          {k, 1, pocetiteracii}],
 TableHeadings → {Automatic, {"xi", "chyba: max|xi-xi-1|"}},
     TableSpacing → {1, 5}, TableAlignments → Right]
```

	x_i	chyba: max $\lvert x_i - x_{i-1} \rvert$
1	{-3.0000, 1.0000, 1.3333}	3.
2	{-3.4167, 1.9333, 0.6667}	0.933333
3	{-2.8500, 2.2333, 0.8389}	0.566667
4	{-2.8611, 1.9722, 1.1278}	0.288889
5	{-3.0708, 1.9189, 1.0370}	0.209722
6	{-3.0388, 2.0209, 0.9494}	0.102037
7	{-2.9694, 2.0256, 0.9940}	0.0693519
8	{-2.9906, 1.9890, 1.0187}	0.036679
9	{-3.0121, 1.9925, 0.9995}	0.0215154
10	{-3.0016, 2.0050, 0.9935}	0.0124624
11	{-2.9955, 2.0019, 1.0011}	0.00766169
12	{-3.0001, 1.9980, 1.0022}	0.00458332
13	{-3.0016, 1.9996, 0.9993}	0.00285369
14	{-2.9998, 2.0008, 0.9993}	0.00183345
15	{-2.9995, 2.0000, 1.0003}	0.00100296
16	{-3.0002, 1.9997, 1.0002}	0.000686736
17	{-3.0002, 2.0000, 0.9999}	0.000332408
18	{-2.9999, 2.0001, 1.0000}	0.000242649
19	{-3.0000, 2.0000, 1.0001}	0.000117451
20	{-3.0000, 2.0000, 1.0000}	0.0000808784

Gauss-Seidlova metóda

Iteračná schéma pre túto metódu má tvar $x^{(k+1)} = -(L + D)^{-1}.U\,.\,x^{(k)} + (L + D)^{-1}.b$. To znamená, že je potrebné zvoliť $H = -(L + D)^{-1}.U$ a $g = (L + D)^{-1}.b$

```
H = -Inverse[L + Diag].(U);
H // MatrixForm
g = Inverse[L + Diag].b;
g // MatrixForm
```

$$\begin{pmatrix} 0 & \frac{1}{4} & -\frac{1}{2} \\ 0 & -\frac{1}{10} & 0 \\ 0 & \frac{1}{20} & -\frac{1}{6} \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ \frac{11}{5} \\ \frac{16}{15} \end{pmatrix}$$

```
Clear[x]
x[0] = {0, 0, 0};
pocetiteracii = 20;
Do[
 x[k + 1] = H.x[k] + g // N, {k, 0, pocetiteracii}]
```

```
TableForm[Table[NumberForm[x[k][[i]], {10, 4}], {k, 1, pocetiteracii}, {i, 1, n}],
 TableHeadings → {Automatic},
    TableSpacing → {1, 5}, TableAlignments → Right]
```

1	-3.0000	2.2000	1.0667
2	-2.9833	1.9800	0.9989
3	-3.0044	2.0020	0.9992
4	-2.9991	1.9998	1.0002
5	-3.0002	2.0000	1.0000
6	-3.0000	2.0000	1.0000
7	-3.0000	2.0000	1.0000
8	-3.0000	2.0000	1.0000
9	-3.0000	2.0000	1.0000
10	-3.0000	2.0000	1.0000
11	-3.0000	2.0000	1.0000
12	-3.0000	2.0000	1.0000
13	-3.0000	2.0000	1.0000
14	-3.0000	2.0000	1.0000
15	-3.0000	2.0000	1.0000
16	-3.0000	2.0000	1.0000
17	-3.0000	2.0000	1.0000
18	-3.0000	2.0000	1.0000
19	-3.0000	2.0000	1.0000
20	-3.0000	2.0000	1.0000

```
TableForm[Table[{NumberForm[x[k], {10, 4}], Max[Abs[x[k] - x[k - 1]]]},
         {k, 1, pocetiteracii}],
 TableHeadings → {Automatic, {"xi", "chyba: max|xi-xi-1|"}},
    TableSpacing → {1, 5}, TableAlignments → Right]
```

	x_i	chyba: max$\lvert x_i - x_{i-1}\rvert$
1	{-3.0000, 2.2000, 1.0667}	3.
2	{-2.9833, 1.9800, 0.9989}	0.22
3	{-3.0044, 2.0020, 0.9992}	0.022
4	{-2.9991, 1.9998, 1.0002}	0.00535185
5	{-3.0002, 2.0000, 1.0000}	0.00107531
6	{-3.0000, 2.0000, 1.0000}	0.000197551
7	{-3.0000, 2.0000, 1.0000}	0.0000347586
8	{-3.0000, 2.0000, 1.0000}	5.97643×10^{-6}
9	{-3.0000, 2.0000, 1.0000}	1.0144×10^{-6}
10	{-3.0000, 2.0000, 1.0000}	1.70901×10^{-7}
11	{-3.0000, 2.0000, 1.0000}	2.86668×10^{-8}
12	{-3.0000, 2.0000, 1.0000}	4.79613×10^{-9}
13	{-3.0000, 2.0000, 1.0000}	8.01188×10^{-10}
14	{-3.0000, 2.0000, 1.0000}	1.33714×10^{-10}
15	{-3.0000, 2.0000, 1.0000}	2.23039×10^{-11}
16	{-3.0000, 2.0000, 1.0000}	3.71925×10^{-12}
17	{-3.0000, 2.0000, 1.0000}	6.20393×10^{-13}
18	{-3.0000, 2.0000, 1.0000}	1.03473×10^{-13}
19	{-3.0000, 2.0000, 1.0000}	1.73195×10^{-14}
20	{-3.0000, 2.0000, 1.0000}	3.10862×10^{-15}

Príklad 2 - 4 x 4 sústava

Príklad 2 - 4 x 4 sústava

Riešte systém rovníc Jacobiho/Gauss-Seidlovou metódou s presnosťou 10^{-2}.

$$
\begin{aligned}
-7 x_1 + 2 x_2 - x_3 + 2 x_4 &= 17 \\
2 x_1 - 7 x_2 + 2 x_3 - x_4 &= 14 \\
-1 x_1 + 2 x_2 - 7 x_3 + 2 x_4 &= -1 \\
2 x_1 - x_2 + 2 x_3 - 7 x_4 &= -34
\end{aligned}
$$

Presné riešenie

```
A = {{-7, 2, -1, 2}, {2, -7, 2, -1}, {-1, 2, -7, 2}, {2, -1, 2, -7}};
b = {17, 14, -1, -34};
n = Length[A];
LinearSolve[A, b] // N
```

```
{-2., -3., 1., 5.}
```

Najskôr upravíme maticu *A* na tvar ostro diagonálnej, alebo symetrickej a pozitívne definitnej matice.
Matica A je ostro diagonálne-dominantná matica, nie sú potrebné žiadne úpravy

```
A // MatrixForm
```

$$
\begin{pmatrix} -7 & 2 & -1 & 2 \\ 2 & -7 & 2 & -1 \\ -1 & 2 & -7 & 2 \\ 2 & -1 & 2 & -7 \end{pmatrix}
$$

Vytvoríme matice potrebné pre dosadenie do schémy

```
Clear[Upom, U, Lpom, L, Diag]
```

```
Upom[i_, j_ /; i < j] := A[[i, j]]
Upom[i_, j_ /; i ≥ j] := 0
U = Table[Upom[i, j], {i, 1, n}, {j, 1, n}];
U // MatrixForm
```

$$
\begin{pmatrix} 0 & 2 & -1 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

```
Lpom[i_, j_ /; i > j] := A[[i, j]]
Lpom[i_, j_ /; i ≤ j] := 0
L = Table[Lpom[i, j], {i, 1, n}, {j, 1, n}];
L // MatrixForm
```

$$
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 2 & -1 & 2 & 0 \end{pmatrix}
$$

```
Diag = DiagonalMatrix[Table[A[[i, i]], {i, 1, n}]];
Diag // MatrixForm
```

$$\begin{pmatrix} -7 & 0 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

Jacobiho metóda

Iteračná schéma pre túto metódu má tvar $x^{(k+1)} = -D^{-1}.(L+U).x^{(k)} + D^{-1}.b$. To znamená, že je potrebné zvoliť $H = -D^{-1}.(L+U)$ a $g = D^{-1}.b$

```
H = -Inverse[Diag].(L + U);
H // MatrixForm
g = Inverse[Diag].b;
g // MatrixForm
```

$$\begin{pmatrix} 0 & \frac{2}{7} & -\frac{1}{7} & \frac{2}{7} \\ \frac{2}{7} & 0 & \frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} & 0 & \frac{2}{7} \\ \frac{2}{7} & -\frac{1}{7} & \frac{2}{7} & 0 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{17}{7} \\ -2 \\ \frac{1}{7} \\ \frac{34}{7} \end{pmatrix}$$

```
Clear[x]
x[0] = {0, 0, 0, 0};
pocetiteracii = 20;
Do[
 x[k + 1] = H.x[k] + g // N, {k, 0, pocetiteracii}]
```

```
TableForm[Table[{NumberForm[x[k], {10, 4}], Max[Abs[x[k] - x[k - 1]]]},
          {k, 1, pocetiteracii}],
 TableHeadings → {Automatic, {"xi", "chyba: max|xi-xi-1|"}},
     TableSpacing → {1, 5}, TableAlignments → Right]
```

	x_i	chyba: $\max\|x_i - x_{i-1}\|$
1	{-2.4286, -2.0000, 0.1429, 4.8571}	4.85714
2	{-1.6327, -3.3469, 1.3061, 4.4898}	1.34694
3	{-2.2886, -2.7347, 0.7026, 5.2420}	0.752187
4	{-1.8126, -3.2020, 1.1862, 4.7947}	0.483549
5	{-2.1430, -2.8639, 0.8568, 5.1356}	0.340929
6	{-1.9019, -3.1011, 1.0980, 4.8988}	0.241201
7	{-2.0718, -2.9295, 0.9282, 5.0705}	0.17167
8	{-1.9495, -3.0511, 1.0505, 4.9489}	0.12235
9	{-2.0364, -2.9638, 0.9636, 5.0362}	0.0872836
10	{-1.9741, -3.0260, 1.0259, 4.9740}	0.0622986
11	{-2.0185, -2.9815, 0.9815, 5.0185}	0.0444791
12	{-1.9868, -3.0132, 1.0132, 4.9868}	0.0317622
13	{-2.0095, -2.9906, 0.9905, 5.0094}	0.0226837
14	{-1.9933, -3.0068, 1.0067, 4.9932}	0.016201
15	{-2.0048, -2.9952, 0.9952, 5.0048}	0.0115715
16	{-1.9966, -3.0034, 1.0034, 4.9966}	0.00826507
17	{-2.0025, -2.9975, 0.9975, 5.0025}	0.0059035
18	{-1.9982, -3.0018, 1.0018, 4.9982}	0.00421673
19	{-2.0013, -2.9987, 0.9987, 5.0013}	0.00301193
20	{-1.9991, -3.0009, 1.0009, 4.9991}	0.00215137

Gauss-Seidlova metóda

Iteračná schéma pre túto metódu má tvar $x^{(k+1)} = -(L + D)^{-1}.U.x^{(k)} + (L + D)^{-1}.b$. To znamená, že je potrebné zvoliť $H = -(L + D)^{-1}.U$ a $g = (L + D)^{-1}.b$

```
H = -Inverse[L + Diag].(U);
H // MatrixForm
g = Inverse[L + Diag].b;
g // MatrixForm
```

$$\begin{pmatrix} 0 & \frac{2}{7} & -\frac{1}{7} & \frac{2}{7} \\ 0 & \frac{4}{49} & \frac{12}{49} & -\frac{3}{49} \\ 0 & -\frac{6}{343} & \frac{31}{343} & \frac{78}{343} \\ 0 & \frac{156}{2401} & -\frac{120}{2401} & \frac{373}{2401} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{17}{7} \\ -\frac{132}{49} \\ -\frac{96}{343} \\ \frac{10728}{2401} \end{pmatrix}$$

```
Clear[x]
x[0] = {0, 0, 0, 0};
pocetiteracii = 15;
Do[
 x[k + 1] = H .x[k] + g // N, {k, 0, pocetiteracii}]
```

```
TableForm[Table[{NumberForm[x[k], {10, 4}], Max[Abs[x[k] - x[k - 1]]]},
          {k, 1, pocetiteracii}],
 TableHeadings → {Automatic, {"xi", "chyba: max|xi-xi-1|"}},
    TableSpacing → {1, 5}, TableAlignments → Right]
```

	x_i	chyba: max$\lvert x_i - x_{i-1} \rvert$
1	{-2.4286, -2.6939, -0.2799, 4.4681}	4.46814
2	{-1.8817, -3.2559, 0.7580, 5.0012}	1.03791
3	{-2.0382, -3.0802, 0.9829, 4.9957}	0.224864
4	{-2.0217, -3.0105, 0.9989, 4.9950}	0.0697499
5	{-2.0043, -3.0008, 0.9989, 4.9986}	0.0174479
6	{-2.0005, -3.0002, 0.9996, 4.9998}	0.00378355
7	{-2.0001, -3.0001, 0.9999, 5.0000}	0.000410946
8	{-2.0000, -3.0000, 1.0000, 5.0000}	0.0000778515
9	{-2.0000, -3.0000, 1.0000, 5.0000}	0.0000220089
10	{-2.0000, -3.0000, 1.0000, 5.0000}	6.30381×10^{-6}
11	{-2.0000, -3.0000, 1.0000, 5.0000}	1.37614×10^{-6}
12	{-2.0000, -3.0000, 1.0000, 5.0000}	2.36355×10^{-7}
13	{-2.0000, -3.0000, 1.0000, 5.0000}	4.54455×10^{-8}
14	{-2.0000, -3.0000, 1.0000, 5.0000}	1.13488×10^{-8}
15	{-2.0000, -3.0000, 1.0000, 5.0000}	2.84856×10^{-9}