

# Multidimensional Skills, Sorting, and Human Capital Accumulation

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# Table of contents

1. Introduction
2. Model
3. Data & Estimation
4. Results
5. Counterfactuals
6. Summary

# Introduction

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# Refresher

Recall:

- study of wage and employment inequality
- classical view of human capital
- heterogeneous skills and requirements
- labour market as mediator

But:

- one catch-all skill
- at odds with intuition
- data much finer

Therefore: heterogeneous, multidimensional worker skills and skill requirements.

Their approach: multidimensional skills (cognitive, manual, interpersonal) and on-the-job learning into search model.

# Model

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# Model

- Match Output is  $f(\mathbf{x}, \mathbf{y})$  and depends on the match.
- Skills adjust gradually:  $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{y})$
- Skills drawn from distribution  $N(\cdot)$
- Exogenous transition rates  $(\lambda_0, \lambda_1, \delta, \mu)$
- Utility  $w - c(\mathbf{x}, \mathbf{y})$ , or  $b(\mathbf{x})$
- $P(\mathbf{x}, \mathbf{y})$  is value of firm-worker pair,  $U(\mathbf{x})$  value of unemployment,  $W$  of wage contract

Worker's share of match surplus is

$$\frac{W - U(\mathbf{x})}{P(\mathbf{x}, \mathbf{y}) - U(\mathbf{x})}$$

Worker gets outside offer, Bertrand competition leads to him staying if  $P(\mathbf{x}, \mathbf{y}) \geq P(\mathbf{x}, \mathbf{y}')$  with new wage  $W' = \min\{P(\mathbf{x}, \mathbf{y}), P(\mathbf{x}, \mathbf{y}')\}$

# Rent sharing and value functions

worker's renegotiated share of match surplus:

$$\sigma(\mathbf{x}, \mathbf{y}, \mathbf{y}') = \frac{P(\mathbf{x}, \mathbf{y}') - U(\mathbf{x})}{P(\mathbf{x}, \mathbf{y}) - U(\mathbf{x})}$$
$$\Leftrightarrow W' = P(\mathbf{x}, \mathbf{y}') = U(\mathbf{x}) + \sigma(\mathbf{x}, \mathbf{y}, \mathbf{y}') [P(\mathbf{x}, \mathbf{y}) - U(\mathbf{x})]$$

assumed to stay constant over time.

Value functions:

$$(r + \mu + \delta)P(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}, \mathbf{y}) - c(\mathbf{x}, \mathbf{y}) + \delta U(\mathbf{x}) + \mathbf{g}(\mathbf{x}, \mathbf{y}) \cdot \nabla_{\mathbf{x}} P(\mathbf{x}, \mathbf{y}) \quad (1)$$

and

$$(r + \mu)U(\mathbf{x}) = b(\mathbf{x}) + \mathbf{g}(\mathbf{x}, \mathbf{0}) \cdot \nabla U(\mathbf{x}) \quad (2)$$

# Wage equation

and the wage implementing the  $W$  solves:

$$(r + \delta + \mu)W(\mathbf{x}, \mathbf{y}, \sigma) = w(\mathbf{x}, \mathbf{y}, \sigma) - c(\mathbf{x}, \mathbf{y}) + \delta U(\mathbf{x}) \quad (3) \\ + \lambda_1 \mathbf{E} \max\{0, \min\{P(\mathbf{x}, \mathbf{y}), P(\mathbf{x}, \mathbf{y}')\} - W(\mathbf{x}, \mathbf{y}, \sigma)\} + \mathbf{g}(\mathbf{x}, \mathbf{y}) \cdot \nabla_{\mathbf{x}} W(\mathbf{x}, \mathbf{y}, \sigma)$$

Combining all equations gives the wage equation:

$$w(\mathbf{x}, \mathbf{y}, \sigma) = \sigma f(\mathbf{x}, \mathbf{y}) + (1 - \sigma)b(\mathbf{x}) + (1 - \sigma)c(\mathbf{x}, \mathbf{y}) \\ - \lambda_1 \mathbf{E} \max\{0, \min\{P(\mathbf{x}, \mathbf{y}') - P(\mathbf{x}, \mathbf{y}), 0\} + (1 - \sigma)(P(\mathbf{x}, \mathbf{y}) - U(\mathbf{x}))\} \\ - (1 - \sigma)(\mathbf{g}(\mathbf{x}, \mathbf{y}) - \mathbf{g}(\mathbf{x}, \mathbf{0})) \cdot \nabla U(\mathbf{x})$$



# Model Analysis - Closed Form

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \dot{x}_C \\ \dot{x}_M \\ \dot{x}_I \\ \dot{x}_T \end{pmatrix} = \begin{pmatrix} \gamma_C^u \max \{y_C - x_C, 0\} + \gamma_C^o \min \{y_C - x_C, 0\} \\ \gamma_M^u \max \{y_M - x_M, 0\} + \gamma_M^o \min \{y_M - x_M, 0\} \\ \gamma_I^u \max \{y_I - x_I, 0\} + \gamma_I^o \min \{y_I - x_I, 0\} \\ g x_T \end{pmatrix}$$

and  $x_T(t) = x_T(0) \cdot e^{gt}$ .

Production function:

$$f(\mathbf{x}, \mathbf{y}) = x_T \cdot \left[ \varphi(\mathbf{y}) - \sum_{k=C,M,I} \kappa_k^u \min \{x_k - y_k, 0\}^2 \right]$$

And

$$b(\mathbf{x}) = b x_T$$

Disutility of work:

$$c(\mathbf{x}, \mathbf{y}) = x_T \times \sum_{k=C,M,I} \kappa_k^0 \max \{x_k - y_k, 0\}^2$$

Only positive if overqualified  $\rightarrow$  utility cost of being under-matched.  
Then the match surplus is:

$$P(\mathbf{x}(t), \mathbf{y}) - U(\mathbf{x}) = x_T(t) \times \left\{ \frac{\varphi(\mathbf{y}) - b}{r + \delta + \mu - g} \right. \\ \left. - \sum_{k=C,M,I} \left( \frac{\kappa_k^u \min \{x_k(t) - y_k, 0\}^2}{r + \delta + \mu - g + 2\gamma_k^u} + \frac{\kappa_k^0 \max \{x_k(t) - y_k, 0\}^2}{r + \delta + \mu - g + 2\gamma_k^0} \right) \right\}$$

First term is the surplus for perfect match, the remaining terms reflect the cost of initial mismatch.

## Data & Estimation

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Worker-level: NLSY79. PCA and exclusion restrictions of ASVAB, Rotter, Rosenberg, health, social...

Occupational-level: O\*NET. Over 200 descriptors reduced to three dimensions by PCA and three exclusion restrictions

1770 males followed for 30 years

# Estimation

- By indirect inference, discrete-time approximation.  
Pre-sampling simulation until actual unemployment rate is reached  $\rightarrow$  initial state
- $\varphi(\mathbf{y}) = \alpha_T + \alpha_C y_C + \alpha_M y_M + \alpha_I y_I$  assumed.
- Initial specific skills are fully observed.
- Model for general skills: unrestricted correlation  
 $x_T(t) =$   
 $\exp(g \cdot t + \zeta_S \cdot \text{Schooling years} + \zeta_C x_C(0) + \zeta_M x_M(0) + \zeta_I x_I(0) + \varepsilon_0)$
- Skill requirements are transforms of O\*NET measures:  
 $y_k = \tilde{y}_k^{\xi_k}$ , with  $\xi_k > 0$
- 32 parameters are estimated by targeting 6 moments (they argue are valid approximations to identification restrictions)

## Results

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# Results

Good model fit to the data and to wage regression coefficients. Then the parameter estimates are:

Table 2: Parameter estimates

production function*							disutility of work*			un. inc.	
$\alpha_T$	$\alpha_C$	$\alpha_M$	$\alpha_I$	$\kappa_C^u$	$\kappa_M^u$	$\kappa_I^u$	$\kappa_C^o$	$\kappa_M^o$	$\kappa_I^o$	$b$	
108.3 (11.4)	117.7 (8.55)	53.8 (2.78)	54.4 (3.91)	3,077.4 (136.6)	473.7 (30.5)	135.5 (7.07)	44.3 (2.08)	201.1 (9.94)	67.7 (3.29)	119.1 (4.99)	
				(76.9)	(20.4)	(2.8)	(1.1)	(8.7)	(1.4)		
skill accumulation function**							general efficiency				
$\gamma_C^u$	$\gamma_C^o$	$\gamma_M^u$	$\gamma_M^o$	$\gamma_I^u$	$\gamma_I^o$	$g$	$\zeta_S$	$\zeta_C$	$\zeta_M$	$\zeta_I$	
0.008 (.001)	0.003 (.000)	0.033 (.002)	0.030 (.002)	0.001 (.001)	5.8e-7 (.001)	0.002 (9e-5)	0.026 (.012)	0.74 (.198)	-0.20 (.102)	0.31 (.060)	
(7.64)	(17.5)	(1.74)	(1.90)	(56.9)	(99,397)						
sampling distribution***							transition rates				
$\xi_C$	$\xi_M$	$\xi_I$	$\rho_{CM}$	$\rho_{CI}$	$\rho_{IM}$	$\beta_C$	$\beta_M$	$\beta_I$	$\lambda_0$	$\lambda_1$	$\delta^{****}$
1.03 (.069)	0.89 (.057)	0.82 (.050)	0.21 (.009)	0.64 (.021)	-0.47 (.015)	2.51 (.118)	1.18 (.043)	3.00 (.123)	0.37 (.019)	0.17 (.008)	0.02 (.001)
			(0.20)	(0.62)	(-0.45)	(0.29)	(0.46)	(0.25)			

## Distributions:

- Employers are looking for specialists, and seem to demand fewer cognitive skills than available
- Evolution is towards workers gaining C and losing M on average
- Already limited degree of specialisation regresses further

## Sorting & Mismatch:

- Impression of positive sorting and increases with exp → comb of workers gradually sorting into jobs for which their skills are suited and adjusting skills to requirements
- Mostly over-skilled in C dimension



# Counterfactuals

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# Counterfactuals

1. How efficient is job acceptance and rejection compared to social planner optimum? Planner internalises future match surplus.

$$(r + \delta + \mu)P^*(x, y) = f(x, y) - c(x, y) + \delta U^*(x) + g(x, y) \cdot \nabla_x P^*(x, y) + \lambda_1 E \max \{P^*(x, y') - P^*(x, y), 0\} \quad (4)$$

and

$$(r + \mu)U^*(x) = b(x) + g(x, 0) \cdot \nabla U^*(x) + \lambda_0 E \max \{P^*(x, y') - U^*(x), 0\} \quad (5)$$

For  $g(x, y)=0$ :  $E2E$  reallocation is efficient,  $U2E$  is not

For  $g(x, y)>0$ :  $U2E$  (though 2 competing effects) and  $E2E$  (short-sighted wage gains) smaller for planner. Output gain is very small

2. Cost of frictions: First eliminate mismatch, then search frictions. Frictionless output is 40% higher, most of which (35%) is due to direction rather than frequency.
3. Cost of early-career mismatch: what if initial job is most preferred job? → career output up by 8-22%. A share is due to initial high unemployment in the data (worst form of mismatch), increases in M and I, but U-shaped in C → connection to graduating in a recession.

## Summary

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The three different skills are very different productive attributes.

- Manual skills: moderate returns and adjust quickly
- Cognitive: much higher returns, slower to adjust
- Interpersonal: moderate returns, pretty much don't adjust
- Cost of skill-mismatch: very high for C, employing under-qualified in C worker much worse than hiring over-qualified C