Exercise

Calculate the length of the curve C,

where C is the curve $t \to (t\cos(t), t\sin(t))$
when $0 \leq t \leq 2\pi$.

[Picture]

Hint 1: there is a formula for calculating the length of ~~the~~ curve. You need to know this one!

It is of the form $\int_{\dots}^{\dots} |f(t)|\,dt$.

Hint 2

$$\int_0^{2\pi} |f(t)|\,dt$$

$$\int_0^{2\pi} |f(t)|\,dt = \int_0^{2\pi} \sqrt{t^2\cos^2(t) + t^2\sin^2(t)}\,dt$$

$$= \int_0^{2\pi} \sqrt{t^2}\,dt$$

Now be careful: $\sqrt{t^2} = |t|$,
but since t is from 0 to 2π, $t \geq 0$.
So:

$$= \int_0^{2\pi} t = \left[t^2/2\right]_0^{2\pi} = 2\pi^2$$

Exercise: Consider the curve given by the equation

$$0 = f(x,y) = (x-y)^2 + 4(x+y) - 4$$

Determine where on this curve $x+y$ is the largest.

Hint: write $g(x,y) = x+y$ and use Lagrange

Hint: calculate $\nabla f = \lambda \nabla g$

Hint: $$\begin{pmatrix} 2(x-y)+4 \\ -2(x-y)+4 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

So $$2(x-y)+4 = \lambda$$
$$-2(x-y)+4 = \lambda$$

several ways to solve this.
One way: add the equations
& substract the equations.

add: $8 = 2\lambda \Rightarrow \lambda = 4$ (you don't need this though)

substract: $4(x-y) = 0$
$\Rightarrow x = y$

You are on the equation

$$(x-y)^2+4(x+y)-4=0$$

2

So substitute ~~x~~. $y=x$

$$(x-x)^2+4(x+x)-4=0 \Rightarrow$$

$$8x=4 \Rightarrow x=1/2$$

So $y=x=1/2$

and the point is $\left(\frac{1}{2},\frac{1}{2}\right)$.

The function is here

$$\cancel{f\left(\tfrac{1}{2},\tfrac{1}{2}\right)=\left(\tfrac{1}{2}-\tfrac{1}{2}\right)^2+4\left(\tfrac{1}{2}+\tfrac{1}{2}\right)-4}$$

$$g\left(\tfrac{1}{2},\tfrac{1}{2}\right)=1.$$

<u>Exercise:</u> On the surface of a sphere of radius 1, (centred at (0, 0, 0)) where is the function $f(x,y,z) = \cancel{2x+2y+} \frac{1}{x} + \frac{1}{8y} + \frac{1}{27z}$

maximal and where minimal?
And what is this minimum and maximum?

<u>Hint:</u> the surface of a sphere has the formula

$$x^2+y^2+z^2=R^2,$$ where R is the radius.

<u>Hint:</u> You need to use Lagrange's method

Let $g(x,y,z) = x^2+y^2+z^2-1$.

we need to find when $\nabla f = \lambda \cdot \nabla g$ for some $\lambda \neq 0$.

Hint:

$$\begin{pmatrix} \cancel{2x}\ -1/x^2 \\ \frac{-1}{8y^2} \\ \frac{-1}{27z^2} \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

So $\quad x^3 = \frac{-1}{2\lambda} \qquad x = \sqrt[3]{\frac{-1}{2\lambda}}$

$y^3 = \frac{-1}{8\cdot 2\lambda} \quad \Rightarrow \quad y = \sqrt[3]{\frac{-1}{8\cdot 2\lambda}}$

$z^3 = \frac{-1}{27\cdot 2\lambda} \qquad z = \sqrt[3]{\frac{-1}{27\cdot 2\lambda}}$

What now?

Hint: the points lie on the surface of the sphere

Hint so $x^2+y^2+z^2=1$ must hold

Hint:

$$x = \sqrt[3]{\frac{-1}{2\lambda}}$$

$$y = \sqrt[3]{\frac{-1}{2\lambda}} \cdot \frac{1}{2}$$

$$z = \sqrt[3]{\frac{-1}{2\lambda}} \cdot \frac{1}{3}$$

$$\Rightarrow \left(\sqrt[3]{\frac{-1}{2\lambda}}\right)^2 + \frac{1}{4}\left(\sqrt[3]{\frac{-1}{2\lambda}}\right) + \frac{1}{9}\left(\sqrt[3]{\frac{-1}{2\lambda}}\right) = 1$$

(Continue) that $\Rightarrow$

$$\left(\sqrt[3]{\frac{-1}{2\lambda}}\right)^2\left(1 + \frac{1}{4} + \frac{1}{9}\right) = 1$$

$$\left(\sqrt[3]{\frac{-1}{2\lambda}}\right)\left(\frac{36}{36} + \frac{9}{36} + \frac{4}{36}\right) = 1$$

$$\Rightarrow \left(\sqrt[3]{\frac{-1}{2\lambda}}\right)^2 = \frac{36}{49} \Rightarrow \sqrt[3]{\frac{-1}{2\lambda}} = \pm\frac{6}{7}$$

'two solutn :

$x = \frac{-6}{7}$ and $x = \frac{6}{7}$

$y = \frac{1}{2} \cdot \frac{-6}{7} = \frac{-3}{7}$ $y = 3/7$

$z = \frac{1}{3} \cdot \frac{-6}{7} = \frac{-2}{7}$ $z = 2/7$

(notice indeed:

$\left(\frac{-6}{7}\right)^2 + \left(\frac{-3}{7}\right)^2 + \left(\frac{-2}{7}\right)^2 = 1$)

which is minimum, which max?
And what are the value?

Simple: substitute in f.

$$f\left(\frac{-6}{7}, \frac{-3}{7}, \frac{-2}{7}\right) = \frac{-7}{6} - \frac{1}{8} \cdot \frac{7}{3} - \frac{1}{27} \cdot \frac{7}{2}$$

$$= \frac{-7}{6} - \frac{7}{24} - \frac{7}{54}$$

$$= \frac{-343}{216} = \frac{-7^3}{2^3 \cdot 3^3} = \left(-\frac{7}{6}\right)^3$$

minimum

$$f\left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right) = \left(7/6\right)^3$$

maximum.

~~and~~

Exercise:

Consider the function

$$f(x,y) = \frac{x^2+y^2}{xy}$$

on the shape $K = \left\{ (x,y) \;\middle|\; \begin{matrix} 0 < x \leq 1 \\ 0 < y \leq 1 \end{matrix} \right\}$

~~(a square,~~ ~~mi~~

Determine where f assumes its minimum, and what this minimum is.

~~Let~~ Hint Calculate ∇f.

Then ...

$$\Big[\quad \frac{\cancel{\partial} df}{dx} = \left(\frac{xy \cdot (2x) - (x^2+y^2) \cdot y}{x^2y^2} \right)$$

$$= \frac{x^2y - y^3}{x^2y^2} = \frac{x^2-y^2}{x^2y}$$

$$\frac{df}{dy} = \frac{x^2-y^2}{xy^2}$$

$$\nabla f = \begin{pmatrix} df/dx \\ df/dy \end{pmatrix} = \ldots$$

Now ~~calc~~ solve $\nabla f = 0$:

$\frac{x^2-y^2}{x^2y} = 0$ since $x,y > 0$ this means

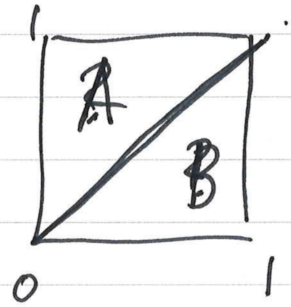
$$x^2 - y^2 = 0$$

$\rightarrow x^2 = y^2 \Rightarrow x = y$ or $x = -y$ (but this is impossible since $x>0, y>0$)

Hint: At $y = x$

$$f(x,x) = \frac{x^2+x^2}{(x\cdot 1)(x\cdot 1)} = \frac{2x^2}{x^2} = \underline{\underline{2}}$$

This is a whole line where the value 2 is assumed.



↑ geogebra picture?

Note that f goes to ∞ if x or y approache 0.

Or you can calculate $f(\frac{1}{2}, 1)$

$$= \frac{\frac{1}{4}+1}{\frac{1}{2}} = \frac{5/4}{1/2} = 5/2 > 2$$

and see that in A values are above the minimum at A.

Solution:

$\nabla f(A) = (5,5,5)$ so the equation is

$$5X + 5Y + 5Z = \delta.$$

Since $A = (1,1,3)$ is on this plane,

$$5\cdot 1 + 5\cdot 1 + 5\cdot 3 = \delta$$
$$25 = \delta$$

So:

$$5X + 5Y + 5Z = 25$$

is the answer.

NOTE: there is also a formula give in the book as: the linearization of f at A:

$$L(x,y,z) = f(A) + f_x(A)(x-x_0) + f_y(A)(y-y_0) + f_z(A)(z-z_0)$$

where $A = (x_0, y_0, z_0)$

which will yield exactly this answer.
However, applying formulas without understanding is like eating your food without chewing!

Exercise: Consider the surface $f(x,y,z)=0$ ①

where $f(x,y,z) = z^2 + x^2 + 3xy + y^2 - z - 11$

a) Check that $A = (1,1,3)$ is on the surface.

b) Give an equation $\alpha X + \beta Y + \gamma Z = \delta$ describing the tangent plane to $f=0$ at A.

Solution: Substitute $x=1, y=1, z=3$ into $z^2+x^2+3xy+y^2-z-11$ and see that it becomes 0.

Hint: there are several ways to go, but in most if not all you need to compute ∇f at the point A.

$\nabla f(A)$: $$\nabla f = \begin{pmatrix} df/dx \\ df/dy \\ df/dz \end{pmatrix} = \begin{pmatrix} 2x+3y \\ 3x+2y \\ 2z-1 \end{pmatrix}$$

$$\nabla f(A) = \begin{pmatrix} 2\cdot 1+3\cdot 1 \\ 3\cdot 1+2\cdot 1 \\ 2\cdot 3-1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$$

How to use $\nabla f(A)$?

The gradient is a vector orthogonal to the plane at that point.

How do you find an equation of a plane, if you have a vector orthogonal to it?

(Turnover)

<u>Exercise</u>: Consider the plane $\mathcal{P}$ which contains the points $A = (1,1,1)$, $B = (2,3,9)$, $C = (3,5,4)$

Try to think of as many ways as possible to determine the equation $\alpha X + \beta Y + \gamma Z = \delta$ of this plane.

<u>Hint</u>:

<u>Method 1</u>: just substitute the points in $\alpha X + \beta Y + \gamma Z = \delta$ and solve for $\alpha, \beta, \gamma, \delta$

<u>Method 2</u>: Make the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and compute $\overrightarrow{AB} \times \overrightarrow{AC}$. Then...

<u>Method 3</u>: find a vector (α, β, γ) which is orthogonal to $\overrightarrow{AB}$, $\overrightarrow{AC}$. So calculate

$$\overrightarrow{AB} \cdot (\alpha, \beta, \gamma)$$
$$\overrightarrow{AC} \cdot (\alpha, \beta, \gamma)$$

and then...

Solution:
$$\left.\begin{aligned} \alpha + \beta + \gamma &= \delta \\ 2\alpha + 3\beta + 9\gamma &= \delta \\ 3\alpha + 5\beta + 4\gamma &= \delta \end{aligned}\right\} \begin{aligned} &R_2 : R_2 - 2R_1 \\ &R_3 : R_3 - 3R_1 \end{aligned} \quad \left.\begin{aligned} \alpha + \beta + \gamma &= \delta \\ \beta + 7\gamma &= \delta \\ 2\beta + \gamma &= \delta \end{aligned}\right\} \ldots$$

sol 2:
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} 1 & 2 & 8 \\ 2 & 4 & 3 \\ e_x & e_y & e_z \end{vmatrix} = (-26, 13, 0) \ldots$$
extra steps plz

Sol 3:
$$(1,2,8) \cdot (\alpha, \beta, \gamma) = \alpha + 2\beta + 8\gamma = 0$$
$$(2,4,3) \cdot (\alpha, \beta, \gamma) = 2\alpha + 4\beta + 3\gamma = 0$$

$$\Rightarrow R_2 : R_2 - 2R_1 \qquad \left.\begin{aligned} \alpha + 2\beta + 8\gamma \\ -13\gamma = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \gamma &= 0 \\ \alpha + 2\beta &= 0 \end{aligned}$$

so pick $\beta = 1$, $\alpha = -2$, $\gamma = 0$

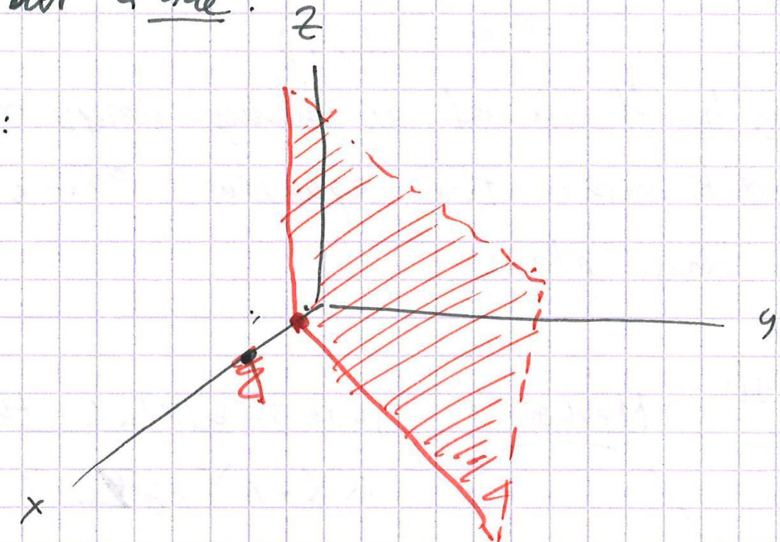
so $-2X + 1 \cdot Y + 0 Z = \delta$

Substitute $(1,1,1)$: $-2 + 1 = \delta \Rightarrow \delta = -1$

so $-2X + Y = -1$ (see other side)

Note: in $\mathbb{R}^3$, the equation ~~is~~ $-2x+y=-1$ is a <u>plane</u> and not a <u>line</u>.

~~Geogr~~ picture:
(geogebra or wolfram alpha! ask me for help if it takes long!)



16

85

~~5 ℓ u = 1500~~

14

~~57 = 14.15~~

~~50 = 1400~~

~~A function f~~

Exercise:

A curve C is given as the image of a function in polar coordinates:

$$f(r, \varphi) = (r\varphi, r+\varphi)$$

where $0 \leq r \leq 4$

$0 \leq \varphi \leq 2\pi$

Give the points 1

$A=(2,2,0)$, $B=(0,3,3)$, $C=(4,0,4)$,

find all point(s) which are equidistant to A, B, C.

Hint: ~~make~~ say $P=(x,y,z)$ is such a point.
Make equations giving the distances being equa

Hint: $|PA| = \sqrt{(x-2)^2+(y-2)^2+(z-0)^2}$ etc.

Hint: $|PA|^2 = |PB|^2 = |PC|^2$

Hint: $\cancel{(x-2)^2}$

$$x^2-4x+4+y^2-4y+4+z^2$$

$$= x^2+y^2-6y+9+z^2-6z+9$$

$$= x^2-8x+16+y^2+z^2-8z+16$$

Hint substract $x^2+y^2+z^2$ from these, and simplify:

$$-4x-4y+8 = -6y-6z+18 = -8x-8z+32$$

substract 8:

$$-4x-4y = -6y-6z+10 = -8x-8z+24$$

divide by -2:

$$2x+2y = 3y+3z-10 = 4x+4z-12$$

2

Hint: Note that you in fact just have 2 equations:

$$2x+2y = 3y+3z-10$$

$$2x+2y = 4x+4z-12$$

$$(\text{or } 3y+3z-10 = 4x+4z-12)$$

Hint: solving this you get

$$2x-y-3z=-10$$
$$-2x+2y-4z=-12$$

this becomes linear algebra! (The solution is intersection of 2 planes, a line thus!)

Hint: $\left(\begin{array}{ccc|c} 2 & -1 & -3 & -10 \\ -2 & 2 & -4 & -12 \end{array}\right) \xrightarrow{R_2 : R_2+R_1}$

$\left(\begin{array}{ccc|c} 2 & -1 & -3 & -10 \\ 0 & 1 & -7 & -22 \end{array}\right) \xrightarrow{R_1 : R_1+R_2}$

$\left(\begin{array}{ccc|c} 2 & 0 & -10 & -32 \\ 0 & 1 & -7 & -22 \end{array}\right) \xrightarrow{R_1 : \frac{1}{2}R_1}$

$\left(\begin{array}{ccc|c} 1 & 0 & -5 & -16 \\ 0 & 1 & -7 & -22 \end{array}\right)$

So $\begin{array}{l} x-5z=-16 \\ y-7z=-22 \end{array} \Longrightarrow \begin{array}{l} x = 5z-16 \\ y = 7z-22 \\ z = \lambda \text{ free} \end{array}$

So: $(x,y,z) = (5\lambda-16, 7\lambda-22, \lambda)$

for any $\lambda \in \mathbb{R}$. A line of points, thus.

Given are 5 points

$A=(0,0,0)$

$B=(2,0,0)$

$C=(4,4,0)$

$D=(4,0,4)$

$E=(0,0,4)$

Find the point(s) which are equidistant to A, B, C, D, E (i.e. have the same distance to A, B, C, D, E .

Hint: write $P=(x,y,z)$ and Make equations of the distance

$$|PA|, \ |PB|, \ |PC|, \ |PD|, \ |PE|$$

Hint: $|PB| = |(x-2,y,z)| = \sqrt{(x-2)^2+y^2+z^2}$

Hint: $|PA|^2=|PB|^2=|PC|^2=|PD|^2=|PE|^2$

Hint: ~~$(x-2)^2+y^2+z^2=$~~

$$x^2+y^2+z^2=(x-2)^2+y^2+z^2=(x-4)^2+(y-4)^2+z^2$$
$$=(x-4)^2+(z-4)^2=(y-4)^2+(z-4)^2$$

Hint:

$0=-4x+4$

$0=-8x+16-8y+16$

$0=-8x+16-8z+16$

$0=-8y+16-8z+16$

Hint:

$x=1$

$8y=-8x+32=-8\cdot 1+32=24 \Rightarrow y=3$

$8z=-8x+32=24 \Rightarrow z=3$

$8y=-8z+32 \Rightarrow 8\cdot 3=-8\cdot 3+32$

$\Rightarrow 24=8$ contradiction :

There is no such point!

$\frac{2}{3}\sqrt{2}\, t\sqrt{t}$

Exercise:

A rocket is flying through space.
At time t it is at location $f(t) = (t\cos t, t\sin t, \cancel{\cdot})$ for $t \geq 0$.
At some point the rocket reaches speed 66.
~~At this moment, the rocket loses a piece of its hull. This piece keeps moving at the same speed and direction as the rocket has at that moment.~~ When is this?

~~What is the location of the piece at time $t = 100$?~~

Hint: compute a formula for the speed of the rocket at time t

Hint: $f'(t) = (\cos t - t\sin t,\ \sin t + t\cos t,\ \sqrt{2}\sqrt{t})$

we need $|f'(t)| = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + 2t}$

simplify this!

Hint: $|f'(t)| = \sqrt{\cos^2 t - 2t\cos t\sin t + t^2\sin^2 t + \sin^2 t + 2t\cos t\sin t + t^2\cos^2 t + 2t}$

Solution:

$$= \sqrt{1 + t^2 + 2t} = \sqrt{(1+t)^2} = |1+t|$$

since $t \geq 0$: $= 1+t$

Continue from here.

$1 + t = 66 \Rightarrow t = 65$

~~At this moment, the location of the rocket is ...~~

~~the speed of the rocket is ...~~

~~thus, the speed of the hull piece is ...~~

~~So, a formula for the location of the hull piece is:~~

(See other side)

At $t=66$, The location of the rocket is $\left(66\cos(66), 66\sin(66), \frac{3}{2}\sqrt{2}(66)^{3/2}\right)$
the velocity is

Exercise: Consider the surface $z = y^3 - 6xy + x^2$ where we restrict ourselves to $x \geq y \geq 0$. (1)
Figure out where z is maximal / minimal on this area.

Hint: "I have no idea how to start"
$\longrightarrow$ You need to calculate the extremal values, and you need to check if the edges of the area ~~are~~ perhaps have some max/min.

Hint:
Its a good idea to draw the area.
Can you reason something about the maximum?

Hint / partial solution

(Geogebra applet I made)

The area is unbounded, and if you go to the right, z gets larger and larger. Easiest to reason this is:
if $y = 0$, then $z = x^2$.
x can be anything (as $x \geq y$) and this makes z as large as you want.

So there is no maximum.

Hint: computation of extremal values

$$\left.\begin{aligned} 0 &= \frac{dz}{dx} = -6y + 2x \\ 0 &= \frac{dz}{dy} = 3y^2 - 6x \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x &= 3y \\ 3y^2 &= 6x \end{aligned}\right\} \Rightarrow 3y^2 = 18y$$

$\Rightarrow \quad y = 0$ or $y = 6 \quad \Rightarrow$ points $(0,0) = A$
$(18,6) = B$

Now figure out if these are local max/min or saddle points!

2

<u>Hint: calculation local max/min</u>

$$H = \frac{d^2z}{dx^2} \cdot \frac{d^2z}{dy^2} - \left(\frac{d^2z}{dxdy}\right)^2$$

$$= \frac{d(-6y+2x)}{dx} \cdot \frac{d(3y^2-6x)}{dy} - \left(\frac{d(3y^2-6x)}{dx}\right)^2$$

$$= 2 \cdot 6y - (-6)^2$$

$$= 12y - 36$$

$(0,0)$: $H = -36 < 0$ saddle point

$(18,6)$: $H = 12 \cdot 6 - 36 = 36 > 0$

$\frac{d^2z}{dx^2} = 2 > 0$ at $(18,6)$, so this is a local minimum

Try to finish the exercise, with solid argumentation on the edges of the area

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<u>Hint:</u> You need to consider the lines
$y=x$ and $y=0$
and analyze the function there separately (only minimum values interest us)

<u>~~4th~~ Hint</u>

$y=0$: $z = x^2$. minimum at $x=0$
$(0,0)$

$y=x$: $z = x^3 - 6x \cdot x + x^2 = x^3 - 5x^2$

minimum? $0 = \frac{d \cdot x^3 - 5x^2}{dx} = 3x^2 - 10x$

$\Rightarrow x = 0$ (had that already) or $x = \frac{10}{3}$

$$\frac{d^2(x^3-5x^2)}{dx^2} = 6x-10$$

so at $x=\frac{10}{3}$ we get $6\cdot\frac{10}{3}-10=10>0$

so at $\left(\frac{10}{3},\frac{10}{3}\right)$ we have a local min.

Try to wrap it up!

Hint/Solution:

Only candidates are

$(0,0) \to z=0$

$(18,6) \to z=6^3-6\cdot18\cdot6+18^2=-108$

$\left(\frac{10}{3},\frac{10}{3}\right) \to z=\left(\frac{10}{3}\right)^3-6\cdot\left(\frac{10}{3}\right)^2+\left(\frac{10}{3}\right)^2=-\frac{500}{27}\approx-18.5$

So minimum is at $\underline{\underline{(18,6)}}$.

Additional ideas/shortcuts:

If you would only have one extremal value, which is a local minimum then that ~~value~~ point is also a global ~~min/max~~ minimum. However, if you also have a saddle point, then that messes up everything. For example, here our function $z=y^3-6xy+x^2$ has unbounded from below values when $x=0$, $y\to-\infty$ (then $z=y^3\to-\infty$).

But, in our particular exercise, this saddle point is on the EDGE. This means that automatically the local minimum will be a global minimum - but this is quite advanced reasoning, which is indeed quicker but not something I expect anyone to come up with

Consider ~~Determine where~~ on the square

$$K := \{ (x,y) \mid 0 \leq x \leq 2\pi, \ 0 \leq y \leq 2\pi \}$$

the function $f(x,y) = (x+1)\sin(xy)$.

Determine the maximal and ~~where~~ minimum it assumes on K.

Hint (You CAN use some ad-hoc methods, but let's do this structurally, in a way which always works.)

Calculate ∇f - what can you do with this?

(And - is determining extremal values enough?

Hint: $\nabla f = \begin{pmatrix} \sin(xy) + y \cdot (x+1)\cos(xy) \\ x(x+1)\cos(xy) \end{pmatrix}$

$\nabla f = 0$ will yield the extremal value.

$$\nabla f = 0 \Rightarrow \begin{cases} \sin(xy) + y(x+1)\cos(xy) = 0 & (1) \\ x(x+1)\cos(xy) = 0 & (2) \end{cases}$$

(2) : $x = 0 \ \vee \ x+1 = 0 \ \vee \ \cos(xy) = 0$

$x = -1$ (out of domain) $\Rightarrow xy = \frac{\pi}{2}$ or $xy = \frac{3\pi}{2}$

~~Case x=0:~~ Now substitute these cases in (1)

$x=0$: $\sin(0\cdot y) + y(0+1)\cos(0y) = 0$

$0 + y = 0$

$\Rightarrow y = 0$

$A = (0,0)$

$xy = \frac{\pi}{2}$: $\sin\left(\frac{\pi}{2}\right) + y(x+y)\cos\left(\frac{\pi}{2}\right) = 0$

$1 + y(x+y)\, 0 = 0$

$1 = 0$

No solution.

$xy = \frac{3\pi}{2}$: $\sin\left(\frac{3\pi}{2}\right) + y(x+y)\cos\left(\frac{3\pi}{2}\right) = 0$

$-1 = 0$

No solution

A is the only extremal value.

Is A now the only maximum or minimum?

No - you have a bounded domain. So you need to check the edges of the square K!

How do you do that?

The edges are the lines

$x = 0$

$x = 2\pi$

$y = 0$

$y = 2\pi$

$\boxed{13}$

so lets look at $f(x,y)$ on these lines:

$x=0$: $f(0,y) = \sin(0) = 0$

$x=2\pi$: Hint $f(2\pi,y) = (2\pi+1)\cdot\sin(2\pi y)$

This is maximal if $\sin(2\pi y)$ is ; (maximal)

So the highest value will be when ~~this is 1~~, $\sin(2\pi)=1$

lowest if $\sin(2\pi)=-1$

so: $2\pi+1$ and $-2\pi-1$.

$y=0$: Hint $f(x,0) = \cancel{\sin}(x+1)\sin(0)=0$

$y=2\pi$: Hint $f(x,2\pi) = (x+1)\sin(2\pi x)$

This has the same max/min

Note: it is not asked __where__, so the final answer is:

maximum $2\pi+1$

minimum $-2\pi-1$

(If you want to know: the maximum is assumed ~~assumed at computed~~: at many values:

~~$\sin(2\pi x) =$~~

$(\frac{1}{4}, 2\pi), (\frac{3}{4}, 2\pi), (\frac{5}{4}, 2\pi), \ldots$

$(2\pi, \frac{1}{4}), (2\pi, \frac{3}{4}), (2\pi, \frac{5}{4}), \ldots$

Similar for the minimum)

BRUNNEN