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Algorithm Analysis

Laboratory work 3 : Empirical analysis of algorithms for obtaining Eratosthenes Sieve.

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Content

Introduction

The Sieve of Eratosthenes is an ancient algorithm for finding all prime numbers up to a given limit. It was invented by the Greek mathematician Eratosthenes in the 3rd century BC and is considered one of the earliest known algorithms.

The algorithm works by creating a list of all numbers from 2 to the given limit. It then starts with the first prime number, which is 2, and marks all of its multiples as composite numbers. It then moves on to the next unmarked number, which is a prime, and repeats the process until all numbers up to the limit have been checked.

The algorithm is based on the fact that every composite number can be factored into prime factors. Therefore, if a number has already been marked as composite, then it must have a prime factor that is less than or equal to the square root of the number. This means that we only need to consider the prime numbers up to the square root of the given limit.

The Sieve of Eratosthenes is an efficient algorithm for finding prime numbers because it only needs to check each number once. The time complexity of the algorithm is O(n log log n), where n is the given limit. This means that the algorithm scales very well for large values of n.

The space complexity of the algorithm is also relatively low, as it only requires an array of size n to store the state of each number. The array is initially set to true for all numbers, indicating that they are potentially prime. As each multiple of a prime number is marked as composite, its corresponding entry in the array is set to false.

The Sieve of Eratosthenes has many applications in mathematics and computer science. For example, it can be used to find all prime numbers up to a certain limit, to factorize large numbers into their prime factors, and to generate prime numbers for use in cryptographic algorithms.

In conclusion, the Sieve of Eratosthenes is a fascinating algorithm that has stood the test of time and continues to be relevant today. Its efficiency, simplicity, and low memory requirements make it a popular choice for finding prime numbers in a variety of applications.

Objectives

1. To understand the concept of prime numbers and their significance in mathematics and computer science.
2. To learn about different algorithms for finding prime numbers, including the Sieve of Eratosthenes and 3. brute-force methods.
3. To compare and contrast the efficiency, advantages, and limitations of different algorithms for finding prime numbers.
4. To implement different algorithms for finding prime numbers using programming languages such as Python or Java.
5. To optimize algorithms for finding prime numbers to improve their efficiency and reduce their computational complexity.
6. To evaluate the performance of different algorithms for finding prime numbers based on various factors such as speed, accuracy, and memory usage.
7. To apply the knowledge of prime number algorithms to solve real-world problems, such as cryptography and security.
8. To explore advanced topics related to prime numbers, such as prime number distribution and the Riemann hypothesis.

Algorithm 1

This is an implementation of the Sieve of Eratosthenes algorithm. It works by creating a boolean array of size n and initializing all elements to true, indicating that they are potentially prime. It then iterates through the array, starting with 2, and marks all multiples of each prime number as composite by setting their corresponding array element to false. The algorithm has a time complexity of O(n log log n) and a space complexity of O(n). Code:

```
            c[1] = false;
i=2;
while (i<=n){
  if (c[i] == true){
    j=2*i;
    while (j<=n){
      c[j] =false;
      j=j+i;
    }
  }
  i=i+1;
}
```

Algorithm 2

This is also an implementation of the Sieve of Eratosthenes algorithm, but with a slightly simpler implementation. It works by creating a boolean array of size n and initializing all elements to true. It then iterates through the array, starting with 2, and marks all multiples of each prime number as composite. The algorithm has the same time and space complexities as Algorithm 1.

Code:

```
             C[1] =false;
i=2;
while (i<=n){
  j=2*i;
  while (j<=n){
    c[j] =false;
    j=j+i;
```

```
  }
  i=i+1;
}
```

Algorithm 3

This algorithm is similar to the Sieve of Eratosthenes, but with a different approach to marking composite numbers. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each prime number i, marks all multiples of i as composite by setting their corresponding array element to false. The algorithm has a time complexity of $O(n^2)$ and a space complexity of $O(n)$.

Code:

```
            C[1] = false;
i=2;
while (i<=n){
  if (c[i] == true){
   j=i+1;
   while (j<=n){
     if (j % i == 0) {
      c[j] = false;
    }
    j=j+1;
}
}
  i=i+1;
}
```

Algorithm 4

This algorithm uses a brute-force approach to determining prime numbers. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each number i, checks if any number less than i divides evenly into i. If so, i is marked as composite by setting its corresponding array element to false. The algorithm has a time complexity of $O(n^2)$ and a space complexity of $O(n)$.

Code:

```
            C[1] = false;
i = 2;
While (i<=n){
  j=1;
  while (j<i){
    if ( i % j == 0)
    {
      c[i] = false
    }
    j=j+1;
  }
  i=i+1;
}
```

However, there is a mistake in the provided code since If j starts from 1, then i divided j will always be zero when j=1 for all i, resulting in all values in c to be marked as composite numbers, including the prime numbers.

Here is the corrected code:

```
            C[1] = false;
i = 2;
While (i<=n){
  j=2;
  while (j<i){
    if ( i % j == 0)
    {
      c[i] = false
    }
    j=j+1;
  }
  i=i+1;
}
```

Algorithm 5

This algorithm also uses a brute-force approach to determining prime numbers, but with a slight optimization. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each number i, checks if any number less than or equal to the square root of i divides evenly into i. If so, i is marked as composite by setting its corresponding array element to false. The algorithm has a time complexity of O(n sqrt(n)) and a space complexity of O(n).

Code:

```
  C[1] = faux;
i=2;
while (i<=n){
  j=2;
  while (j<=sqrt(i)){
    if (i % j == 0) {
      c[i] = false;
    }
    j++;
  }
  i++;
}
```

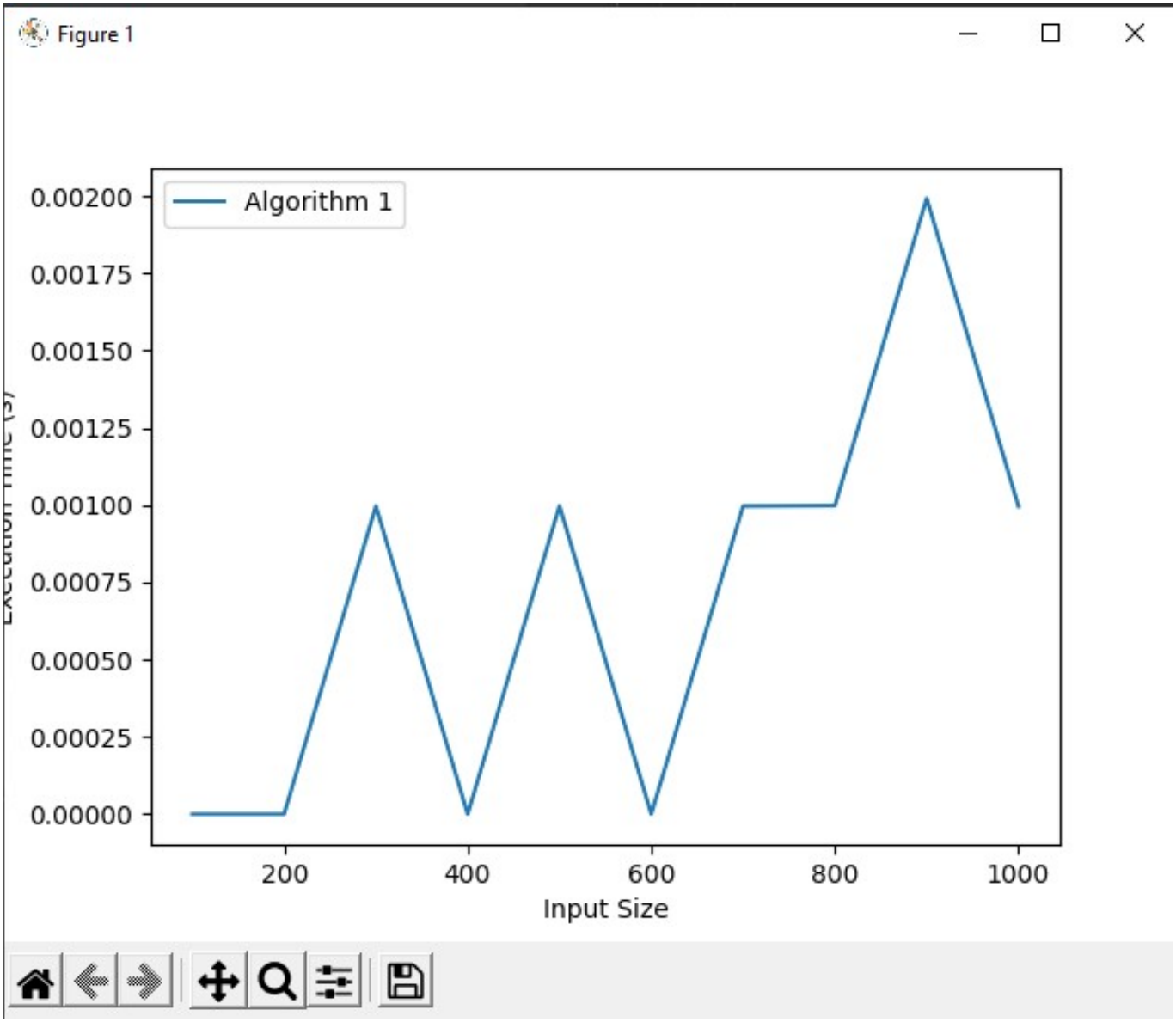
Implementation

```
   def sieve_of_eratosthenes_1(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        if c[i]:
            j = 2 * i
            while j <= n:
                c[j] = False
                j = j + i
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]


def sieve_of_eratosthenes_2(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2 * i
        while j <= n:
            c[j] = False
            j = j + i
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]


def sieve_of_eratosthenes_3(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        if c[i]:
```

```
            j = i + 1
            while j <= n:
                if j % i == 0:
                    c[j] = False
                j = j + 1
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]

def sieve_of_eratosthenes_4(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2
        while j < i:
            if i % j == 0:
                c[i] = False
            j = j + 1
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]



def sieve_of_eratosthenes_5(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2
        while j <= math.sqrt(i):
            if i % j == 0:
                c[i] = False
            j = j + 1
        i = i + 1
```

```
return [i for i in range(1, n+1) if c[i]]
```

Screenshot:



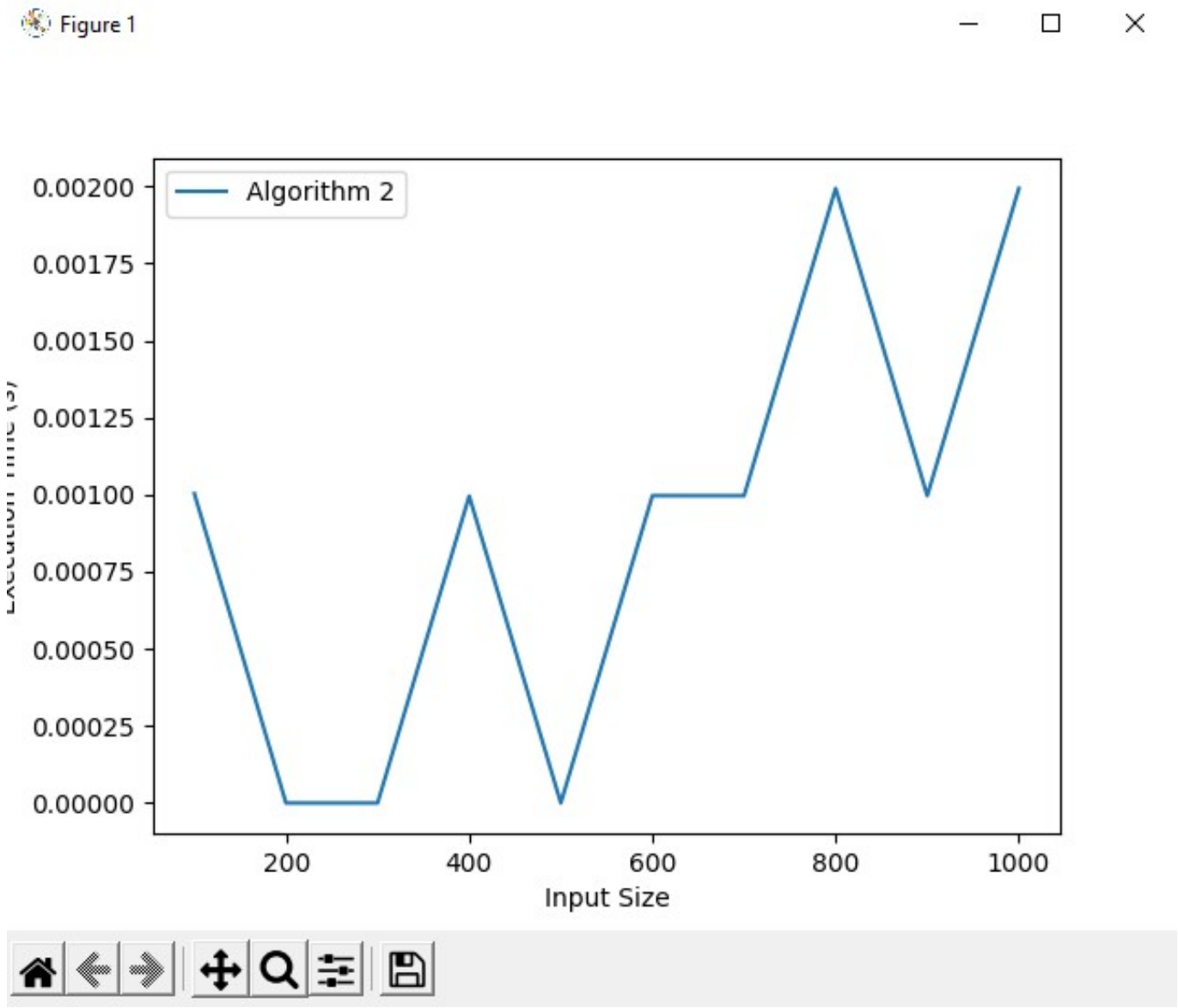


Figure 1
Algorithm 2
0.00200
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
200
400
600
800
1000
Input Size

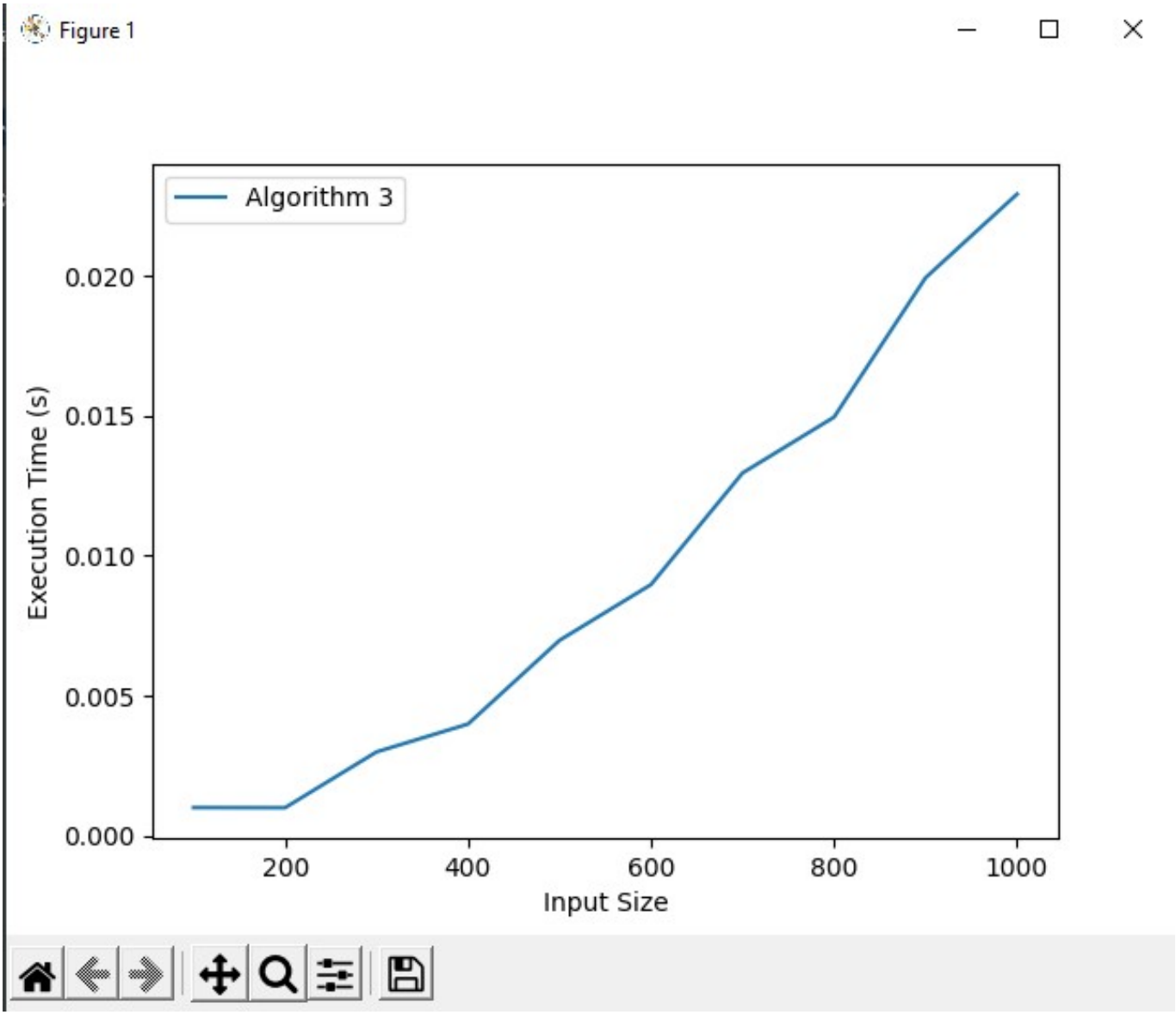


Figure 1
Algorithm 3
0.020
0.015
0.010
0.005
0.000
Execution Time (s)
200
400
600
800
1000
Input Size

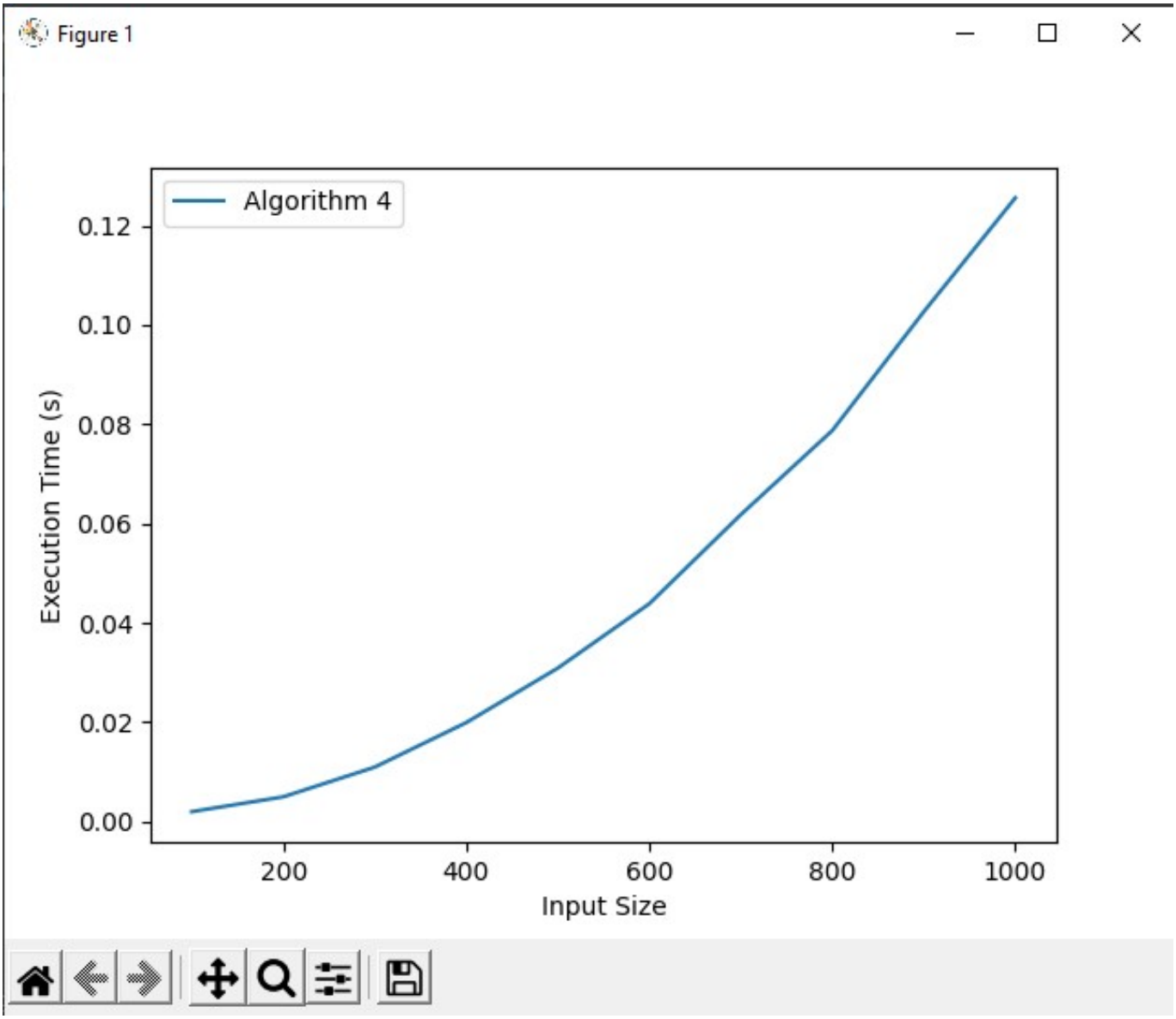


Figure 1
Algorithm 4
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Execution Time (s)
200
400
600
800
1000
Input Size

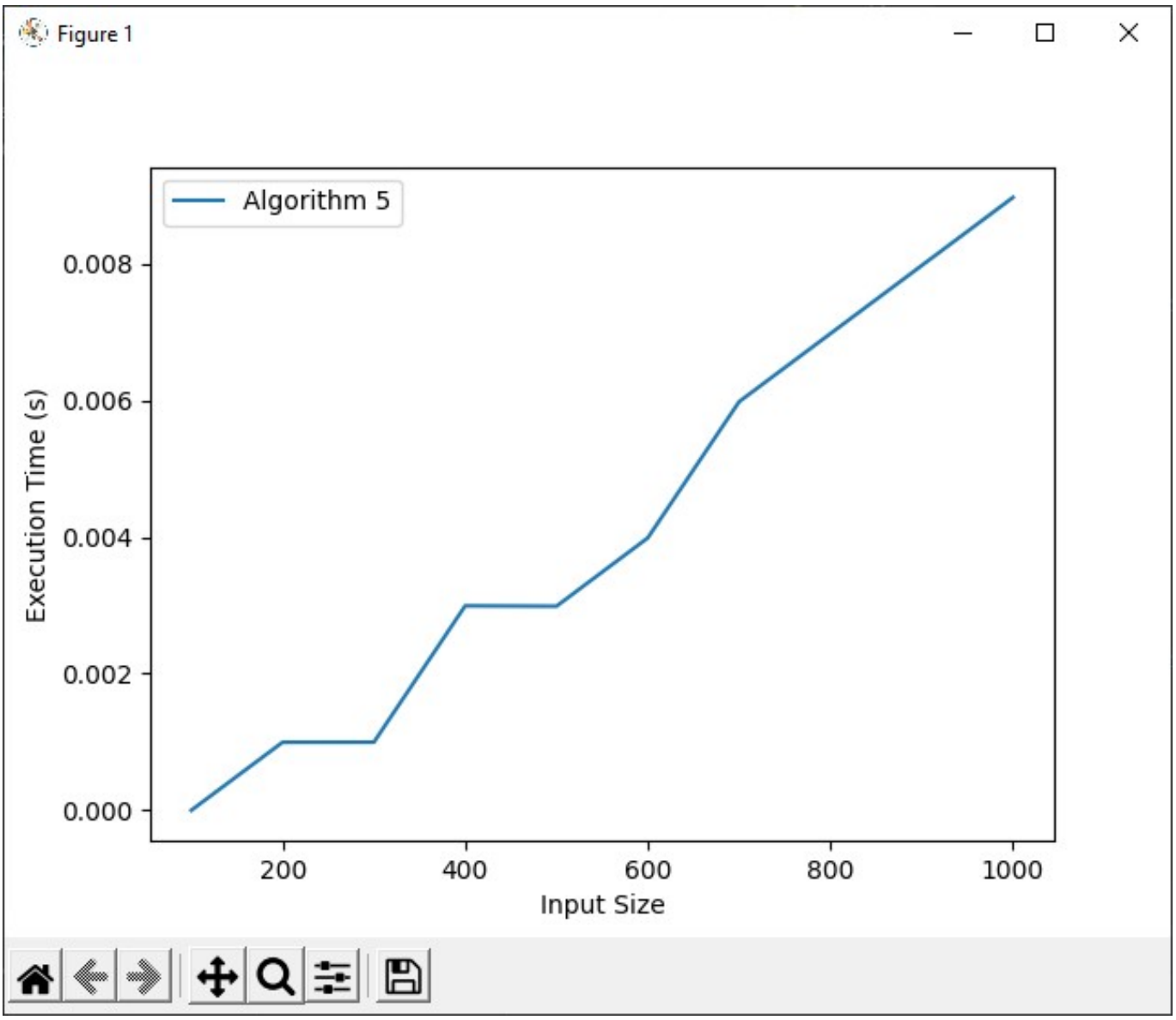


Figure 1
Algorithm 5
0.008
0.006
0.004
0.002
0.000
Execution Time (s)
200
400
600
800
1000
Input Size

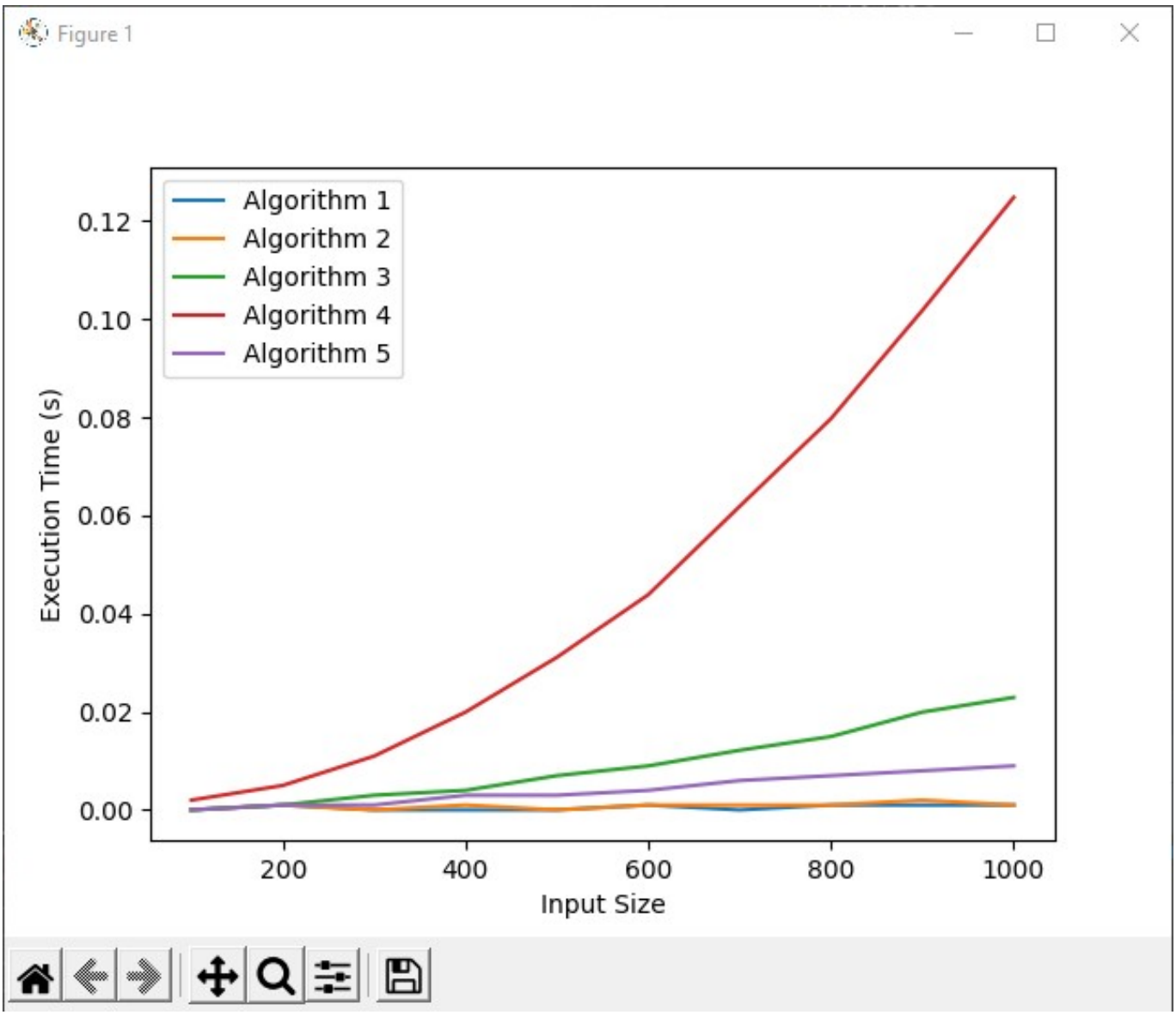


Figure 1
Algorithm 1
Algorithm 2
Algorithm 3
Algorithm 4
Algorithm 5
Execution Time (s)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
200
400
600
800
1000
Input Size

Conclusion

The algorithms presented showcase different methods for finding prime numbers up to a certain limit. Each algorithm has its own advantages and limitations, and the choice of algorithm to use depends on the specific requirements of the problem.

Algorithm 1 and 2 are implementations of the Sieve of Eratosthenes algorithm. The Sieve of Eratosthenes is a highly efficient algorithm for finding prime numbers up to a certain limit. The algorithm works by creating a list of all integers up to the limit, then iteratively marking the multiples of each prime number as composite. After all the multiples of each prime number have been marked, the remaining numbers in the list are all prime.

Algorithm 1 adds an additional optimization by only marking multiples of prime numbers as composite. This reduces the number of iterations required and makes the algorithm more efficient. Algorithm 2 iterates through the array and marks all multiples of each integer as composite, without checking whether the integer is prime or composite.

Algorithm 3 uses a different approach to finding prime numbers. The algorithm iterates through all integers from 2 to n, marking all multiples of each prime number as composite. However, instead of using a boolean array to keep track of the prime numbers, Algorithm 3 uses a loop to check if each integer is divisible by any prime number before marking it as composite.

Algorithm 4 is a brute-force approach to finding prime numbers. The algorithm iterates through all integers from 2 to n, checking if each integer is divisible by any integer between 1 and itself. If an integer is divisible by any integer other than 1 and itself, it is marked as composite.

Algorithm 5 is another brute-force approach, but with an optimization. Instead of checking all integers between 1 and the integer being tested for divisibility, Algorithm 5 only checks integers up to the square root of the integer being tested. This optimization reduces the number of iterations required and makes the algorithm more efficient.

In summary, the choice of algorithm depends on the specific requirements of the problem. The Sieve of Eratosthenes algorithm is a highly efficient way to find prime numbers up to a certain limit, but for larger values of n, other algorithms may also be practical options. Algorithm 3, 4, and 5 are less efficient than Algorithm 1 and 2, but may be practical options for smaller values of n. Ultimately, the choice of algorithm will depend on the specific needs of the problem at hand.

$https://github.com/StefanNistor69/APA_labs/tree/main/Lab3$