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Algorithm Analysis

Laboratory work 5 :Empirical analysis of algorithms: Dijkstra Algorithm, Floyd-Warshall Algorithm

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Content

Introduction

Dijkstra's algorithm and Floyd-Warshall algorithm are two popular algorithms used to solve shortest path problems in graphs.

Dijkstra's algorithm is a single-source shortest path algorithm, which means it finds the shortest path from a single source vertex to all other vertices in a weighted graph. The algorithm maintains a set of visited vertices and a set of unvisited vertices. It starts at the source vertex and iteratively selects the unvisited vertex with the smallest tentative distance, updating the distance of its neighboring vertices. Dijkstra's algorithm is particularly useful for finding the shortest path in a graph with positive edge weights, and it has a time complexity of O(—E—+—V—log—V—), where —E— is the number of edges and —V— is the number of vertices in the graph.

Floyd-Warshall algorithm, on the other hand, is an all-pairs shortest path algorithm, which means it finds the shortest path between all pairs of vertices in a weighted graph. The algorithm maintains a distance matrix, which stores the shortest path distances between all pairs of vertices in the graph. It iteratively updates the distance matrix by considering all intermediate vertices between any two pairs of vertices. Floyd-Warshall algorithm is particularly useful for finding the shortest path in a graph with both positive and negative edge weights, and it has a time complexity of $O(—V—^3), where |V| is the number of vertices in the graph.$

Both Dijkstra's and Floyd-Warshall algorithms are widely used in various fields such as network routing, transportation planning, and computer graphics. Choosing the appropriate algorithm depends on the characteristics of the graph and the specific problem being solved.

Dijkstra's algorithm and Floyd-Warshall algorithm are fundamental algorithms for solving the shortest path problem in a graph. They are widely used in many real-world applications such as network routing, GPS navigation, airline scheduling, and more.

Dijkstra's algorithm is particularly suitable for finding the shortest path in a graph with positive edge weights. The algorithm is widely used in network routing protocols, where the weights of the edges represent the costs of the network links. Dijkstra's algorithm has a time complexity of O(—E—+—V—log—V—), where —E— is the number of edges and —V— is the number of vertices in the graph. The time complexity can be further improved to O(—E—+—V—) using a priority queue data structure.

In summary, Dijkstra's algorithm and Floyd-Warshall algorithm are powerful tools for solving the shortest path problem in graphs. They have their own strengths and weaknesses, and choosing the appropriate algorithm depends on the characteristics of the graph and the specific problem being solved.

Objectives

1. Implement Dijkstra's algorithm for finding the shortest path from a single source node to all other nodes in a graph with non-negative edge weights.
2. Implement the Floyd-Warshall algorithm for finding the shortest paths between every pair of nodes in a weighted, directed graph.
3. Generate random graphs to be used as input for both Dijkstra's and Floyd-Warshall algorithms, ensuring various sizes and edge weights.
4. Compare the execution time of Dijkstra's and Floyd-Warshall algorithms as the number of nodes in the input graph increases.
5. Visualize the input graph and the resulting shortest path trees produced by both Dijkstra's and Floyd-Warshall algorithms using the NetworkX and Matplotlib libraries.

Dijkstra

Dijkstra's Algorithm: Dijkstra's algorithm is a widely-used graph traversal algorithm for finding the shortest path from a starting node to all other nodes in a graph with non-negative edge weights. The algorithm begins by initializing the starting node with a distance of 0 and all other nodes with a distance of infinity. At each step, the algorithm selects the unvisited node with the smallest known distance and updates the distances of its neighboring nodes. It does so by checking if the sum of the current node's distance and the edge weight to the neighboring node is less than the current distance assigned to the neighbor. If this condition is true, the algorithm updates the neighbor's distance with the new, smaller value. This process is repeated until all nodes are visited. By the end of the algorithm, the shortest path from the starting node to every other node in the graph is found. Code:

```
 function Dijkstra(Graph, source):

for each vertex v in Graph.Vertices:
     dist[v] ← INFINITY
    prev[v] ← UNDEFINED
   add v to Q
dist[source] ← 0

while Q is not empty:
    u ← vertex in Q with min dist[u]
    remove u from Q

    for each neighbor v of u still in Q:
        alt ← dist[u] + Graph.Edges(u, v)
        if alt < dist[v]:
            dist[v] ← alt
            prev[v] ← u

return dist[], prev[]
```

Floyd-Warshall

Floyd-Warshall Algorithm: The Floyd-Warshall algorithm is an all-pairs shortest path algorithm that finds the shortest paths between every pair of nodes in a weighted, directed graph. It works by iterating through all possible node combinations and updating the distance matrix based on the shortest known path between each node pair. The algorithm begins by initializing a distance matrix, where the diagonal elements are set to 0, and the off-diagonal elements are set to the corresponding edge weights or infinity if no direct edge exists between the node pair. The algorithm then iteratively checks if a shorter path between a pair of nodes can be found by traversing through an intermediate node. If a shorter path is discovered, the distance matrix is updated with the new, shorter path value. This process is repeated for all possible intermediate nodes. Upon completion, the distance matrix contains the shortest path distances between every pair of nodes in the graph.

Code:

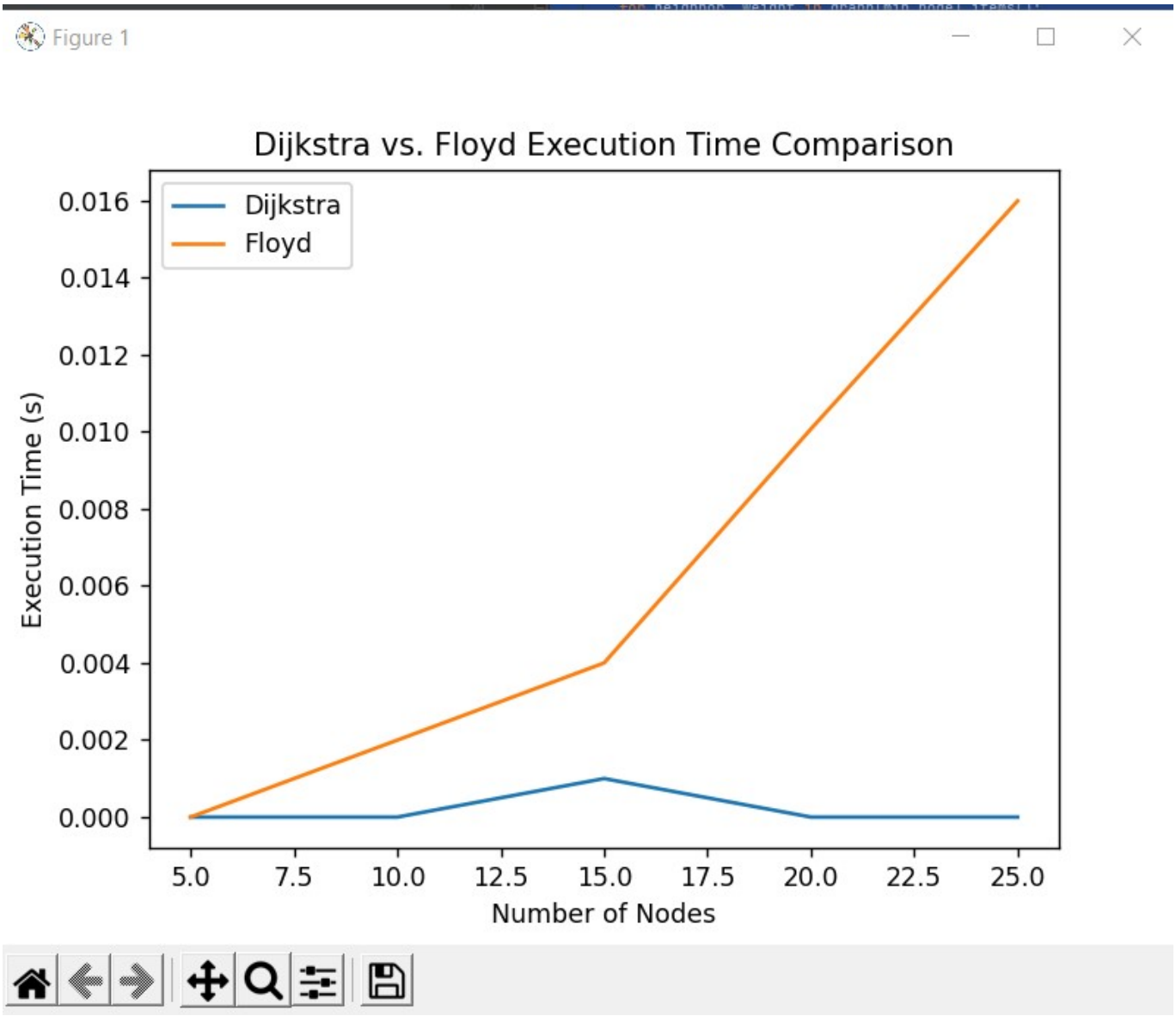
```
      let dist be a |V| × |V| array of minimum distances initialized to infinity
for each vertex v
   dist[v][v] ← 0
for each edge (u,v)
   dist[u][v] ← w(u,v)  // the weight of the edge (u,v)
for k from 1 to |V|
   for i from 1 to |V|
      for j from 1 to |V|
         if dist[i][j] > dist[i][k] + dist[k][j]
            dist[i][j] ← dist[i][k] + dist[k][j]
```

Implementation

```
def dijkstra(graph, start):
    dist = {node: float('inf') for node in graph}
    dist[start] = 0
    visited = set()

    while len(visited) != len(graph):
        min_node = None
        for node in dist:
            if node not in visited:
                if min_node is None or dist[node] < dist[min_node]:
                    min_node = node
        visited.add(min_node)

        for neighbor, weight in graph[min_node].items():
            new_dist = dist[min_node] + weight
            if new_dist < dist[neighbor]:
                dist[neighbor] = new_dist
    return dist

def floyd(graph):
    nodes = list(graph.keys())
    n = len(nodes)
    dist = np.full((n, n), np.inf)

    for i, node in enumerate(nodes):
        dist[i, i] = 0
        for neighbor, weight in graph[node].items():
            j = nodes.index(neighbor)
            dist[i, j] = weight
```

```
for k in range(n):
    for i in range(n):
        for j in range(n):
            if dist[i, k] + dist[k, j] < dist[i, j]:
                dist[i, j] = dist[i, k] + dist[k, j]

return {nodes[i]: {nodes[j]: dist[i, j] for j in range(n)} for i in range(n)}
```

Screenshot:



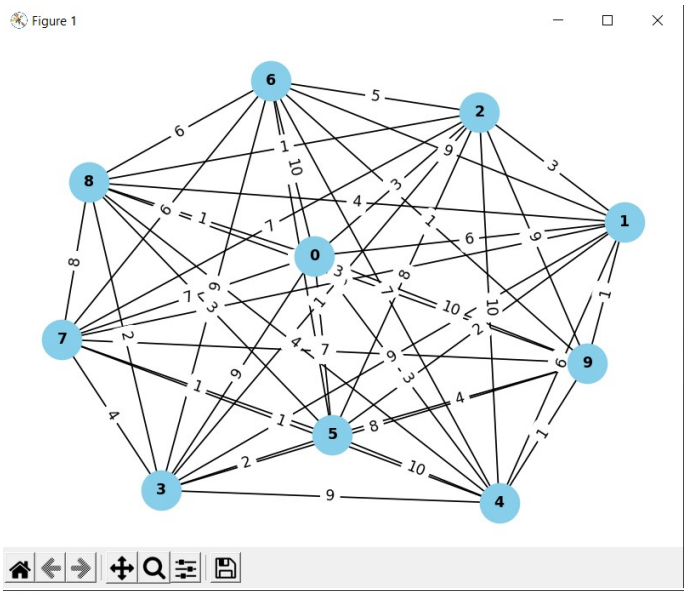


Figure 1

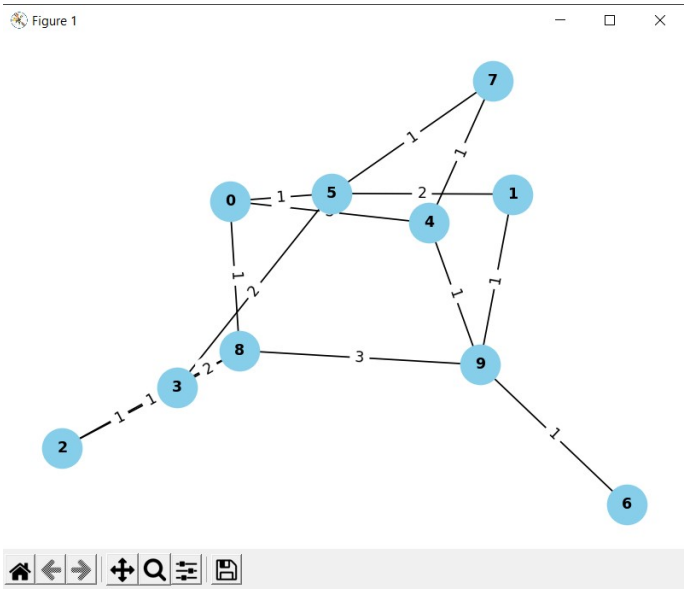


Figure 1

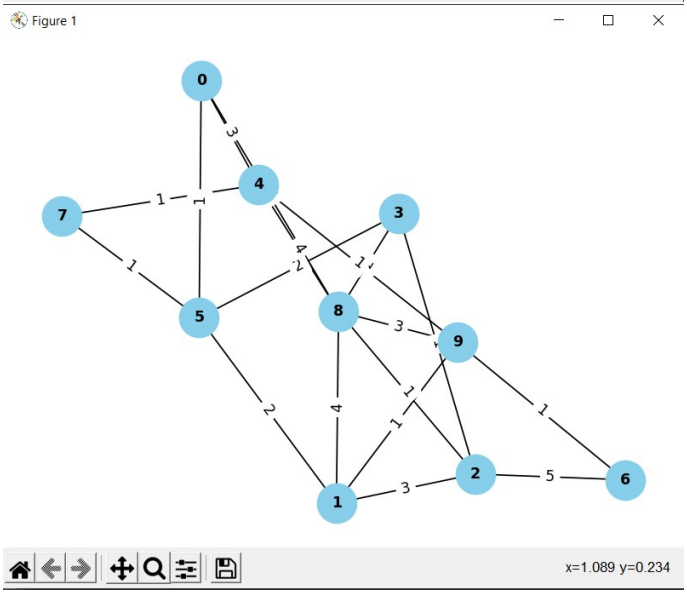


Figure 1
x=1.089 y=0.234

Conclusion

In conclusion, Dijkstra's algorithm and Floyd-Warshall algorithm are two popular and widely used algorithms in graph theory.

Dijkstra's algorithm is a single-source shortest path algorithm that computes the shortest path from a single source vertex to all other vertices in a weighted graph. It works by iteratively selecting the vertex with the smallest distance and relaxing its outgoing edges. It is efficient in finding the shortest path in sparse graphs with non-negative edge weights.

On the other hand, Floyd-Warshall algorithm is a dynamic programming algorithm that computes the shortest paths between all pairs of vertices in a weighted graph. It works by considering all possible intermediate vertices in a path and updates the shortest path distances accordingly. It is more efficient than running Dijkstra's algorithm for every vertex pair in dense graphs with non-negative edge weights.

Both algorithms have their advantages and limitations, and the choice of which algorithm to use depends on the specific problem and characteristics of the graph. However, they are both powerful tools in solving shortest path problems and have contributed significantly to the development of graph theory and its applications.

One of the main advantages of Dijkstra's algorithm is its efficiency in finding the shortest path from a single source vertex to all other vertices in a graph. This makes it ideal for problems that involve finding the shortest path between a fixed source and various destinations. However, it does not work well with negative edge weights, and its time complexity can be prohibitive in dense graphs.

In contrast, the Floyd-Warshall algorithm is more efficient than running Dijkstra's algorithm for every vertex pair, making it ideal for problems that require finding the shortest path between all pairs of vertices in a graph. It also works well with negative edge weights, but its time complexity can be prohibitive in large graphs.

It's worth noting that both algorithms are designed for graphs with non-negative edge weights. When dealing with graphs with negative edge weights, Bellman-Ford algorithm is more appropriate, as it can detect negative cycles and report them.

In summary, Dijkstra's algorithm and Floyd-Warshall algorithm are powerful tools in solving shortest path problems. They have contributed significantly to the development of graph theory and its applications, and they will continue to play a vital role in the future of graph theory and its related fields.

$https://github.com/StefanNistor69/APA_labs/tree/main/Lab5$