— SigSys I notation
— SysID notation (adapted)

$f(x)$	continuous argument
$f[n]$	discrete argument

time signal

DTFT $\rightleftarrows$ IDTFT

frequency domain

complex number
periodic (because of sampling in time domain)

continuous

only (real) magnitude
periodic

real, even symmetric

energy spectral density

autocorrelation

DTFT

discrete time, continuous, periodic freq.

finite energy

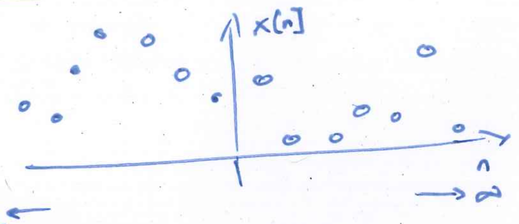
$$x[n] \overset{2.11}{=} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

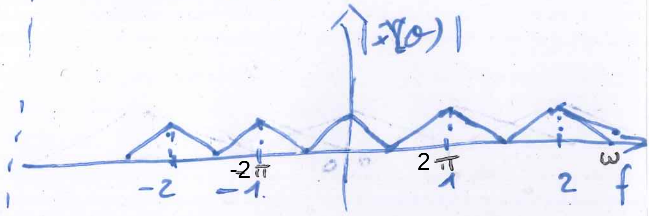
$$x[n] \overset{SS1,53}{=} \int_0^1 \hat{x}(\theta)\, e^{2\pi i n\theta}\, d\theta$$

$$X(\omega) \overset{2.10}{=} \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\hat{x}(\theta) \overset{SS1,60}{=} \sum_{n=-\infty}^{\infty} x[n]\, e^{-2\pi i n\theta}$$

$$\theta = fT = \frac{f}{f_s}$$





1-periodisch in θ

$x[n]$ endlich, nicht länger N: taste $\hat{x}$ ab: $\hat{x}\left(\frac{k}{N}\right)$ $k\in\{0..N-1\}$ und $\hat{x}[k+N]=\hat{x}[k]$ (view as x[n] periodic)

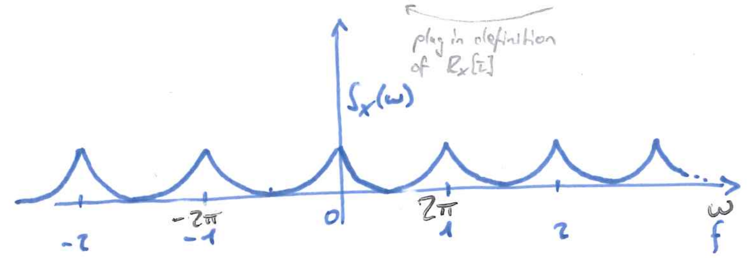
$|\cdot|^2$ loose information?

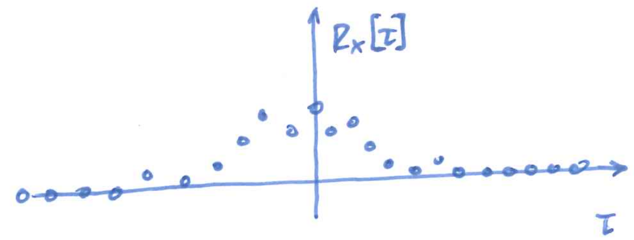
$$S_x(\omega) \overset{2.12}{=} |X(\omega)|^2 \overset{2.13\ \text{notes}}{=} \sum_{\tau=-\infty}^{\infty} R_x[\tau]\, e^{-j\omega\tau}$$

DTFT

$$R_x[\tau] \overset{2.13}{=} \sum_{n=-\infty}^{\infty} x[n]\, x[n-\tau]$$

plug in definition of $R_x[\tau]$





DFT

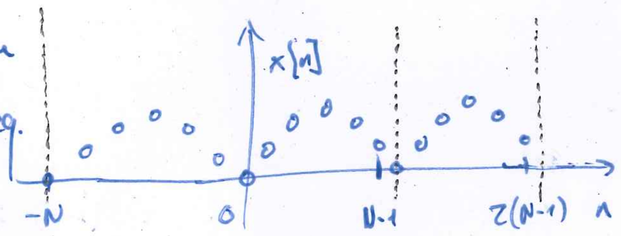
discrete, periodic time
discrete, periodic freq.

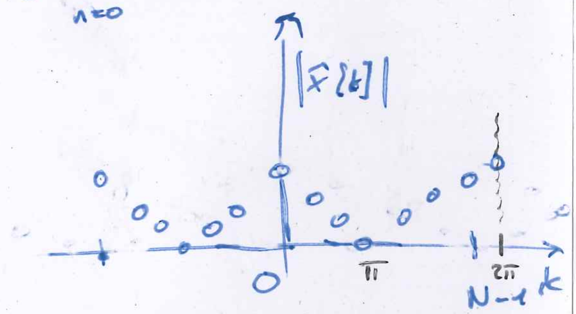
infinite energy

$$x[n] \overset{SS1,68}{=} \frac{1}{N}\sum_{k=0}^{N-1} \hat{x}[k]\, e^{2\pi i \frac{kn}{N}}$$

DFT $\rightleftarrows$ IDFT

$$\hat{x}[k] \overset{SS1,68}{=} \sum_{n=0}^{N-1} x[n]\, e^{-2\pi i \frac{kn}{N}}$$





$$x[n] \overset{2.18}{=} \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\omega_n k}, \quad \omega_n = \frac{2\pi n}{N}$$

$$X[k] \overset{2.18}{=} \sum_{n=0}^{N-1} x[n]\, e^{-j\omega_k n}, \quad \omega_k = \frac{2\pi k}{N}$$

$$\boxed{k\in\{0,\dots,N-1\}}$$

$\Updownarrow$

$\omega_k \in \{0, 2\pi(1-\frac{1}{N})\}$

$$\| \; X(e^{j\omega_k})$$

power spectral density PSD

$\frac{1}{N}|\cdot|^2$ loose information

$$\phi_x[k] \overset{2.19}{=} \frac{1}{N}|X[k]|^2 \overset{2.19}{=} \sum_{\tau=0}^{N-1} R_x[\tau]\, e^{-j\omega_k\tau}$$

$$\| \; \phi_x(e^{j\omega_k})$$

DTFT

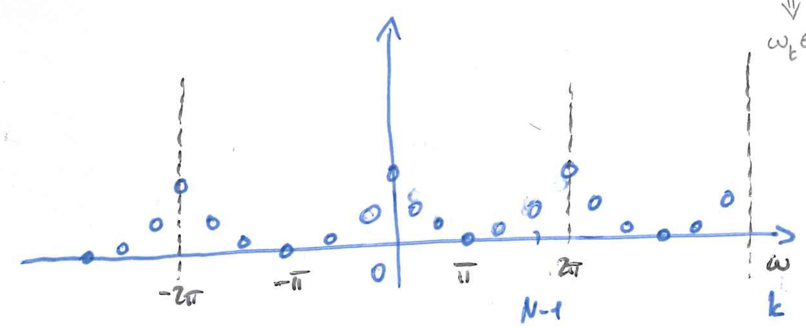
$$R_x[\tau] \overset{2.19}{=} \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, x[n-\tau]$$

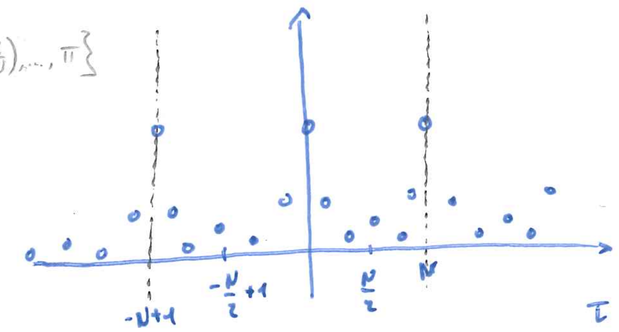
$$\omega_k = \frac{2\pi k}{N}$$

$$\boxed{k\in\{-\tfrac{N}{2}+1,\dots,\tfrac{N}{2}\}}$$

$\Updownarrow$

$\omega_k \in \{-\pi(1-\frac{2}{N}),\dots,\pi\}$





discrete

$$\frac{2\pi(N-1)}{N} = \frac{2\pi N}{N} - \frac{2\pi}{N} = 2\pi\left(1-\frac{1}{N}\right)$$

$$\frac{\frac{-N+2}{2}\cdot 2\pi}{N} = \frac{2\pi - N\pi}{N} = \pi\left(\frac{2}{N}-\frac{N}{N}\right) = \pi\left(\frac{2}{N}-1\right) = -\pi\left(1-\frac{2}{N}\right)$$