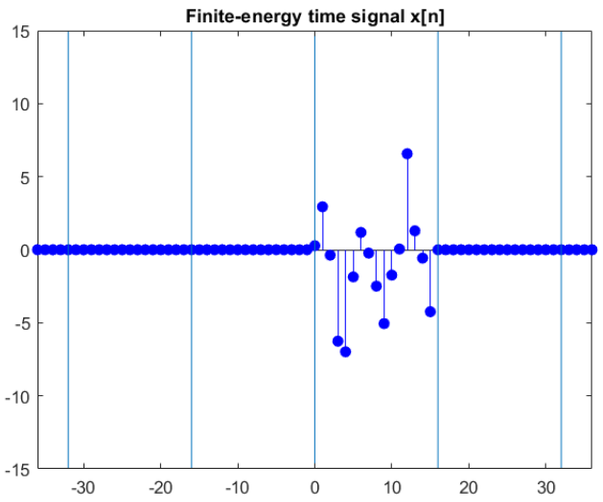
Nonperiodic Time Domain Signal

$$x[n] = \int_0^1 \hat{x}(\theta) e^{2\pi i n\theta}\, d\theta$$

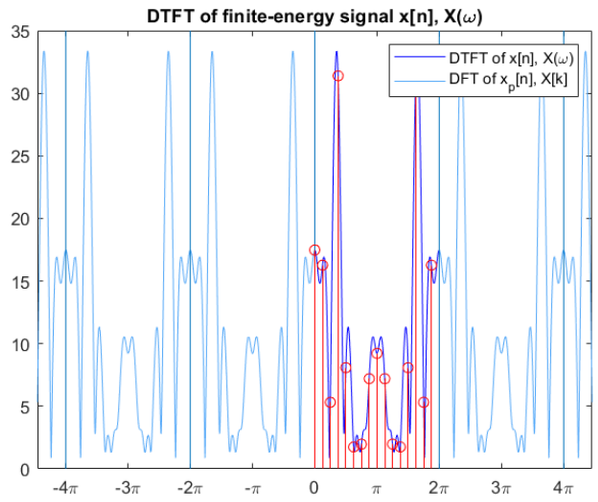
$$x(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega k}\, d\omega$$



Discrete Time Fourier Transform (DTFT)

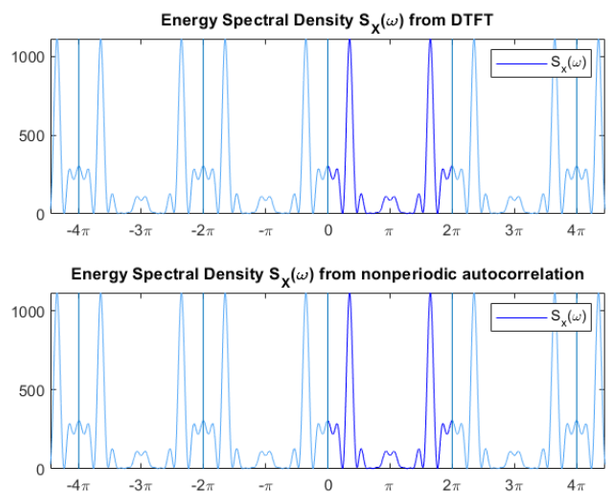
$$\hat{x}(\theta) = \sum_{n=-\infty}^{\infty} x[n] e^{-2\pi i n\theta}$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} x(k) e^{-j\omega k}$$



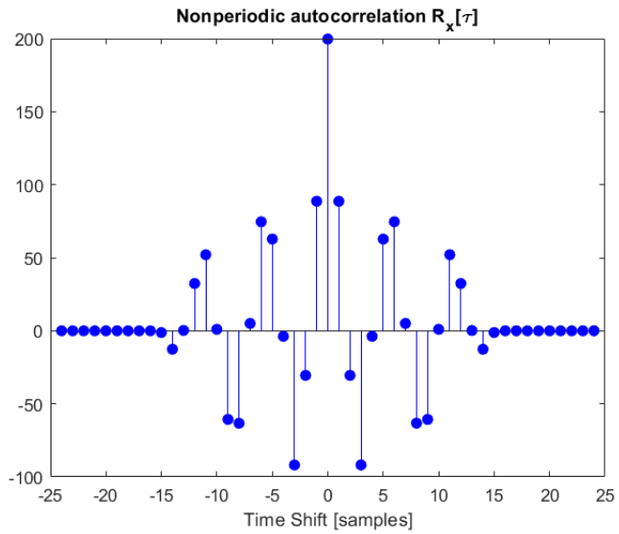
Energy Spectral Density

$$S_x(\omega) = |X(\omega)|^2 = \sum_{\tau=-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau}$$



Nonperiodic Autocorrelation

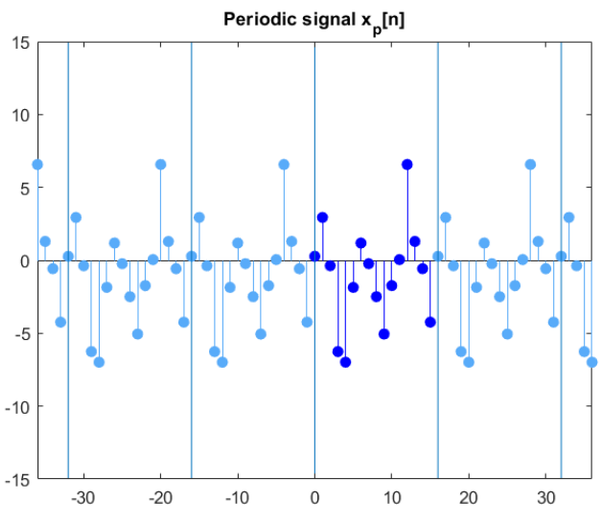
$$R_x(\tau) = \sum_{k=-\infty}^{\infty} x(k)x(k-\tau)$$



Periodic Time Domain Signal

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} \hat{x}[k] e^{2\pi i n\frac{k}{N}}$$

$$x(k) = \frac{1}{N}\sum_{n=0}^{N-1} X(\omega_n) e^{j\omega_n k}, \quad \omega_n = 2\pi\frac{n}{N}$$



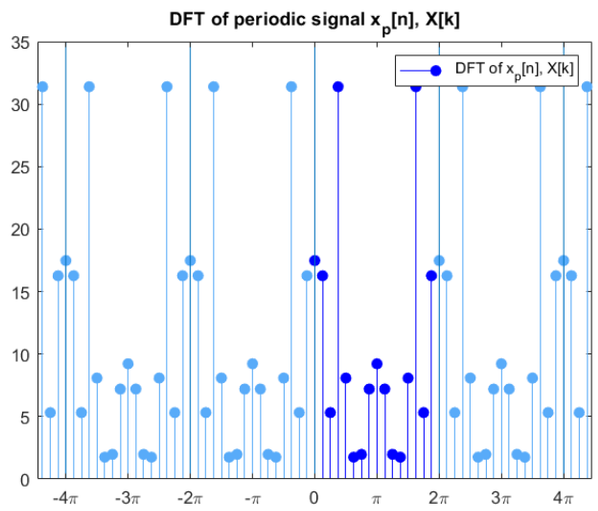
Discrete Fourier Transform (DFT)

$$\hat{x}[k] = \sum_{n=0}^{N-1} x[n] e^{-2\pi i k\frac{n}{N}}$$

$$k \in \{0, \dots, N-1\}$$

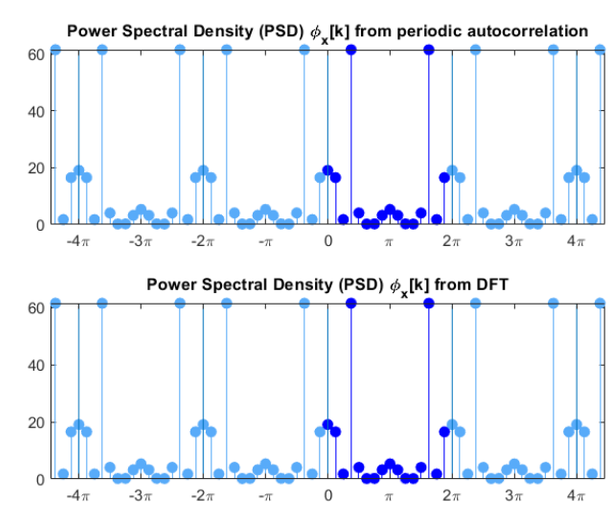
$$X(\omega_n) = \sum_{k=0}^{N-1} x(k) e^{-j\omega_n k}, \quad \omega_n = 2\pi\frac{n}{N}$$

$$\omega_n \in \left\{0, \dots, 2\pi\left(1-\frac{1}{N}\right)\right\}$$



Power Spectral Density (PSD)

$$\phi(\omega_n) = \frac{1}{N}|X(\omega_n)|^2 = \sum_{\tau=0}^{N-1} R_x(\tau) e^{-j\omega_n\tau}$$



Periodic Autocorrelation

$$R_x(\tau) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)x(k-\tau)$$