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Regularization

F. Thamm, Z. Yang, K. Breininger, S. Jaganathan, C. Liu, L. Rist, N. Maul, L. Folle, N. Gourmelon, Z. Pan, M. Zinnen, K. Packhäuser, B. Geissler, S. Sun, M. Vornehm, J. Wolf, E. Wittmann, W. Karbole, N. Panchi

Pattern Recognition Lab, Friedrich-Alexander University of Erlangen-Nürnberg

May 20, 2024



Tasks in this exercise

1. Optimization Constraints: Augmenting the loss function
2. Dropout **Layer**
3. Batch Normalization **Layer**
4. LeNet: Put everything together (**optional**)
5. RNN layer: Elman Unit
6. LSTM layer: Backpropagation at its best! (**optional**)



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Optimization Constraints: Loss function augmentation



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 - Implement constraints as separate classes
- **Independent** of loss function
- Constraints **only need** current weights
- Add constraint objects in the optimizer
- Since constraints generate part of the loss:
- Change Neural Network container class (and associated classes) to “channel” and gather **regularization loss** for **all layers**

Workflow

- Forward pass
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- Backward pass
- In each trainable layer, include **the gradient of norm** when calculating **update**

L₂ regularization

- Forward pass:

$$\tilde{L}(\mathbf{w}) = L(\mathbf{w}) + \lambda \|\mathbf{w}\|_2^2$$

- Backward pass:

$$\mathbf{w}^{(k+1)} = \underbrace{(1 - \eta \lambda) \mathbf{w}^{(k)}}_{\text{Shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$

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L_2 regularization

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- Notice for matrices we compute here the Frobenius norm, not the Spectral norm.
- The influence of constraints is controlled via λ . Because `lambda` is a python keyword, you want to use e.g. `alpha` instead.

L_1 regularization

- Forward pass:

$$\tilde{L}(\mathbf{w}) = L(\mathbf{w}) + \lambda \|\mathbf{w}\|_1$$

- Backward pass:

$$\mathbf{w}^{(k+1)} = \underbrace{\mathbf{w}^{(k)} - \eta \lambda \text{sign}(\mathbf{w}^{(k)})}_{\text{Other shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$



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Dropout



Method

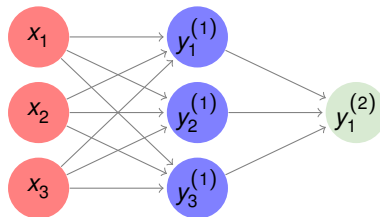


Figure: Dropout

- Implement this as a **fixed-function layer**

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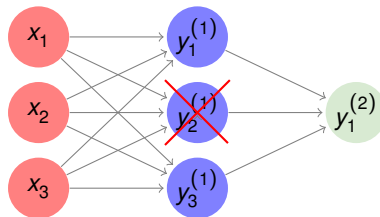


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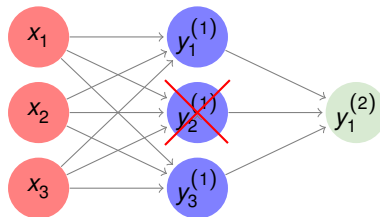


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Inverted Dropout

- Can we get rid of the dropout layer at test-time?
- Change the behavior during training
- Multiply activations in forward-pass **only during training** by $\frac{1}{p}$
- Note: the backward pass has to be adapted as well!



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Batch normalization



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- β, γ and μ_B, σ_B^2 have same **dimension** to be able to preserve **identity**
- Notice that β is a **bias**

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- Therefore a **moving average** is common:

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- Moving average **decay** α (e.g. 0.8)
- The exponent (k) and (k-1) are iteration-indices!

Backward pass

- Gradient **with respect to weights** is simply:

$$\frac{\partial L}{\partial \gamma} = \sum_{b=1}^B \frac{\partial L}{\partial \hat{\mathbf{Y}}_b} \tilde{\mathbf{X}}_b = \sum_{b=1}^B \mathbf{E}_b \tilde{\mathbf{X}}_b$$

- For the **bias** likewise we have:

$$\frac{\partial L}{\partial \beta} = \sum_{b=1}^B \frac{\partial L}{\partial \hat{\mathbf{Y}}_b} = \sum_{b=1}^B \mathbf{E}_b$$

Backward pass

The **gradient with respect to the input** is more complicated, but here it is:

$$\frac{\partial L}{\partial \tilde{\mathbf{X}}} = \frac{\partial L}{\partial \hat{\mathbf{Y}}} \odot \gamma$$

$$\frac{\partial L}{\partial \sigma_B^2} = \sum_{b=1}^B \frac{\partial L}{\partial \tilde{\mathbf{X}}_b} \odot (\mathbf{x}_b - \mu_B) \odot \frac{-1}{2} (\sigma_B^2 + \epsilon)^{\frac{-3}{2}}$$

$$\frac{\partial L}{\partial \mu_B} = \left(\sum_{b=1}^B \frac{\partial L}{\partial \tilde{\mathbf{X}}_b} \odot \frac{-1}{\sqrt{\sigma_B^2 + \epsilon}} \right) + \underbrace{\frac{\partial L}{\partial \sigma_B^2} \odot \frac{\sum_{b=1}^B -2(\mathbf{x}_b - \mu_B)}{B}}_0$$

$$\frac{\partial L}{\partial \mathbf{X}} = \frac{\partial L}{\partial \tilde{\mathbf{X}}} \odot \frac{1}{\sqrt{\sigma_B^2 + \epsilon}} + \frac{\partial L}{\partial \sigma_B^2} \odot \frac{2(\mathbf{X} - \mu_B)}{B} + \frac{\partial L}{\partial \mu_B} \odot \frac{1}{B}$$

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 - we can **reshape** the $B \times H \times M \times N$ tensor to $B \times H \times M \cdot N$
 - because of our format we have to **transpose** from $B \times H \times M \cdot N$ to $B \times M \cdot N \times H$
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- ... and do the **same** in the **backward pass**



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LeNet (optional)



LeNet architecture

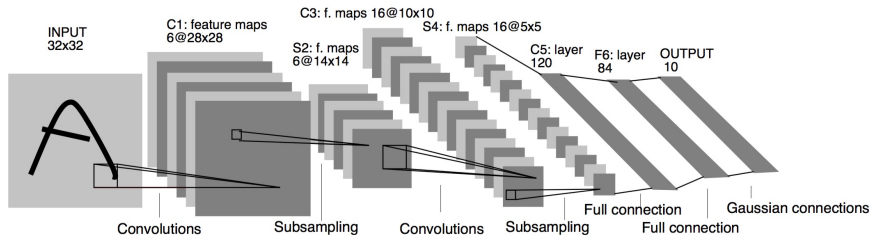


Figure: LeNet

Modified LeNet architecture

Deviations

- Input is 28×28
- Our conv only supports “same” padding - so C3 has **larger activation maps**
- Input to **C5** is also **larger**
- We only implemented ReLUs, so **no** TanH
- We also use the implemented SoftMax **instead of** RBF units

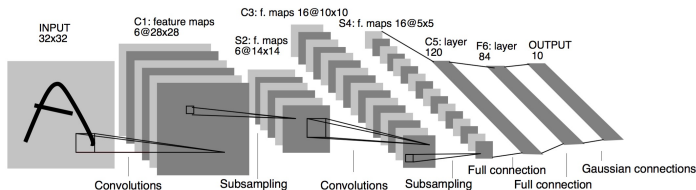


Figure: LeNet



Thanks for listening.
Any questions?