FRIEDRICH-ALEXANDER-
UNIVERSITÄT
ERLANGEN-NÜRNBERG
SCHOOL OF ENGINEERING

Recurrent Neural Networks

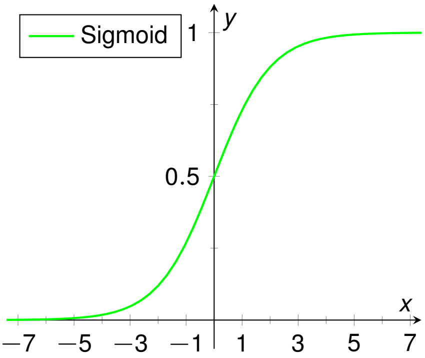
F. Thamm, Z. Yang, K. Breininger, S. Jaganathan, C. Liu, L. Rist, N. Maul, L. Folle, N. Gourmelon, Z. Pan, M. Zinnen, K. Packhäuser, B. Geissler, S. Sun, M. Vornehm, J. Wolf, E. Wittmann, W. Karbole, N. Panchi

Pattern Recognition Lab, Friedrich-Alexander University of Erlangen-Nï¿½rnberg

May 20, 2021

Activation Functions

Sigmoid Activation Function



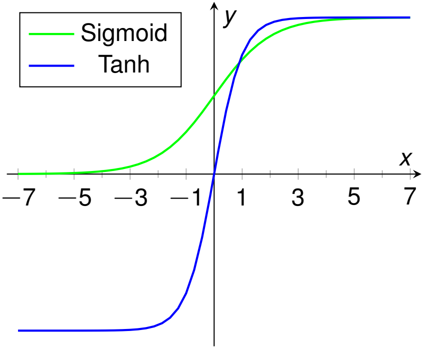
Sigmoid (logistic function)

$$f(x) = \frac{1}{1 + exp(-x)}$$

$$f'(x) = f(x)(1 - f(x))$$

➔ Observe that the derivative can be solely expressed in terms of the activation!

Tanh Activation Function



Tanh

$$f(x) = tanh(x)$$
$$f'(x) = 1 - f(x)^2$$

➔ The derivative is still a function of the activation!

Elman Recurrent Neural Network

General strategy

- We interpret the **batch** dimension as **time** dimension now

General strategy

- We interpret the **batch** dimension as **time** dimension now

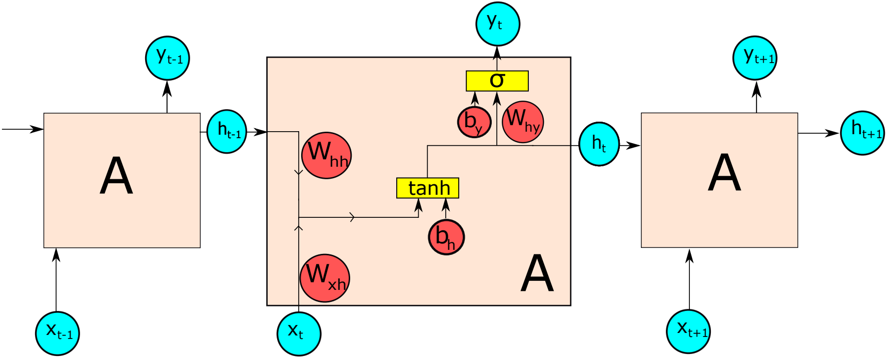
➔ Samples are correlated in this dimension

General strategy

- We interpret the **batch** dimension as **time** dimension now

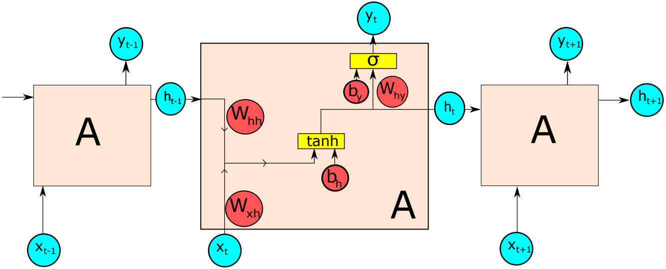
➔ Samples are correlated in this dimension

- This allows to **reuse** loss functions, optimizers, initializers, activation functions and the Neural Network class



y_t
y_{t-1}
σ
y_{t+1}
b_y
W_{hy}
h_{t-1}
h_t
h_{t+1}
W_{hh}
A
tanh
A
b_h
W_{xh}
A
x_{t-1}
x_t
x_{t+1}

Elman RNN Cell



Output formula:

$$\mathbf{y}_t = \sigma\left(\mathbf{h}_t \cdot \mathbf{W}_{hy} + \mathbf{b}_y\right)$$

$\mathbf{W}_{hy}$: Weight matrix for current hidden state $\mathbf{h}_t$

$\mathbf{b}_y$: Output bias

A word on software engineering

- In terms of **encapsulation** - how good was the idea to demand exposition of the weights as member?

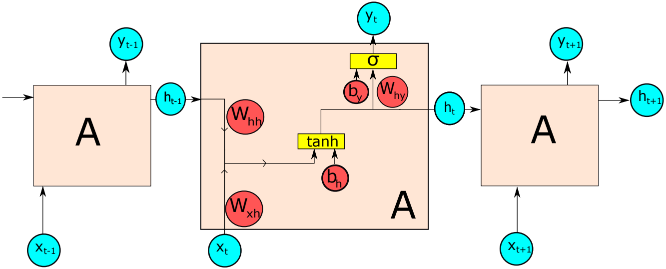
A word on software engineering

- In terms of **encapsulation** - how good was the idea to demand exposition of the weights as member?
- Suppose we implement the RNN cell as **composite** structure
- **Getters** and **Setters** provide us the flexibility to do so

A word on software engineering

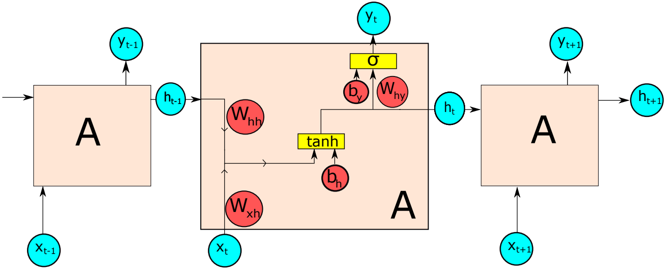
- In terms of **encapsulation** - how good was the idea to demand exposition of the weights as member?
- Suppose we implement the RNN cell as **composite** structure
- **Getters** and **Setters** provide us the flexibility to do so
- Takeaway? Not doing **proper software engineering** most of the time will demand a price at some point.

Elman RNN Cell



$$\mathbf{h}_t = \tanh\left(\mathbf{h}_{t-1} \cdot \mathbf{W}_{hh} + \mathbf{x}_t \cdot \mathbf{W}_{xh} + \mathbf{b}_h\right)$$

Elman RNN Cell



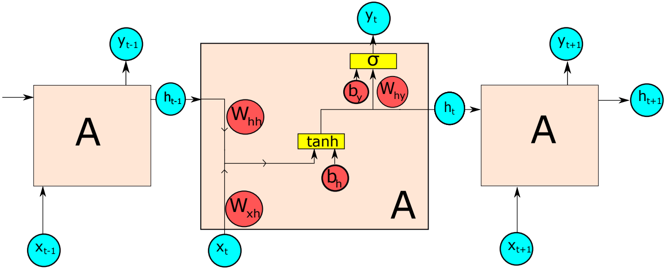
$$\mathbf{h}_t = \tanh\left(\mathbf{h}_{t-1} \cdot \mathbf{W}_{hh} + \mathbf{x}_t \cdot \mathbf{W}_{xh} + \mathbf{b}_h\right)$$

$\mathbf{W}_{hh}$: Weight matrix for previous hidden state $\mathbf{h}_{t-1}$

$\mathbf{W}_{xh}$: Weight matrix for current input $\mathbf{x}_t$

$\mathbf{b}_h$: Update bias

Elman RNN Cell



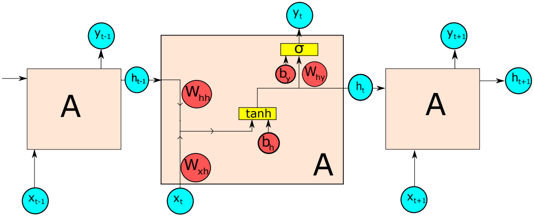
$$\mathbf{h}_t = \tanh\left(\tilde{\mathbf{x}}_t \cdot \mathbf{W}_h\right)$$

$\mathbf{W}_h$: Weight matrix of a fully connected layer

$\tilde{\mathbf{x}}_t$: Concatenation of $\mathbf{x}_t$, $\mathbf{h}_{t-1}$ and a 1

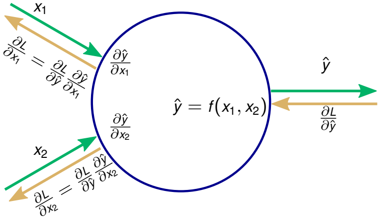
Different from output: Not processed independently!

Backward



- Most gradients are handled by the **embedded layers**
- **Store and feed the values for backprop (input tensors, activations) externally** to the embedded layers because of **multiple forward calls**
- We need gradients through **summation, multiplication and copying**

Backward



Sum

$$f(x_1, x_2) = x_1 + x_2$$
$$\frac{\partial \hat{y}}{\partial x_1} = 1$$

Gradient is **copying** $\frac{\partial L}{\partial \hat{y}}$

Multiply

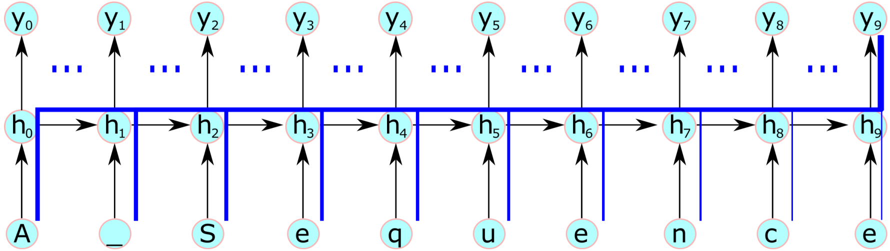
$$f(x_1, x_2) = x_1 \cdot x_2$$
$$\frac{\partial \hat{y}}{\partial x_1} = x_2$$

Gradient is $\cdot$ with **switched inputs**

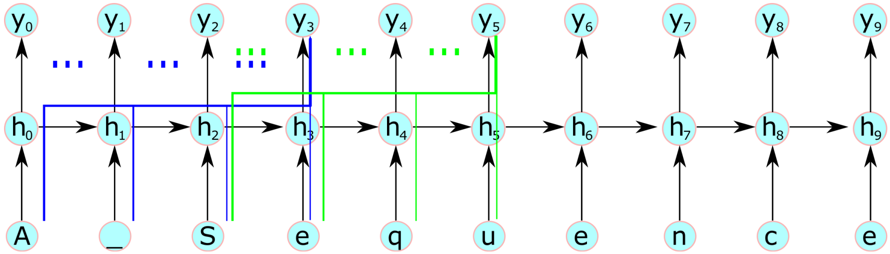
Copy

Backward pass of sum

So the gradient is a sum!

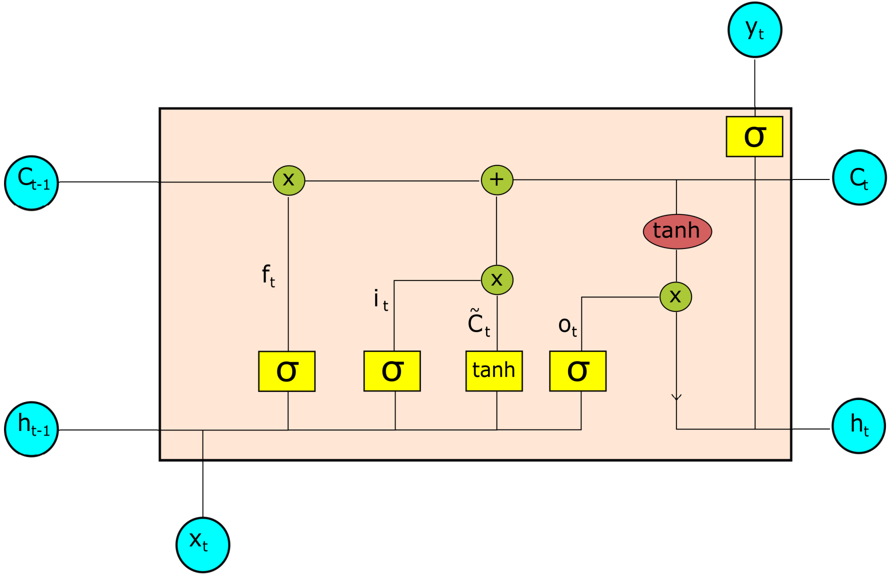


- Implemented by passing the whole sequence as a **batch**



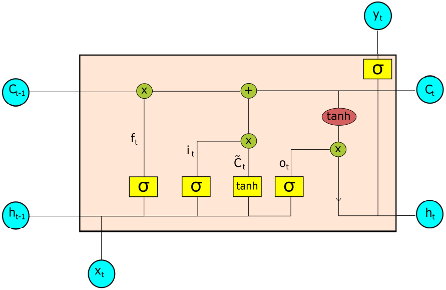
- Implemented by passing **overlapping** parts as a **batch**
- We need to implement memory **between states**
- Simply store the **last hidden state** and implement a **method** switching whether this state is reused in subsequent forward passes.
- Data has to be fed in **accordingly**!
- Referencing the TBPPT Algorithm presented in the lecture: k_1 is always the sequence length and k_2 is always the TBPTT length.

Long Short-Term Memory (optional)



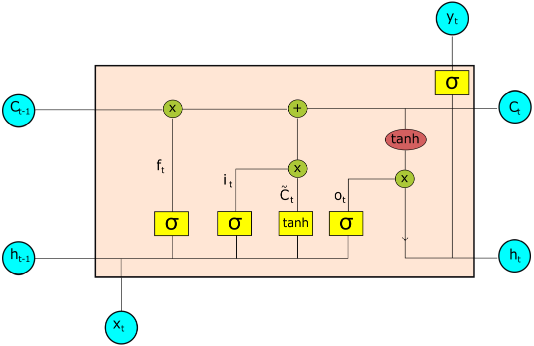
y_t
σ
C_{t-1}
x
+
C_t
tanh
f_t
x
i_t
$\tilde{C}_t$
o_t
x
σ
σ
tanh
σ
h_{t-1}
h_t
x_t

Forward



- We can reuse a **fully connected** layer again to for the **output**
- The **concatenation** is also **analogous** to the RNN
- The gates σ and the yellow $\mathtt{tanh}$ can be a single **fully connected** layer with an output size of $4 \cdot$ **dim(hidden state)**
- Remember that we have to pass the vectors of the input tensor **sequentially**

Backward



- Most gradients are again handled by the **embedded layers**
- Again **store and feed the values for backprop externally** to the embedded layers

Thanks for listening.

Any questions?