Assessed coursework for CS2013 in October/November 2016

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Question about Formal Languages

a) L is a context free language of strings consisting of an arbitrary number of the symbol ‘a’ followed by an arbitrary number of the symbol ‘b’. It is limited to the symbols a, b and λ.

b) The regular expression denoting L is: (a)* + (b)*

c) Statement: no string in the language L has ‘ba’ as a substring

Base Step

P(n) /= include substring ‘ba’
P(n) = P(0) = λ
We could also use P(n) as P(1) to get e.g. a
P(1) = aS or Sb or a or b or λ
These do not include the substring ‘ba’

Induction Step:

P(n+1) /= include substring ‘ba’
Case 1: P(n+1) = ab
Case 2: P(n+1) = aabb
In terms of the result, P(n) = P(n+1) as neither has the required substring.

If the theorem holds for any string longer than 2, it holds for any string. This is because there is no way for the substring ‘ba’ to be generated in any future n results.

Question about Logic

a) Yes, this idea is expressible in First Order Predicate Logic.

-(Ax(F(x)))
OR
Ex F(x) & Ax Ay ((F(x) & F(y)) → x = y)
OR
Ex Ey (F(x) & F(y) & x ≠ y & Av (F(v) → (z = x OR z = y)))
OR
Ex Ey Ez (F(x) & F(y) & F(z) & x ≠ y & x ≠ z & y ≠ z & Av (F(v) → (v = x OR v = y OR v = z)))
A simpler way:
-(Ax(F(x))) $\vee \exists!x\, F(x) \vee \exists 2!x\, F(x) \vee \exists 3!x\, F(x)$

b) Yes, this idea is expressible in First Order Predicate Logic.

Using the inverse of 'even'

-(Ax (G(x) ↔ (Ey x=2y)))

c) No, this idea is not expressible in First Order Predicate Logic unless we allow infinite conjunctions.

If y = there exists finitely many x such that H(x)
Then –y = There exists infinitely many, and such a formula does not exist.

Therefore, disproving either one automatically proves both of the statements above.

Compactness Theorem: "If all finite subsets of a set S of FOPL formulas are satisfiable, then S itself is satisfiable."

We can prove this with a simple example like: ExH(x)

With this example, we can define a qn where Ex1 until Exn all have different values until n elements.
With this information, we can define S = {p} $\cup$ {q_n: n>=1}, where p has a model of arbitrary finite size. Now, if all finite subsets of S are satisfiable, it follows that S itself is satisfiable. Therefore the model M of S has infinitely many elements, concluding that no FOPL formula can state "there are infinitely many things that have the property H.