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Knowledge-Based Systems

A Formal Introduction to Description Logic (II)

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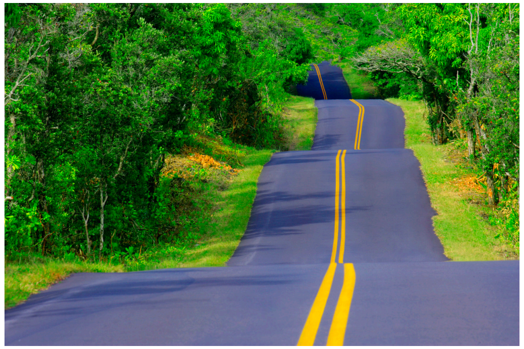
Roadmap



- Foundation
  - KR, ontology and rule; set theory
- Knowledge capture
- Knowledge representation
  - Ontology: Semantic Web standards RDF and OWL, Description Logics
  - Rule: Jess
- Knowledge reasoning
  - Ontology: formal semantics, tableaux algorithm
  - Rule: forward chaining, backward chaining
- Knowledge reuse and evaluation
- Meeting the real world
  - Jess and Java, Uncertainty, Invited talk

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Schema in a Knowledge Based System

1) Allow schema constraints, such as DisjointClasses (UndgStudent MastStudent)

UndgStudent ID	Name	take-course
csd:p001	John	csd:cs3014
~~csd:p002~~	Tom	csd:cs3025

MastStudent ID	Name	take-course
csd:p008	Yuan	csd:cs5010
~~csd:p002~~	Tom	csd:cs5017

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Schema in a Knowledge Based System

2) Allow some reasoning based on axioms (open world assumption), such as SubClassOf (MastStudent Student)

Student ID	Name	take-course
csd:p001	John	csd:cs3015
csd:p002	Tom	csd:cs3025

MastStudent ID	Name	take-course
csd:p008	Yuan	csd:cs5010
csd:p002	Tom	csd:cs5017

thus all the students include csd:p001, csd:p002, and csd:p008

rdf:range and Foreign Key: Revisit

- They are quite similar but not exactly the same,
  - due to the difference between open and closed world assumptions

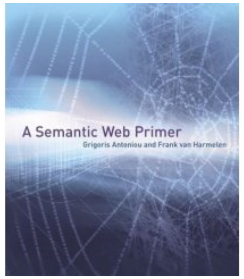
Student ID	Name	take-course
p001	John	~~cs3015~~
p002	Tom	cs3025

Course ID	Title	coordinator
cs3017	AIS	AS
cs3025	KBS	JP

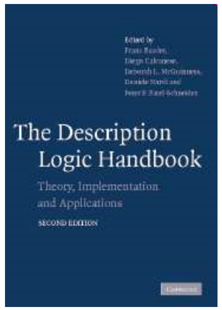
- Semantics of rdfs:range ([rdfs3])
  - [p rdfs:range D .] [a p b .] => [b rdf:type D .]

Lecture Outline

- Motivation
- Introduction to Semantics of DL axioms
- More detailed discussions on DL Semantics
- Practical



[Chapter 4]



[Sections: 2.2.1.1, 2.2.1.2, 2.2.2.1 2.2.2.2]

Motivations:

- DL representation and reasoning
  - DL syntax
  - Semantics
  - Reasoning
- The role of semantics
  - To give formal meaning for the valid sentences/axioms
  - To provide a foundation for defining reasoning problems

Lecture Outline

- Motivation
- Introduction to Semantics of DL axioms
  - The big picture
- More detailed discussions on DL Semantics
- Practical

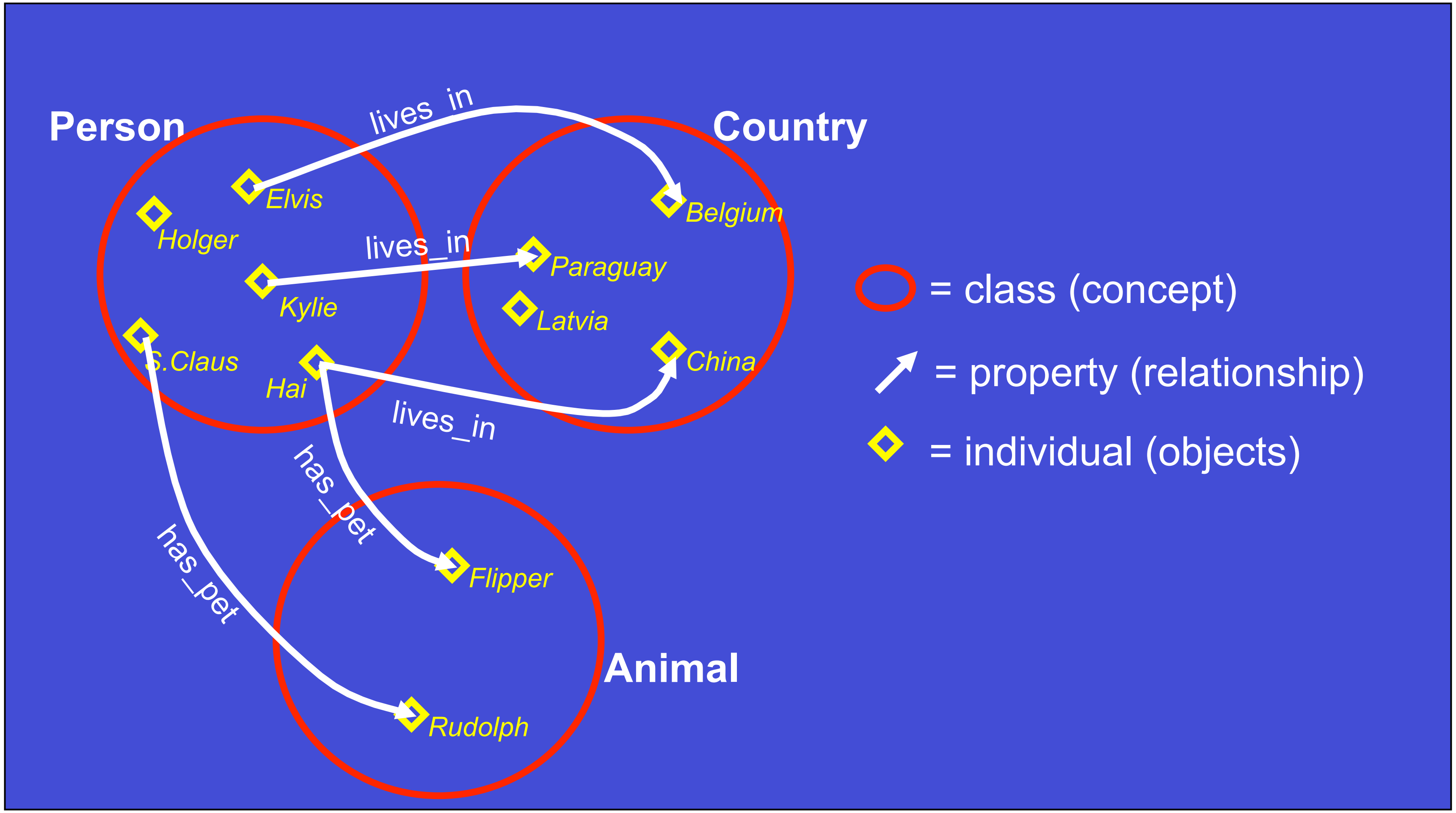
DL Interpretations

- An interpretation **I** is written as $(\Delta^I, \bullet^I)$
  - Δ^I is the **domain** (similar to universal set)
  - $\bullet^I$ is the **interpretation function**
    - all individuals are members of the domain: $o^I \in \Delta^I$
    - all classes are subsets of the domain $A^I \subseteq \Delta^I$
      - e.g., $Employee^I$= {E1, E2, E3, E4}
    - all properties are subsets $R^I \subseteq \Delta^I \times \Delta^I$

      e.g., $Works\text{-}for^I$= {<E1,P1>, <E2,P1>, <E2,P2>, <E3,P1>, <E3,P2>, <E4,P2>}

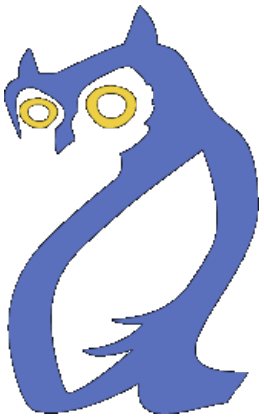
- Domain is a mathematical representation of the world
- Interpretation function allows us to consider all possible assignment of class and property memberships
  - all possible databases for the given schema

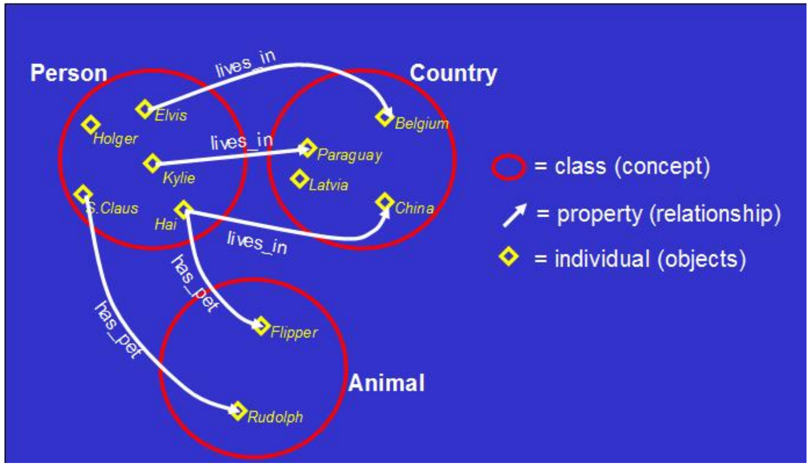
Example: DL Interpretations



[Picture Credit: Protégé Team]

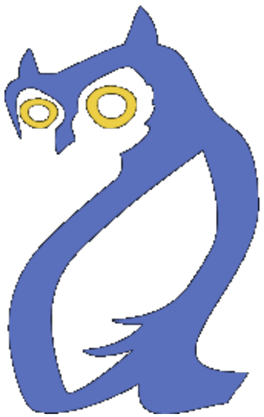
Example: DL Interpretations (II)

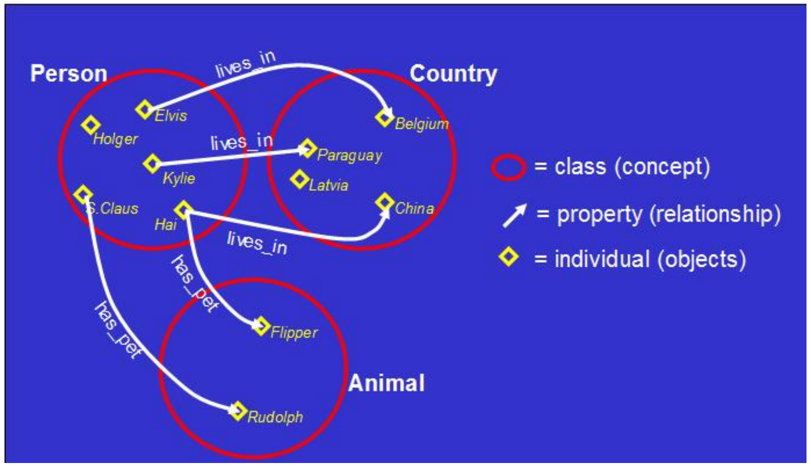




- Δ^I = {Elvis, Holger, …}
- Named objects
  - Elvis^I = Elvis
  - Holger^I = Holger
  - …
- Named classes
  - Animal^I = {Flipper, Rudolph}
  - Person^I = {Elvis,Holger,Kylie,Hai,S.Claus}
  - Country^I = {Belgium,Paraguar,Latvia, China}
- Named properties
  - has_pet^I = {<Hai,Plipper>, <S.Claus,Rudolph>}
  - lives_in^I = {<Elvis,Brlgium>, <Kylie,Paraguar>, <Hai, China> }

Example: DL Interpretations (III)

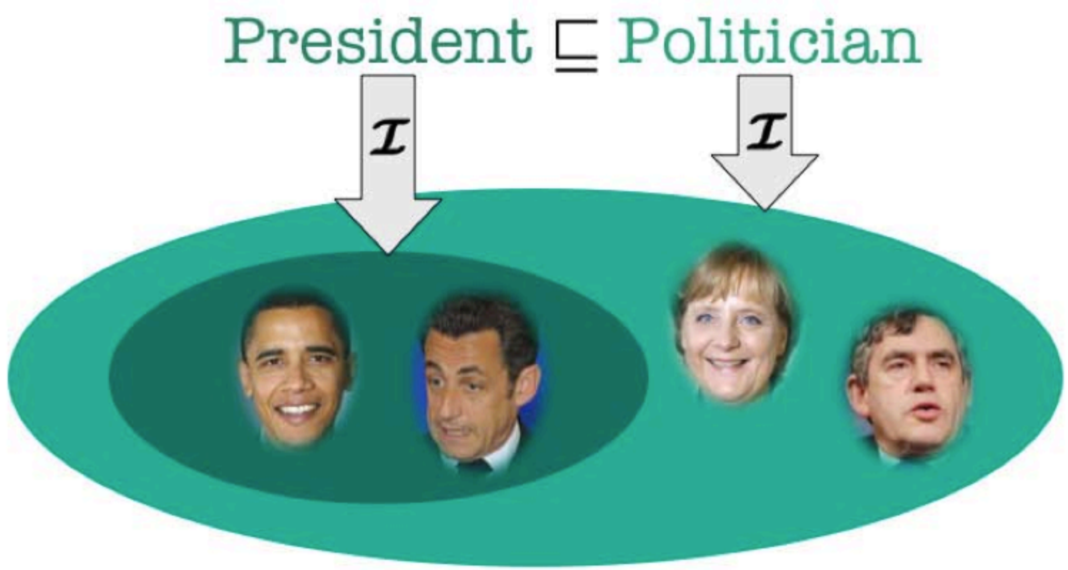




- Suppose we extend the vocabulary with
  - Young
- Given the following interpretation of Young:
  - $Young^I$ ={Holger, Hai, Kylie, Flipper}
  - How about the interepretation of the OWL class description?
    - Young ⊓ Person = {Holger, Hai, Kylie}
    - ∃has_pet.Young = {Hai}

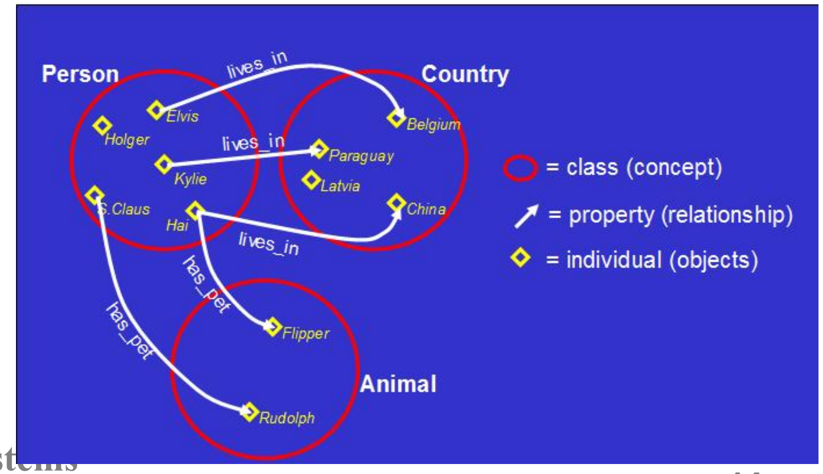
Axioms

- Axioms are used to “filter out” invalid interpretations from valid ones
  - An interpretation I is a model for an ontology O if it satisfies all its axioms
  - An ontology O is consistent if it has some model (valid interpretation).



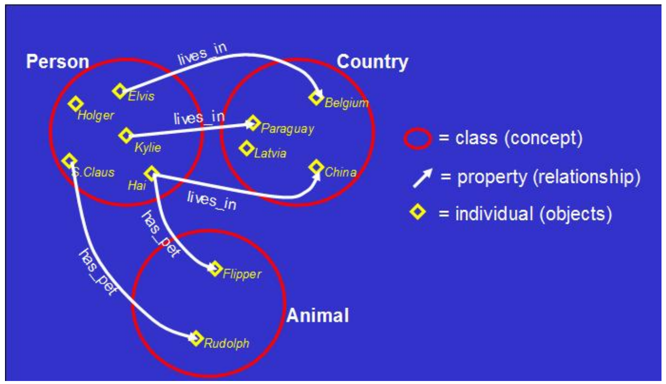
Interpretations of Class Axioms

- Class inclusion axioms
  - An interpretation **I** satisfies a class inclusion axiom $C \sqsubseteq D$ if $C^I \subseteq D^I$
- Class equivalence axioms
  - An interpretation **I** satisfies a class equivalence axiom $C \equiv D$ if $C^I = D^I$
- Does the given interpretation satisfy the following class axioms?
  - Person $\sqsubseteq$ Animal **false**
  - Person $\equiv$ Animal **false**
  - Person $\sqsubseteq \neg$Animal **true**



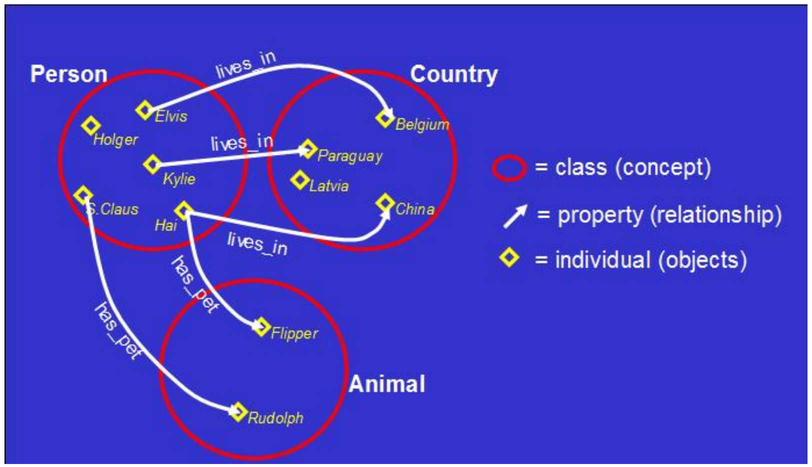
Interpretations of Property Axioms

- Property inclusion axioms
  - An interpretation **I** satisfies a property inclusion axiom $R1 \sqsubseteq R2$ if $R1^I \subseteq R2^I$
- Property equivalence axioms
  - An interpretation **I** satisfies a property equivalence axiom $R1 \equiv R2$ if $R1^I = R2^I$
- Does the given interpretation satisfy the following property axioms?
  - $has\_pet \sqsubseteq lives\_in$ **false**
  - $has\_pet \equiv lives\_in$ **false**
  - $\exists has\_pet.\top \sqsubseteq Person$ **true**
  - $\exists has\_pet^{-}.\top \sqsubseteq Animal$ **true**



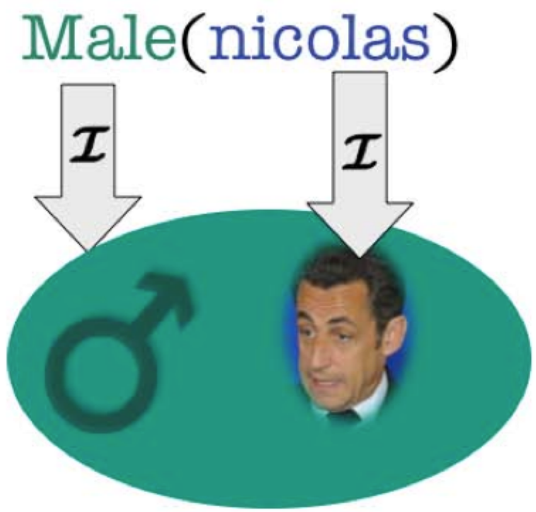
Interpretations of Property Axioms (II)

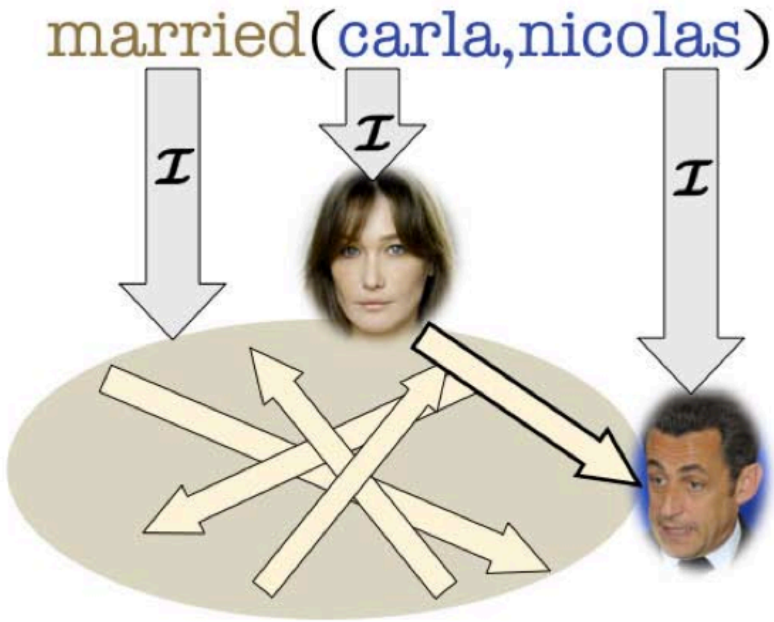
- Transitive Property axioms
  - An interpretation **I** satisfies a transitive property axiom Trans(R) if, for any a, b, c, $\langle a,b\rangle \in R^{I}$ and $\langle b,c\rangle \in R^{I}$ implies $\langle a.c\rangle \in R^{I}$
- Functional Property axioms
  - An interpretation **I** satisfies a functional property axiom Func(R) if, for all x, $\#\{y \mid \langle x,y\rangle \in R^{I}\} \leq 1$
- Does the given interpretation satisfy the following axioms?
  - Func(lives_in) **true**



Interpretations of Individual Axioms

- Class assertions
  - An interpretation **I** satisfies a class assertion a:C if $a^I \in C^I$
- Property assertions
  - An interpretation **I** satisfies a property assertion <a,b>:R if $< a^I , b^I > \in R^I$

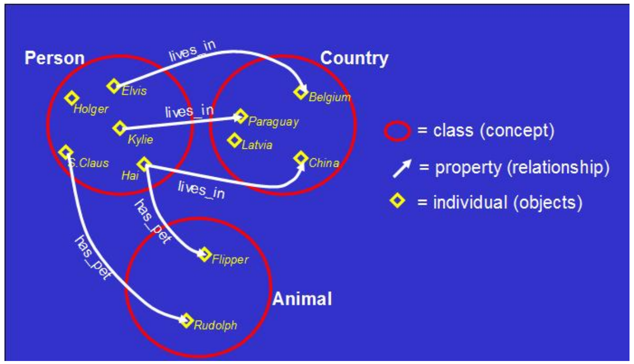




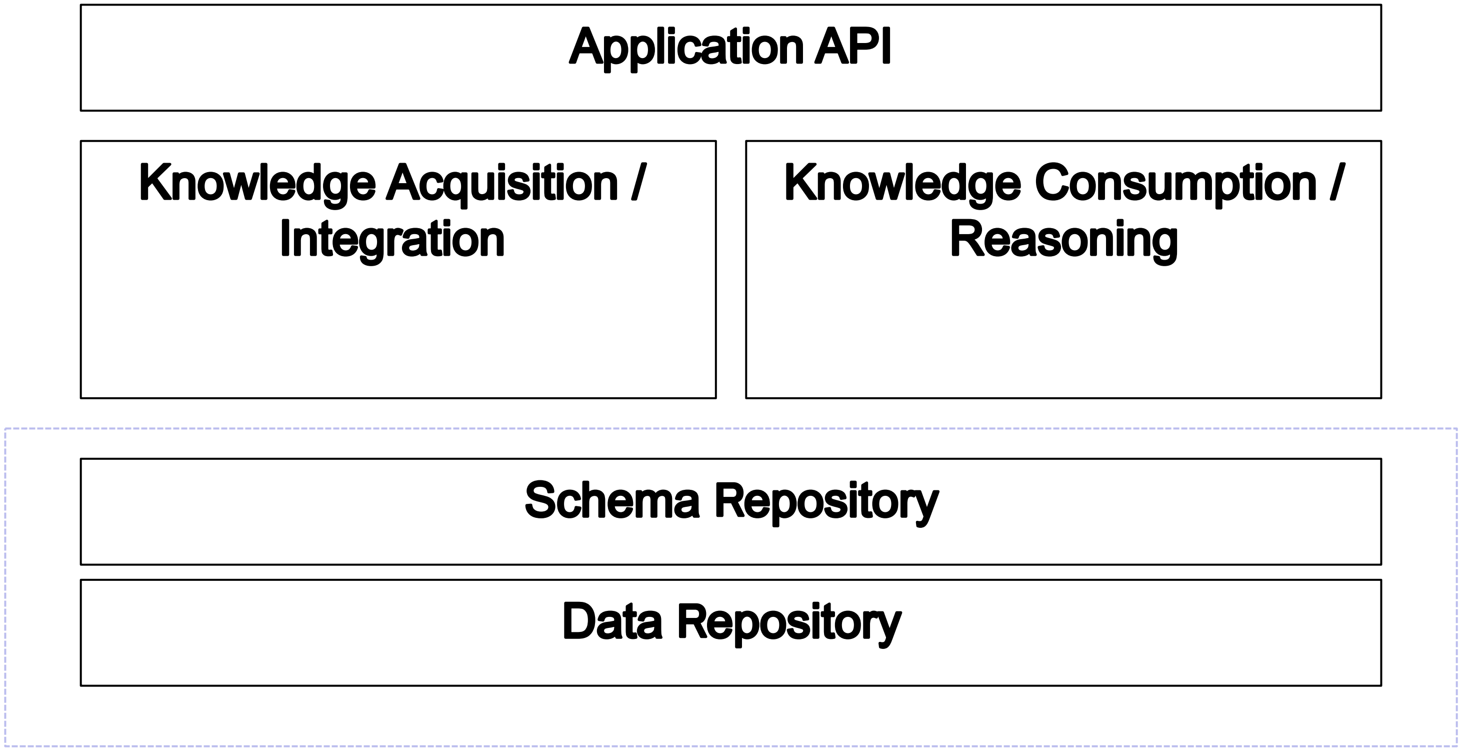
Interpretations of Individual Axioms

- Class assertions
  - An interpretation **I** satisfies a class assertion a:C if $a^{I} \in C^{I}$
- Property assertions
  - An interpretation **I** satisfies a property assertion <a,b>:R if $< a^{I}, b^{I} > \in R^{I}$
- Equality assertions
  - An interpretation **I** satisfies an equality assertion a=b if $a^{I} = b^{I}$
- Inequality assertions
  - An interpretation **I** satisfies an inequality assertion a≠b if $a^{I} \neq b^{I}$

- Does the given interpretation satisfy the following individual axioms
  - UK:Country **false**
  - <Hai,Flipper>:has_pet **true**
  - Hai=Flipper **false**
  - Hai≠Flipper **true**



Architecture of Knowledge Based Systems



Ontology and Reasoning

- Ontology contains
  - knowledge and data that
  - we know that we know
  - we know that we don't know or partially know
- Reasoning helps to find out
  - things that we might not know that we know

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Interpretations of Ontologies

- An interpretation **I** satisfies an ontology O if **I** satisfies **all** axioms in O
  - **I** is called an interpretation of O
- An ontology O is called **consistent** if there exists (at least) **one** interpretation that satisfies O
- A class C is **satisfiable** (w.r.t an ontology O) if there exists **one** interpretation **I** of O, such that C^{I} is not empty
- **Entailment (|=)**: given an axiom α, we say an ontology O entails the axiom α if and only if **all** interpretation **I** of O satisfy α.

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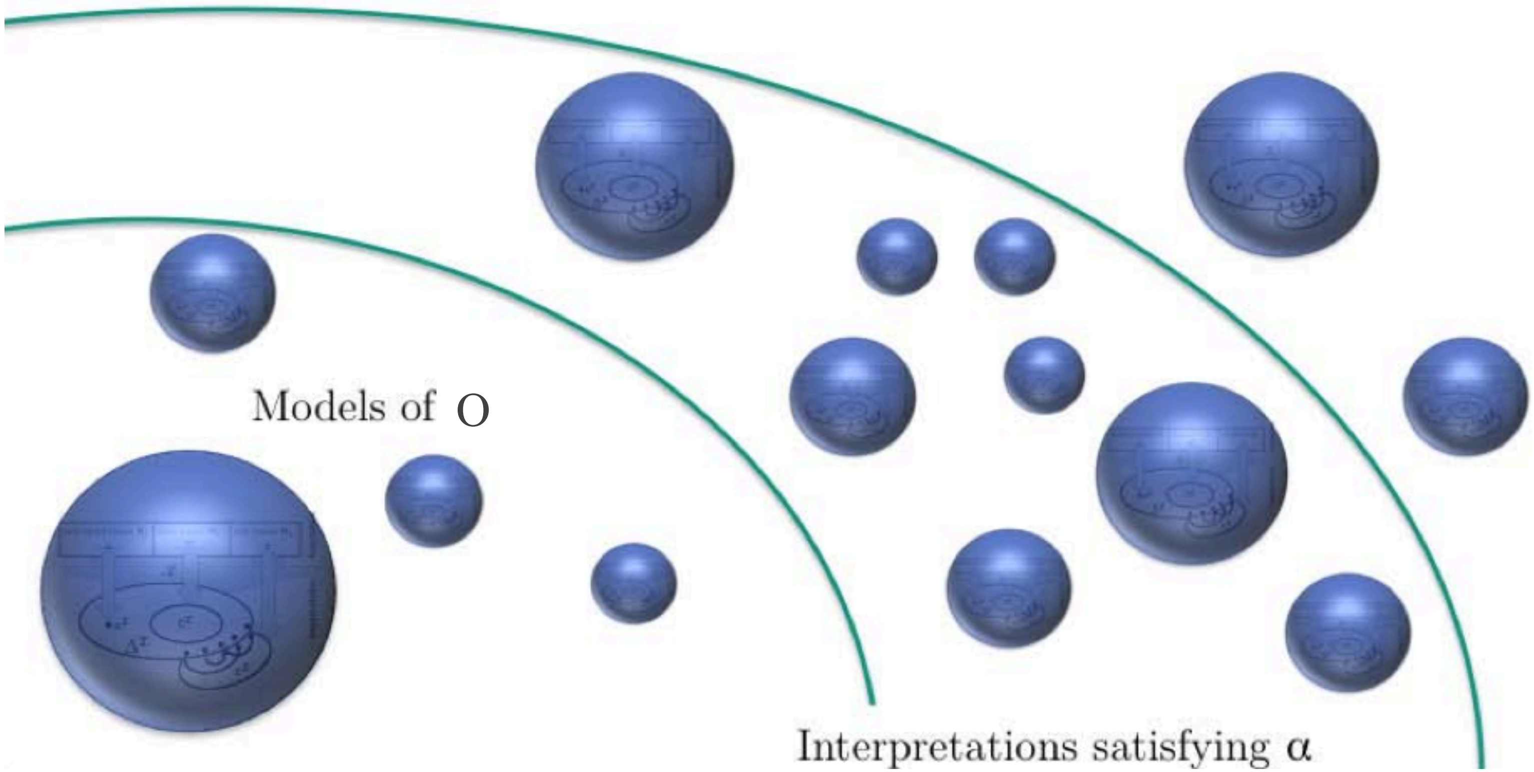
Entailments of Axioms

- **Entailment (|=)**: given an axiom α, we say an ontology O entails the axiom α if and only if **all** interpretation **I** of O satisfy α.



Standards DL Reasoning Services

- "Easier" reasoning services
  - whether O is consistent
  - whether a given class is satisfiable
- "Harder" reasoning services
  - whether O entails a class inclusion axiom
    - such as Class (English partial People)
  - whether O entails an individual axiom
    - such as Individual (Bill type (English))

Lecture Outline

- Motivation
- Introduction to Semantics of DL Axioms
- More detailed discussions on DL Semantics
- Practical

“Harder” Reasoning Services

How to support the “harder” reasoning services

- by proofs
  - with the help of class equivalence (next slide)
- by reduction to the “easier” reasoning services

Class Equivalence

Two class descriptions are called "equivalent" (written as C ≡ D) if for every single interpretation I we have $C^I=D^I$.

$$C \sqcap D \equiv D \sqcap C \qquad C \sqcup D \equiv D \sqcup C$$
$$(C \sqcap D) \sqcap E \equiv C \sqcap (D \sqcap E) \qquad (C \sqcup D) \sqcup E \equiv C \sqcup (D \sqcup E)$$
$$C \sqcap C \equiv C \qquad C \sqcup C \equiv C$$

$$(C \sqcup D) \sqcap E \equiv (C \sqcap E) \sqcup (D \sqcap E) \qquad (C \sqcup D) \sqcap C \equiv C$$
$$(C \sqcap D) \sqcup E \equiv (C \sqcup E) \sqcap (D \sqcup E) \qquad (C \sqcap D) \sqcup C \equiv C$$

$$\neg\neg C \equiv C$$
$$\neg(C \sqcap D) \equiv \neg D \sqcup \neg C$$
$$\neg(C \sqcup D) \equiv \neg D \sqcap \neg C$$

$$\neg\exists r.C \equiv \forall r.\neg C$$
$$\neg\forall r.C \equiv \exists r.\neg C$$
$$\neg\leqslant nr.C \equiv \geqslant(n+1)r.C$$
$$\neg\geqslant(n+1)r.C \equiv \leqslant nr.C$$

$$\geqslant 0r.C \equiv \top$$
$$\geqslant 1r.C \equiv \exists r.C$$
$$\leqslant 0r.C \equiv \forall r.\neg C$$

Prove it!

Prove if the following equivalences hold

$$C \sqcap D \equiv D \sqcap C$$

$(C \sqcap D)^I = C^I \cap D^I$

$(D \sqcap C)^I = D^I \cap C^I = C^I \cap D^I$

$$(C \sqcap D) \sqcap E \equiv C \sqcap (D \sqcap E)$$

$$C \sqcap (D \sqcup E) \equiv (C \sqcap D) \sqcup E$$

Class Subsumption Checking

- Given an ontology O, check if, for every interpretation **I** of O, **I** |= $C^I \subseteq D^I$

- Question: Given the following ontology O,
  - Class (C complete complementOf (
    restriction (eats someValuesFrom (Plant))))
  - Class (D complete restriction (eats
    allValuesFrom (complementOf (Plant)))

  Does O entail Class(C partial D)?

Does O entails $\neg\exists eats.Plant \sqsubseteq \forall eats.(\neg Plant)$? **true**

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Class Instance Checking

- Given an ontology O, a class C and an individual x, check if for every interpretation **I** of O, $x^{\mathbf{I}}$ is in $C^{\mathbf{I}}$

- Question: given the following ontology O,
  - Class (OldLady partial

    restriction (hasPet allValuesFrom (Cat)))
  - Individual (Minnie type (OldLady)

    value (hasPet Tom))

- Does O entail Individual (Tom type (Cat)) ?

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Class Instance Checking

- Can we reduce class instance checking to another reasoning task?
- How about Ontology Consistency Checking
  - If O entails C(x), then in every interpretation I of O, we have x^I is in C^I
  - It means O U O{¬C(x)} is inconsistent

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Class Instance Checking

- Question: given the following ontology O,
  - OldLady $\sqsubseteq$ $\forall$hasPet.Cat
  - OldLady(Minnie)
  - hasPet(Minnie, Tom)

    $\neg$Cat(Tom)
- Does O entail Individual (Tom type (Cat)) ?

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Practical

- Interpretations in DL
- Reasoning based on understanding of DL interpretations

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After-Lecture Exercise

Class(Animal partial)

Class(Plant partial)

DisjointClasses(Animal Plant)

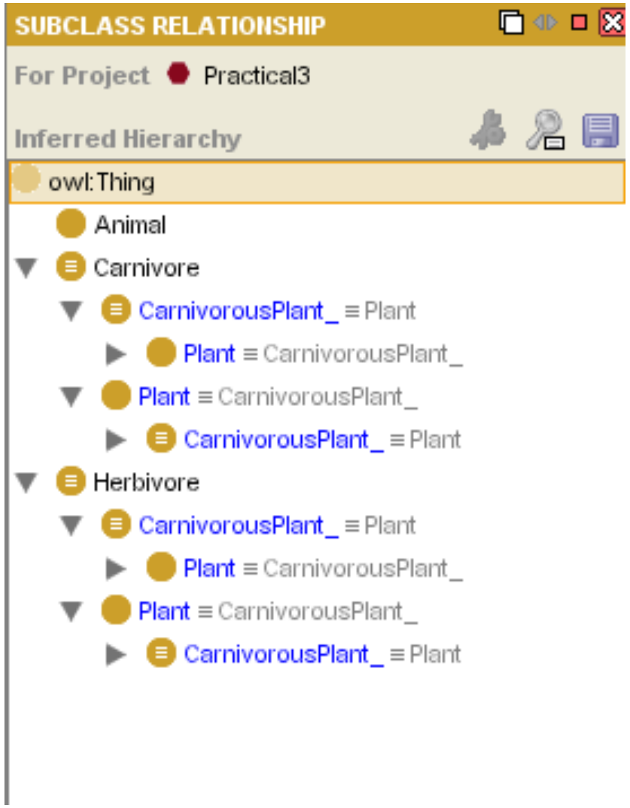
ObjectProperty(eats domain(Animal))

Class(Herbivore complete restriction(eats allValuesFrom(Plant)))

Class(Carnivore complete restriction(eats allValuesFrom(Animal)))

Class(CarnivorousPlant complete intersectionOf(Plant Carnivore))

After-Lecture Exercise



- Input the above ontology in Protégé
- Feel free to send me an email on:
  - Why Plant is a subclass of Herbivore and Carnivore?
  - How to solve the problem?
  - Are there any other problems in your revised ontology?

Summary

- DL semantics
  - Interpretation
  - Interpretations of descriptions
  - Interpretations of axioms
  - Interpretations of ontologies

- Reasoning is based on semantics

"Billy, I'm not going to argue the semantics of biting. Whether or not you penetrated skin, I'm calling your mother."