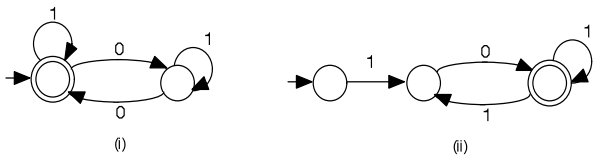
CS3012 Formal Languages

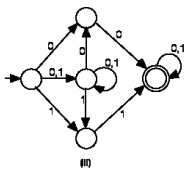
Exercises 2: Finite State Automata

1. Describe in words the languages accepted by each of the following FSAs:



Answer (i):
strings of 0's and 1's with an even number of 0's

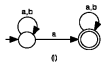
Answer (ii):
strings of 0's and 1's, starting with a 10, and no pairs of 0's

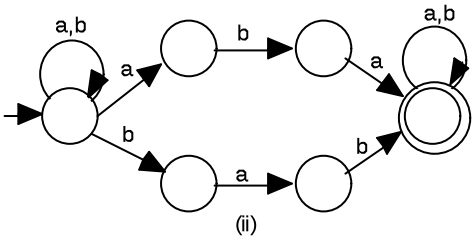


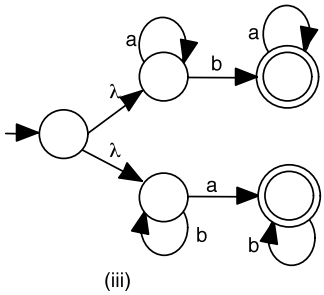
Answer (iii):
strings of 0's and 1's with either a pair of 0's or a pair of 1's

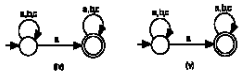
2. Give a formal definition for the FSAs which define the following languages
(i.e. specify Q, I, F, T, E):

Answer: (I show the diagram, but you can easily convert to QIFTE)









3. Give a formal definition for a FSA accepting the infinite set of strings representing numbers divisible by 2. (i.e. 0,2,4,6,8,10,12,….)

Answer:
Just check the last digit, make sure it's even (need two states)
Let E=0,2,4,6,8
Let O=1,3,5,7,9

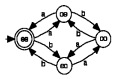
Then QIFTE=({1,2},{1},{2},{O,E},{(1,O,1),(1,E,2),(2,O,1),(2,E,2)})

4. Give a formal definition for a FSA accepting the infinite set of strings representing numbers divisible by 3. (this one is tricky, try (on paper) doing a division of 3 into some large number *x*, and see if the process you are using to deal with each new digit of *x* could be automated by a machine.)

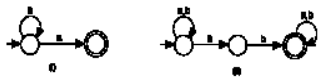
Answer:
(rough definition)
3 states 0,1,2
from 0 to 0: label 0,3,6,9
from 0 to 1: label 1,4,7
from 0 to 2: label 2,5,8
from 1 to 0: label 2,5,8
from 1 to 1: label 0,3,6,9
from 1 to 2: label 1,4,7
from 2 to 0: label 1,4,7
from 2 to 1: label 2,5,8
from 2 to 2: label 0,3,6,9

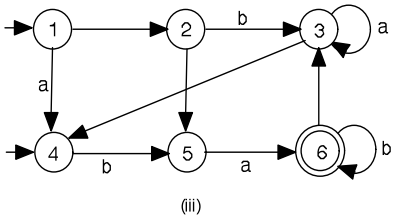
5. Give a formal definition for a FSA accepting the set of strings over {*a*, *b*} which contain an even number of *a*'s and an even number of *b*'s. (this one is tricky, try doing it manually for some random string of a's and b's. What do you need to remember as you're going through the string? How many states will the FSA need, to remember this?)

Answer:

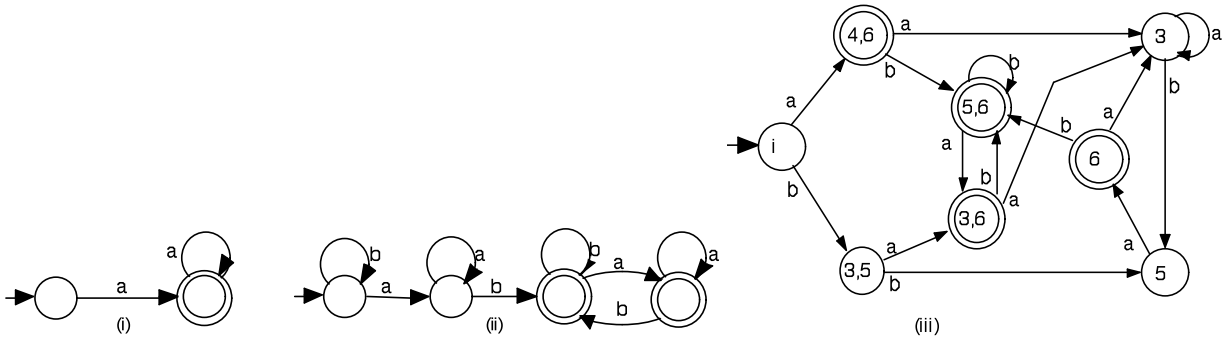


6. Give a formal definition for a DFSA for the following, using the "NDFSA -> DFSA" algorithm from lectures (i.e. in your answer show the workings of the table used for removing edge choices, and specify the final Q, I, F, T, E):





Answer



I'	**F'**	**E'**	**S**	**Q'**	**X**	**t**	**S'**
1		({1},a,{1,2})	~~{1}~~	{1}	{1}	a	{1,2}

	{1,3}	({1},b,{1})	~~{1,2}~~	{1,2}		b	{1}
	{1,2,3}	({1,2},a{1,2})	~~{1,3}~~	{1,3}	{1,2}	a	{1,2}
		({1,2},b,{1,3})	~~{1,2,3}~~	{1,2,3}		b	{1,3}
		({1,3},a,{1,2,3})			{1,3}	a	{1,2,3}
		({1,3},b,{1,3})				b	{1,3}
		({1,2,3},a,{1,2,3})			{1,2,3}	a	{1,2,3}
		({1,2,3},b,{1,3})				b	{1,3}

Answer (iii)

I'	F'	E'	S	Q'	X	T	S'
{1}	{4,6}	({1},a,{4,6})	~~{1}~~	{1}	{1}	a	{4,6}
		({1},b,{3,5})	~~{4,6}~~	{4,6}		b	{3,5}
		({4,6},a,{3})	~~{3,5}~~	{3,5}	{4,6}	a	{3}
		({4,6},b,{5,6})	~~{3}~~	{3}		b	{5,6}
	{5,6}	({3,5},a,{3,6})	~~{5,6}~~	{5,6}	{3,5}	a	{3,6}
	{3,6}	({3,5},b,{5})	~~{3,6}~~	{3,6}		b	{5}
		({5,6},a,{6})	~~{5}~~	{5}	{5,6}	a	{6}
	{6}	({5,6},b,{5,6})	~~{6}~~	{6}		b	{5,6}
		({3,6},a,{3})			{3,6}	a	{3}
		({3,6},b,{1,5,6})				b	{5,6}
		({5},a,{6})			{5}	a	{6}
						b	{}
		({6},a,{3})			{6}	a	{3}
		({6},b,{5,6})				b	{5,6}
		({3},a,{3})			{3}	a	{3}
		({3},b,{5})				b	{5}

7. (very simple) Give a method of constructing a FSA B from a FSA A, such that:
L(B) = {*w* | there is a path from *p* to *q* labelled *w* in A, for some states *p* and *q*}

Answer:
Make all states start and finish states.