CS3012 Formal Languages
Exercises 1: Alphabets, Strings and Languages

w, *u* and *v* are strings
T is an alphabet
A and B are languages

1. What is the value of $|\lambda|$?

Answer:
   0

2. What can you say about λw and $w\lambda$?

Answer:
   $\lambda w = w\lambda$

3. What is the relationship between $|uv|$, $|u|$ and $|v|$?

Answer:
   $|uv| = |u| + |v|$

4. State a necessary and sufficient condition on T for T^* to be finite. If T^* is finite, list its members.

Answer:
   T={} or T={labda}

5. Give an example to show that $uv = vu$ is not always true.
   Give a *non-trivial* example to show it is not always false.
   (in this case trivial means an example using λ)

Answer:
   $u = ab, v = ba, uv = abba \neq baab = vu$.
   $u = baba, v = ba, uv = bababa = vu$

6. Give necessary and sufficient conditions on *u* and *v* for $uv = vu$.

Answer:
   There is a string α such that $u = \alpha^n$ and $v = \alpha^m$, for some *n* and *m*

7. List all the (i) prefixes, (ii) suffixes, and (iii) substrings of *abb*.

Answer: (i) λ, a, ab, abb.
   (ii) λ, b, bb, abb.
   (iii) $\lambda, a, b, ab, bb, abb$.

8.   If A = {*a*, *ba*} and B = {*a*, λ, *bb*}, list, in lexical order, the elements of (i) A+B and (ii) AB, and the
      first 12 strings in A*.

Answer: (i) λ, *a*, *ba*, *bb*.
(ii)   *a*, *aa*, *ba*, *abb*, *baa*, *babb*.
(iii)   λ, *a*, *aa*, *ba*, *aaa*, *aba*, *baa*, *aaaa*, *aaba*, *abaa*, *baaa*, *baba*.

9.   Which ordering scheme (dictionary or lexical) is most useful for infinite sets, and why?
(give an example of the first 10 elements of some infinite language in both orderings, to illustrate the
      advantage of one scheme)

Answer: Lexical is most useful,
In dictionary, strings <u>not</u> listed in finite time

10. Under what conditions are A* and A+ equal?

Answer:
λ ∈ A

11. If T has only two symbols, show that every string over T of length 4 or more must have two adjacent
      non-empty equal substrings.
      (hint: for starters you can write out all of the strings of length four and see if it is true for them)

Answer: strings length 4 beginning *a* are: <u>aa</u>aa, <u>aa</u>ab, <u>a</u>ab<u>a</u>, <u>a</u>ab<u>b</u>, ab<u>aa</u>, a<u>b</u>a<u>b</u>, a<u>bb</u>a, a<u>bb</u>b.
similar for strings beginning with *b*.
Any string of length greater than 4 must contain a substring of length 4.

12. Let T be the alphabet {*a,b,c*}, and λ be the empty string.
      (i)   Write down the first five elements of T* in lexical order and dictionary order.

Answer:
Lexical: λ, *a*, *b*, *c*, *aa*
Dictionary: λ, *a*, *aa*, *aaa*, *aaaa*

      (ii)   Write down the first five elements of T+ in lexical order and dictionary order.

Answer:
Lexical: *a*, *b*, *c*, *aa*, *ab*
Dictionary: *a*, *aa*, *aaa*, *aaaa*, *aaaaa*

13. If I have a machine with 1GB of memory, how many possible states can the machine be in?

Answer:
1 GB = 2^10 MB
= 2^20 KB
= 2^30 Bytes
= 2^33 bits
states = 2^(2^33)