Exercises for CS2013

Logic Practical 2

Nov 13, 2013

Introduction

This practical covers a range of aspects of Predicate Logic.

Do as much as you can during the session, and what remains do as homework.

If you work best on your own, fine. But, if you work better with others, form peer groups. See to it that you individually know what to do and how, for at the "end of the day" the exam is you and the exam.

Predicate Logic

Vacuous Quantification

1. Formally evaluate the following expression with respect to model **M1** = (**D**,**I**) where **D**={bill, jill, mary} and **I**(jill') = jill, **I**(mary') = mary, **I**(bill') = bill, **I**(B)={bill, jill}, **I**(G)={mary}, **I**(A)={(bill, jill) , (mary, bill)}. Show the set of expressions of the form ∀x [B(bill) → A(bill, jill)](x := bill), ∀x [B(bill) → A(bill, jill)](x := jill), ∀x [B(bill) → A(bill, jill)](x := mary).

   ∀x [B(bill) → A(bill,jill)]

Empty Domain

2. Formally evaluate the following expression with respect to model **M2** = (**D**,**I**) where **D**={ }, **I**(B)={}, **I**(G)={}, **I**(A)={}.

   ∀x [B(x) → A(x,x)]

False Antecedent

3. Formally evaluate the following expression with respect to model **M3** = (**D**,**I**) where **D**={bill, jill, mary} and **I**(jill') = jill, **I**(mary') = mary, **I**(bill') = bill, **I**(B)={}, **I**(G)={mary}, **I**(A)={(bill, jill) , (mary,bill)}.

   ∀x [B(x) → A(x,x)]

Create a model

4. Provide a model **M4** = (**D**,**I**) where **D** has 6 elements, proper names have their usual interpretations, the interpretation of B, **I**(B), has at least three elements of **D**, and the interpretation of A, **I**(A), has three pairs from **D**. The

model should be such that the following expressions have the indicated truth values.

a. $\exists x\, \exists y\, \mathbf{(}B(x) \wedge A(x,y)\mathbf{)}$ **true**
b. $\forall x\, \exists y\, \mathbf{(}B(x) \wedge A(x,y)\mathbf{)}$ **false**
c. $\forall x\, \exists y\, \mathbf{(}B(x) \rightarrow A(x,y)\mathbf{)}$ **false**
d. $\exists x\, \exists y\, \mathbf{(}B(x) \rightarrow A(x,y)\mathbf{)}$ **true**

Formal Evaluation of Nested Quantifiers

5. Formally evaluate the following expression in **M5** = (**D**,**I**) where **D**={bill, jill, mary}, **I**(jill') = jill, **I**(mary') = mary, **I**(bill') = bill, **I**(B) ={(bill, jill), (mary,bill), (jill, jill)}, **I**(A)={(bill, bill), (mary,jill), (jill, jill)}

$$\forall x\, \exists y\, (B(x,y) \rightarrow \exists z\, A(x,z))$$

Quantifier Equivalences

6. Suppose for a given domain D = {a,b,c,…} that the following quantifier expansions hold:

$\forall x\, P(x) \Leftrightarrow P(\mathrm{a}) \wedge P(\mathrm{b}) \wedge P(\mathrm{c}) \wedge \ldots$
$\exists x\, P(x) \Leftrightarrow P(\mathrm{a}) \vee P(\mathrm{b}) \vee P(\mathrm{c}) \vee \ldots$

Prove that the following quantifier equivalences hold using some of the propositional equivalence laws:

$\forall x\, P(x) \Leftrightarrow \neg \exists x\, \neg P(x)$

$\exists x\, P(x) \Leftrightarrow \neg \forall x\, \neg P(x)$

Quantifier Definition

7. Define a quantifier in Predicate Logic for the following two quantifiers:

At least three

Exactly three