CS1512
Foundations of Computing Science 2

Tutorial Solutions: Week 4

1.

If someone tells you that the probability of event X happening is y (i.e. P(X) = y), what are the possible sources of y?

Possible sources are:

- 'a priori': if it is believed that you can generate a sufficiently good model purely from theoretical considerations (e.g. fair dice);
- experiment: if a large number of experiments have been carried out;
- belief: i.e. subjective estimate

2.

A fair coin is tossed 4 times. How many possible outcomes are there for this experiment? List them, writing 'H' for heads and 'T' for tails. What is the probability of getting:

(a) 4 heads?
(b) no heads?
(c) exactly 3 heads?
(d) at least 3 heads?
(e) a run of 3 or more heads (that is, 3 or more in a row)?
(f) at least 2 tails?

Let X be the number of heads minus the number of tails. For each value k of the random variable X write down the probability P(X = k).

There are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ outcomes. The sample space has 16 elements, namely:

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

Assumption: All 16 outcomes are equally likely

(a) Probability is 1/16.
(b) Again 1/16.
(c) This event, as a subset of the sample space above, has 4 elements.
The probability is 4/16 = 1/4.
(d) Just by counting, we get probability 5/16 — or add (a) and (c).
(e) Here it is certainly best to look at the list above and count.
We have {HHHH, HHHT, THHH}. Probability is 3/16.
(f) Probability is 11/16.

Let X be the number of heads minus the number of tails. For each value k of the random variable X write down the probability P(X = k).

The possible values for the difference are:

-4 0 heads and 4 tails
-2 1 head and 2 tails
0 2 heads and 2 tails
2 3 heads and 1 tail
4 4 heads and 0 tails

By counting, the probability distribution for the random variable X is:

k	-4	-2	0	2	4
$P(X = k)$	1/16	1/4	3/8	1/4	1/16

3.
A bag contains 2 red balls, 3 white balls and 5 blue balls. A ball is withdrawn, its colour noted, replaced, and a second ball is drawn. What is the probability of a red ball being drawn followed by a blue ball? What is the probability of getting a red ball and a blue ball if the order in which they are drawn is not taken into account? What are the two probabilities if the first ball is ***not*** *replaced?*

a. Red ball followed by blue ball, with replacement, order is important:

Second ball

First ball

	r	r	w	w	w	b	b	b	b	b
r	rr	rr	rw	rw	rw	rb	rb	rb	rb	rb
r	rr	rr	rw	rw	rw	rb	rb	rb	rb	rb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb	wb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb	wb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb	wb
b	br	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb	bb

Look for number of rb's = 10; total number of possibilities = 100
Probability = 10/100 = 1/10
Could also compute from P(rb) = P(r)*P(b) = 2/10 * 5/10 (since trials are independent)

b. Red ball and blue ball, with replacement, order is not important:

Same table
Look for number of rb's and br's = 10 + 10; total number of possibilities = 100
Probability = 20/100 = 2/10
Could also compute from P(rb) = P(r)*P(b) + P(b)*P(r) = 2/10*5/10 + 5/10+2/10

c1. Red ball followed by blue ball, without replacement, order is important:

Second ball

First ball

	r	w	w	w	b	b	b	b	b
r	rr	rw	rw	rw	rb	rb	rb	rb	rb
r	rr	rw	rw	rw	rb	rb	rb	rb	rb
w	wr	ww	ww	ww	wb	wb	wb	wb	wb
w	wr	ww	ww	ww	wb	wb	wb	wb	wb
w	wr	ww	ww	ww	wb	wb	wb	wb	wb
b	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	bw	bw	bw	bb	bb	bb	bb	bb
b	br	bw	bw	bw	bb	bb	bb	bb	bb

Look for number of rb's = 10; total number of possibilities = 90
Probability = 10/90 = 1/9
Could also compute from P(rb) = P(r)*P(b|r) = 2/10 * 5/9 (since trials are not independent)

c2. Red ball and blue ball, without replacement, order is not important:

rb and br are exclusive, so probability of either rb or br is the sum of the two. We already have P(rb); get P(br) similarly:

Second ball

First ball

	r	r	w	w	w	b	b	b	b
r	rr	rr	rw	rw	rw	rb	rb	rb	rb
r	rr	rr	rw	rw	rw	rb	rb	rb	rb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb
w	wr	wr	ww	ww	ww	wb	wb	wb	wb
b	br	br	bw	bw	bw	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb
b	br	br	bw	bw	bw	bb	bb	bb	bb

Look for number of br's = 10; total number of possibilities = 90
Probability = 10/90 = 1/9

Could also compute from P(br) = P(b)*P(r|b) = 5/10 * 2/9

P(r,b) = P(rb) + P(br) = 2/9

4.

What is the probability of getting a total score of 3 from throwing two dice? Of getting 5? Of getting 7? 9? 11? What is the probability of getting a total score which is odd.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

p(3) = 2/36
p(5) = 4/36
p(7) = 6/36
p(9) = 4/36
p(11) = 2/36

p(3 or 5 or 7 or 9 or 11) = p(3) + p(5) + p(7) + p(9) + p(11) = ½

5.

Three cards are drawn at random without replacement from a well=shuffled pack of 52 cards. What is the probability that they are:

- *all spades?*
- *all Aces?*

Probability of three spades:

p(S) = 13/52 (13 spades in the pack)
p(S|S) = 12/51
p(S| S and S) = 11/50
p(S and S and S) = p(S!S and S) *p(S|S) *p(S)
= 11/50 * 12/51 * 13/52
= 11/850 (after simplification)

Probability of three Aces = 4/52 * 3/51 2/50 = 3/16575