Under Actuated Robotics - Workshop 2

Stefan Ravn van Overeem – stvan13@student.sdu.dk

October 8, 2015

1 Derivation of State Space Representation

The equations of motion for the pendulum can be expressed as:

$$m \cdot l^2 \cdot \ddot{\theta} + b \cdot \dot{\theta} + m \cdot g \cdot l \cdot sin(\theta) = u$$

We assume small angle, and thus get

$$m \cdot l^2 \cdot \ddot{\theta} + b \cdot \dot{\theta} + m \cdot g \cdot l \cdot \theta = u$$

We write $\ddot{\theta}$ as $\dot{x_2}$, $\dot{\theta}$ as x_2 and θ as x_1
Thus we can write

$$\dot{x_1} = x_2$$
$$\dot{x_2} = -\frac{g}{l} \cdot x_1 - \frac{b}{m \cdot l^2} \cdot x_2 + \frac{u}{m \cdot l^2}$$

We can thus write the state space equation

$$\dot{x} = A \cdot x + B \cdot u$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{b}{m \cdot l^2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix} \cdot u$$

Thus we get $A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{b}{m \cdot l^2} \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix}$

To get the output to equal our state, we set the C matrix to the identity matrix, and the D matrix to 0.

We control the input signal U using a fullstate feedback controller, of the form $u = -K \cdot x$, where K is a matrix

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

K_1 and K_2 is determined using LQR.

This is done in python and can be seen in SimplePendulumController.py