Relational Concept Analysis

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FCA and complex relations

Concepts in multi-relational or n-ary relationals datasets

- Power contexts families [Wille, 2002]
- Triadic [Lehmann and Wille, 1995], Polyadic Concept Analysis [Voutsadakis, 2002]
- Relational Concept Analysis (RCA) [Huchard et al., 2007, Hacene et al., 2013]
- Cubes of Concepts [Ferré et al., 2012]
- Machine learning with Galois lattices and graphs [Liquière and Sallantin, 1998]
- Graph pattern structures [Ganter and Kuznetsov, 2001]
- Relational, windowed structures [Kötters, 2013]
- Graph-FCA [Ferré, 2015]

Relational Concept Analysis (RCA)

Principles

- Extends the purpose of FCA for taking into account object categories and links between objects
- Main principles:
  - a relational model based on the entity-relationship model (with binary relationships)
  - integrate relations in formal contexts between objects as *relational* attributes
  - various operators (quantifiers) inspired by description logics
  - iterative and tunable process
- RCA provides a set of interconnected lattices
- Produced structures can be represented as ontology concepts within a knowledge representation formalism such as description logics (DLs)

Relational Context Family (RCF)

A RCF $\mathcal{F}$ is a pair (K, R) with:

- K is a set of object-attribute contexts $K_i = (O_i, A_i, I_i)$
- R is a set of object-object contexts $R_j = (O_k, O_l, I_j)$
  - (O_k, O_l) are the object sets of formal contexts $(K_k, K_l) \in K^2$
  - $I_j \subseteq O_k \times O_l$
  - K_k is the *source/domain context*
  - K_l is the *target/range context*
  - we may have $K_k = K_l$

A few examples of RCF

Royal Relational Context Family (1 category, 2 symmetric relations)

- **object-attribute context**
  - Royal
- **object-object contexts**
  - hasParent $\subseteq$ Royal $\times$ Royal
  - isParentOf $\subseteq$ Royal $\times$ Royal

Drone Relational Context Family (2 categories, 1 relation)

- **object-attribute contexts**
  - DroneFleet
  - Drone
- **object-object context**
  - contains $\subseteq$ DroneFleet $\times$ Drone

A few examples of RCF

Dish Relational Context Family (3 categories, 3 relations forming a cycle)

- **object-attribute contexts**
  - Dish
  - Cereal
  - Country
- **object-object contexts**
  - hasMainCereal $\subseteq$ Dish $\times$ Cereal
  - isProducedIn $\subseteq$ Cereal $\times$ Country
  - eatLotOf $\subseteq$ Country $\times$ Dish

A few examples of RCF

Pesticide plants Relational Context Family (3 categories, 2 relations)

- **object-attribute contexts**
  - Protected organism
  - Plant
  - Pest
- **object-object contexts**
  - isProtectedBy $\subseteq$ ProtectedOrganism $\times$ Plant
  - protectsAgainst $\subseteq$ Plant $\times$ Pest

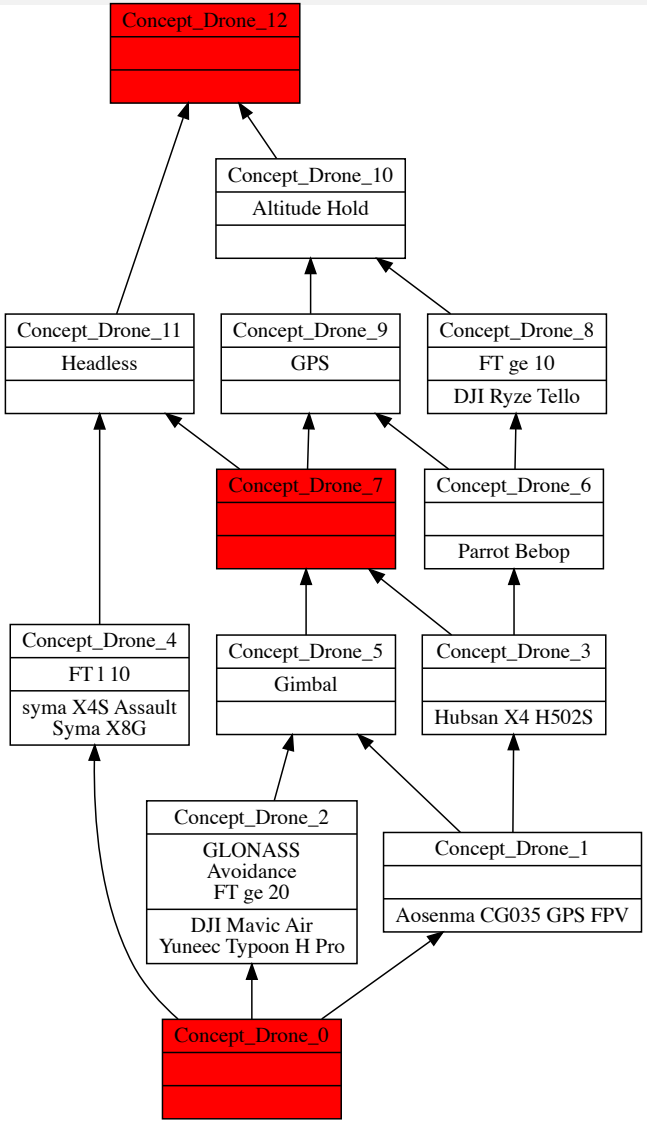
Drones

Drone	Gimbal	GPS	GLONASS	Avoidance	Headless	Altitude Hold	FT l 10	FT ge 10	FT ge 20
Syma X4S Assault 					×		×		
Syma X8G 					×		×		
Parrot Bebop 		×				×		×	
DJI Ryze Tello 						×		×	
Hubsan X4 H502S 		×			×	×		×	
Aosenma CG035 GPS FPV 	×	×			×	×		×	
DJI Mavic Air 	×	×	×	×	×	×			×
Yuneec Typoon H Pro 	×	×	×	×	×	×			×

https://www.thedronechart.com

Hum, typo, here. Typoon in World of Warcraft? Teaspoon?

Concept Lattice and AOC-poset)



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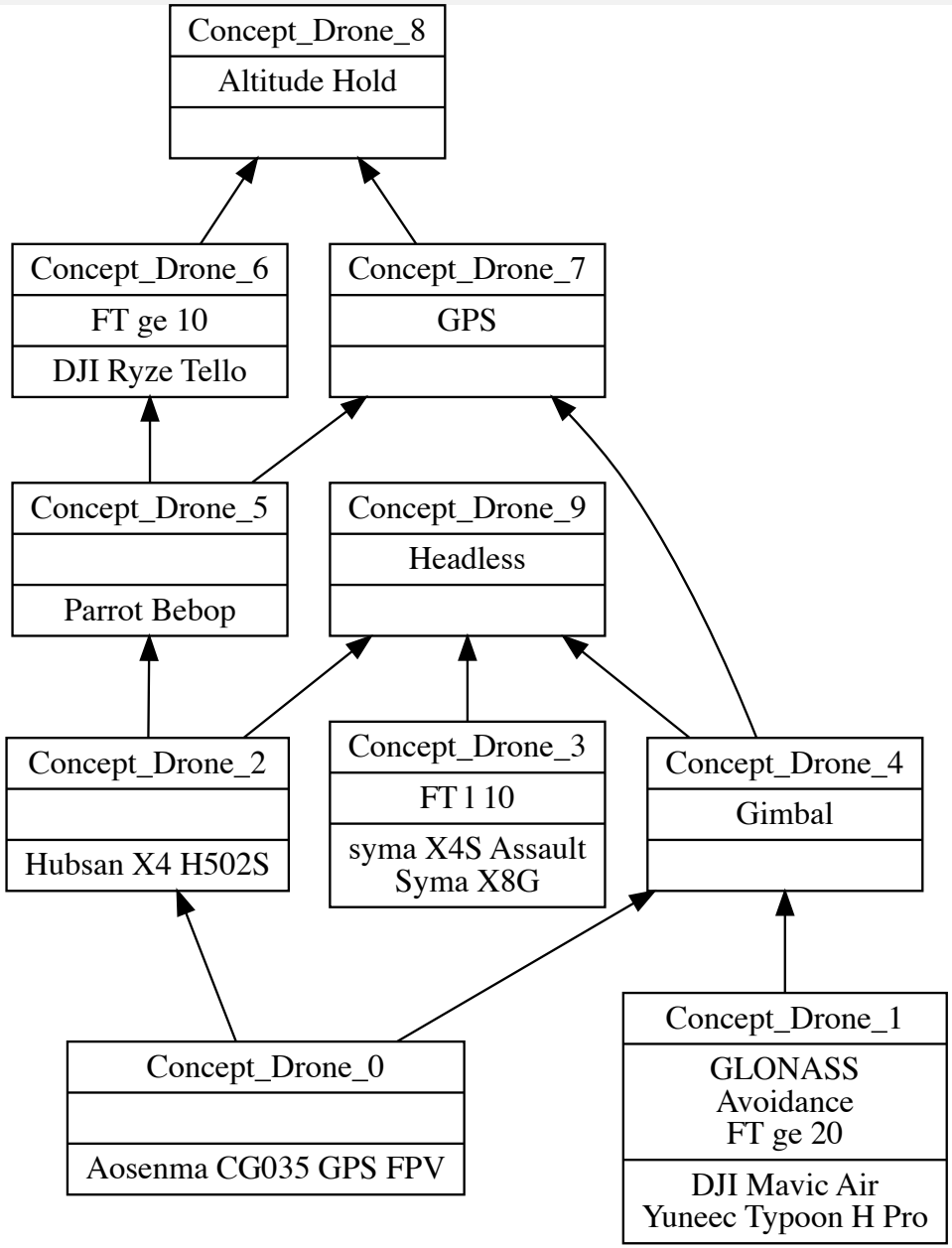
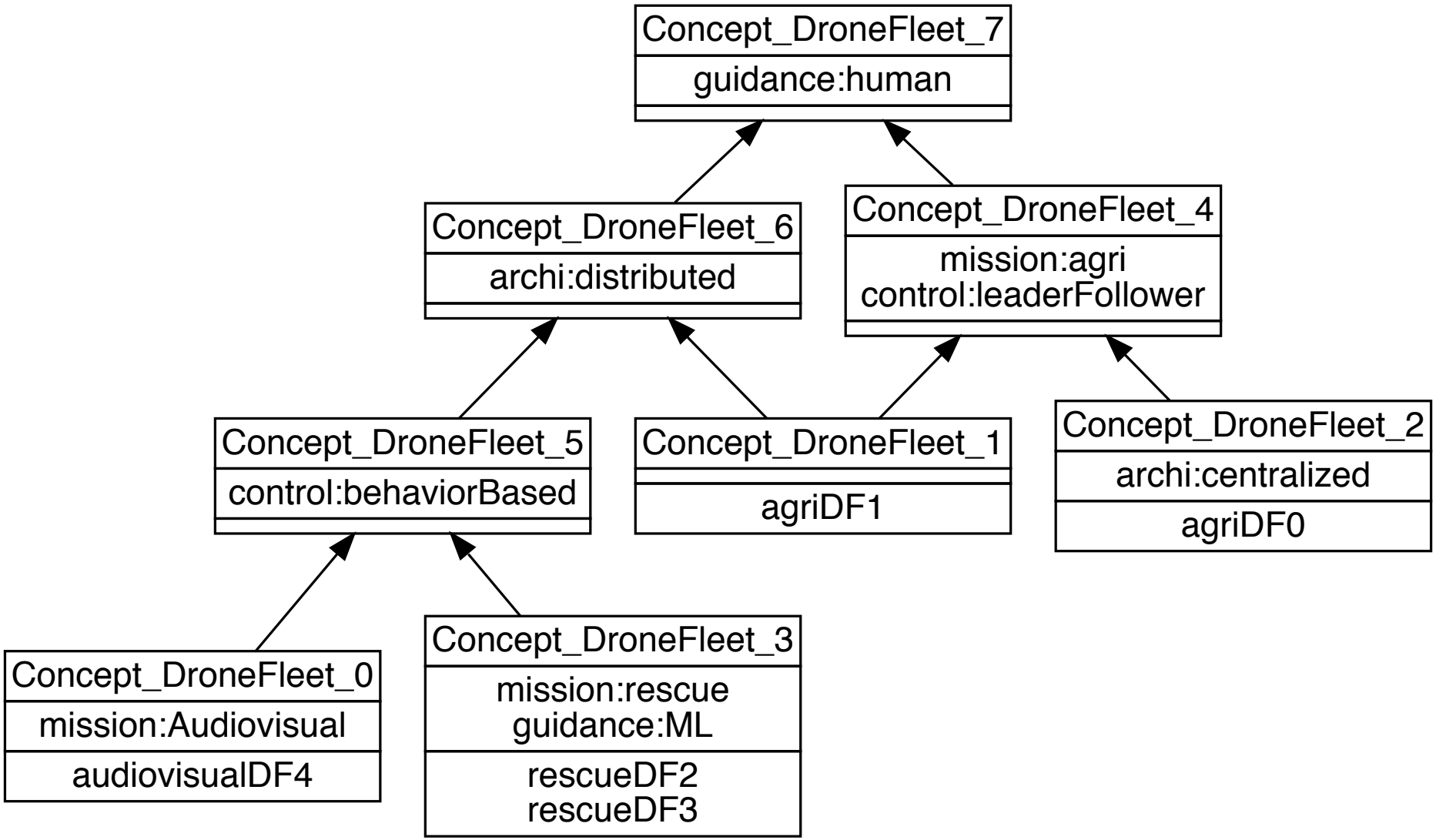


Drone fleet (Formal context)

DroneFleet	mission:agri	mission:rescue	mission:Audiovisual	archi:centralized	archi:distributed	guidance:human	guidance:ML	control:leaderFollower	control:behaviorBased
agriDF0	×			×		×		×	
agriDF1	×				×	×		×	
rescueDF2		×			×	×	×		×
rescueDF3		×			×	×	×		×
audiovisualDF4			×		×	×			×

Drone fleet (AOC-poset)



Drone fleet 2 Drone (Relational Context)

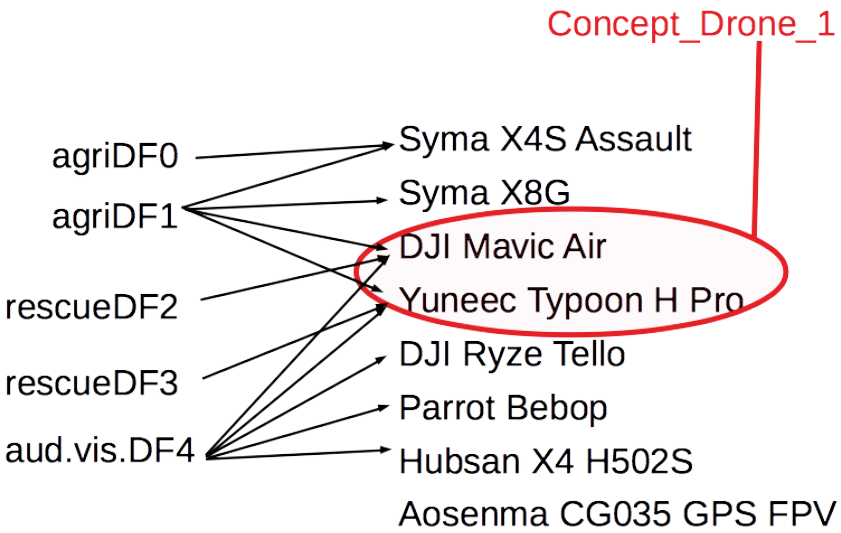
contains	Syma X4S Assault	Syma X8G	Parrot Bebop	DJI Ryze Tello	Hubsan X4 H502S	Aosenma CG035 GPS FPV	DJI Mavic Air	Yuneec Typoon H Pro
agriDF0	x							
agriDF1	x	x					x	x
rescueDF2							x	
rescueDF3								x
audiovisualDF4			x	x	x		x	x

$\Rightarrow$ `rescueDF2` and `rescueDF3` do not share concrete drone types

Understanding scaling quantifiers

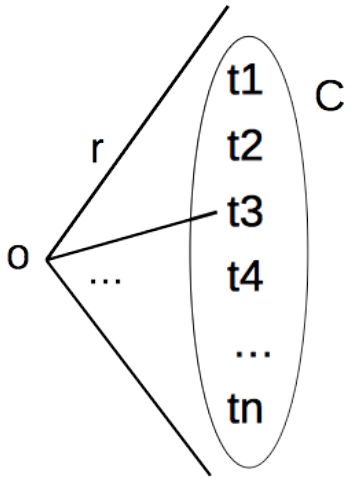
`rescueDF2` and `rescueDF3` do not share concrete drone types, but they share the fact that all their drones with `GLONASS`, `GPS`, `FT` $\geq$ 20, etc.

Relational attribute: $\exists\forall contains(Concept\_Drone\_1)$

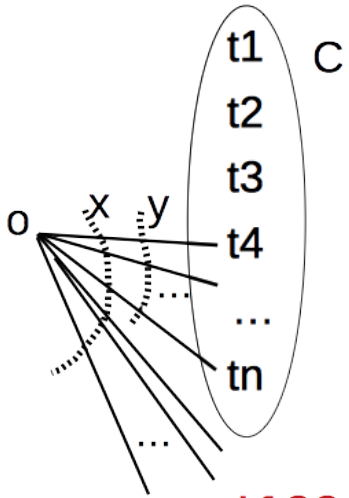


Drone Fleet	Examples of Relational Attributes
agriDF1, rescueDF2	∃ contains (Concept_Drone_1)
rescueDF2, rescueDF3	∃∀ contains (Concept_Drone_1)
agriDF1	∃∀≥50% contains (Concept_Drone_1)
aud.vis.DF4	∃∀≥40% contains (Concept_Drone_1)
agriDF1, aud.vis.DF4	∃⊇ contains (Concept_Drone_1)
rescueDF2, rescueDF3	∃⊇≥50% contains (Concept_Drone_1)

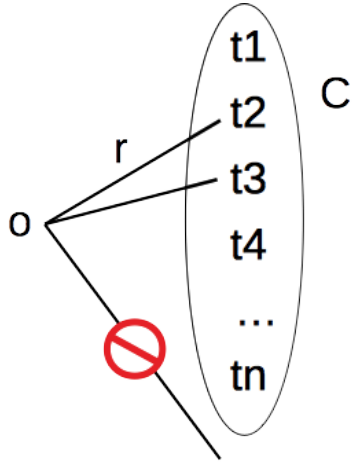
Understanding scaling quantifiers



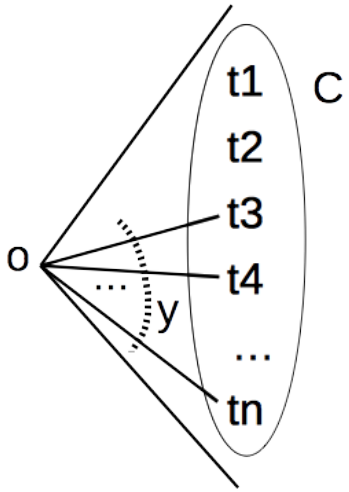
$\exists$ r (C)



$\exists\forall \geq y*100/x\%$ r (C)

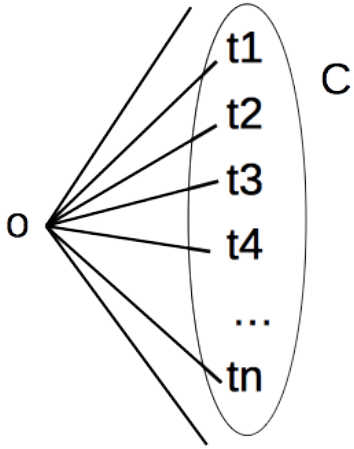


$\exists\forall$ r (C)



|C| = x

$\exists\supseteq \geq y*100/x\%$ r (C)

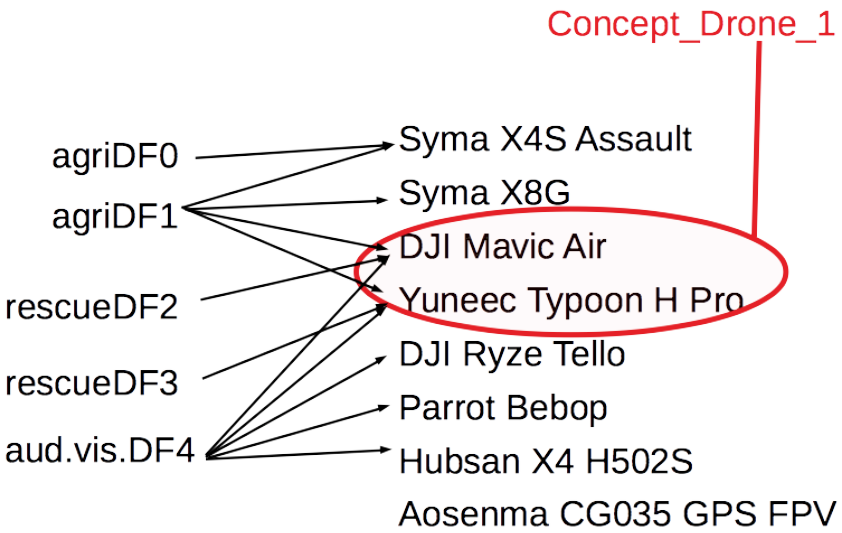


$\exists\supseteq$ r (C)

Understanding scaling quantifiers (again ...)

`rescueDF2` and `rescueDF3` do not share concrete drone types, but they share the fact that all their drones with `GLONASS`, `GPS`, `FT` $\geq$ 20, etc.

Relational attribute: $\exists\forall contains(Concept\_Drone\_1)$



Drone Fleet	Examples of Relational Attributes
agriDF1, rescueDF2	∃ contains (Concept_Drone_1)
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agriDF1	∃∀≥50% contains (Concept_Drone_1)
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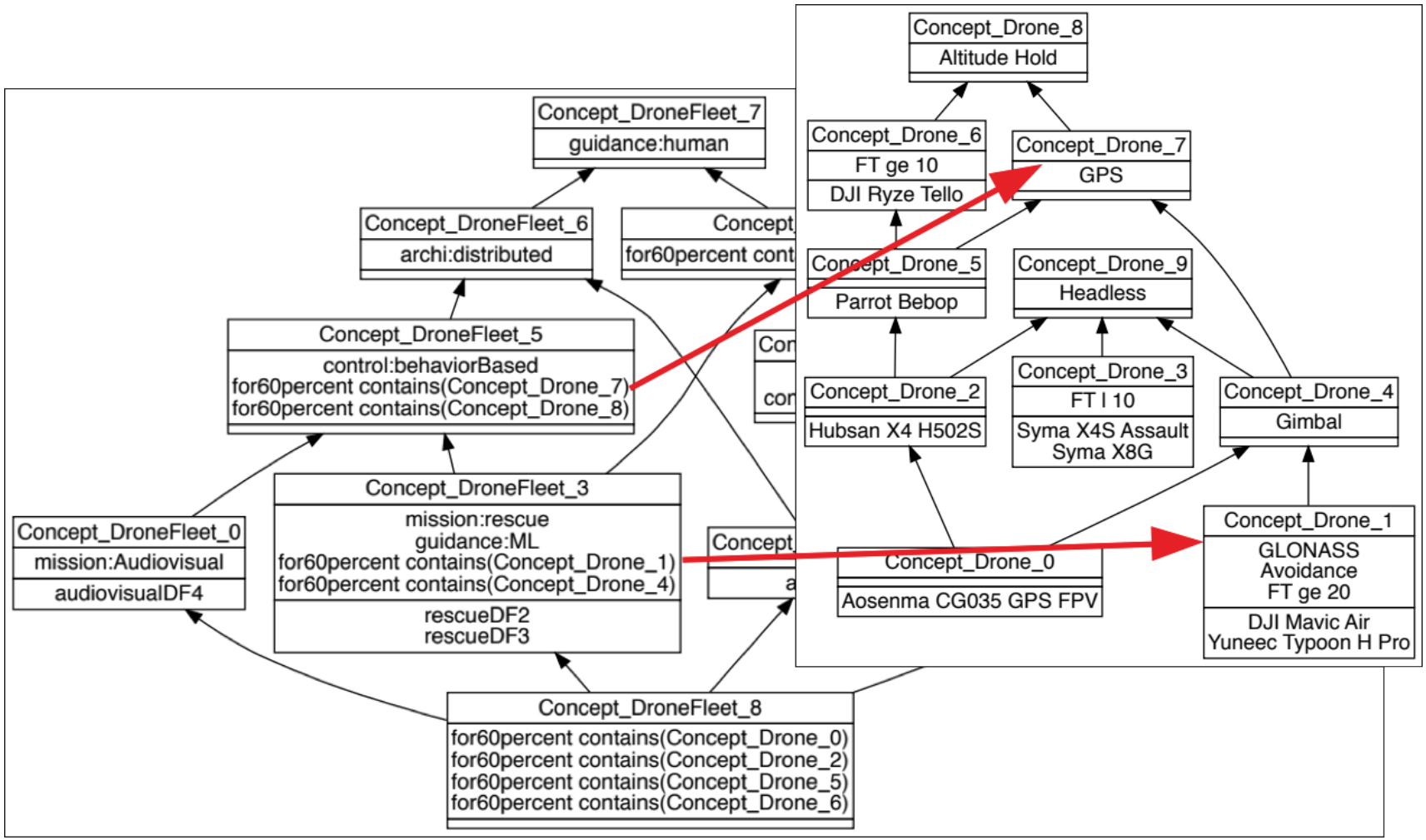
agriDF1, aud.vis.DF4	∃⊇ contains (Concept_Drone_1)
rescueDF2, rescueDF3	∃⊇≥50% contains (Concept_Drone_1)

Drone fleet 2 Drone (Extended Relational Context)

DroneFleet + $\exists\forall_{\geq 60\%}$ Contains	mission:agri	mission:rescue	mission:Audiovisual	archi:centralized	archi:distributed	guidance:human	guidance:ML	control:leaderFollower	control:behaviorBased	$\geq_{60\%}$AE contains(Concept_Drone_0)	$\geq_{60\%}$AE contains(Concept_Drone_1)	$\geq_{60\%}$AE contains(Concept_Drone_3)	$\geq_{60\%}$AE contains(Concept_Drone_2)	$\geq_{60\%}$AE contains(Concept_Drone_4)	$\geq_{60\%}$AE contains(Concept_Drone_5)	$\geq_{60\%}$AE contains(Concept_Drone_6)	$\geq_{60\%}$AE contains(Concept_Drone_7)	$\geq_{60\%}$AE contains(Concept_Drone_9)	$\geq_{60\%}$AE contains(Concept_Drone_8)
agriDF0	×			×		×		×				×						×	
agriDF1	×				×	×		×										×	
rescueDF2		×			×	×	×		×		×			×			×	×	×
rescueDF3		×			×	×	×		×		×			×			×	×	×
audiovisualDF4			×		×	×			×								×		×

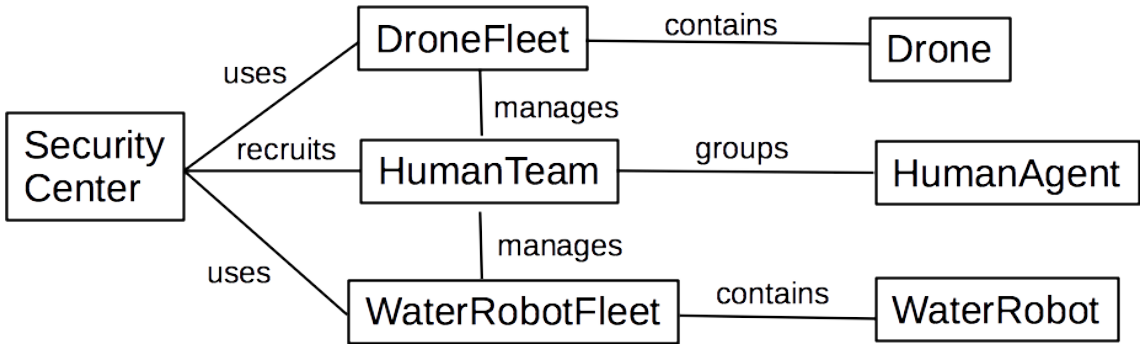
Drone fleets extended by relations to their drones



Rescue fleets have a majority of drones with GLONASS,

Avoidance system and Flight Time $\geq$ 20mn

RCA in the general case



An iterative process

- Complex model with paths and cycles of any length
- Objects groups (concepts) are propagated along the paths and the cycles, step after step
- The process stops when no new concept appears

Tool

- http://dataqual.engees.unistra.fr/logiciels/rcaExplore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