Element Stiffness Matrix

$$[k^e] = \frac{E}{1-\nu^2}\begin{bmatrix} \frac{3-\nu}{6} & \frac{1+\nu}{8} & -\frac{3+\nu}{12} & \frac{-1+3\nu}{8} & \frac{-3+\nu}{12} & -\frac{1+\nu}{8} & \frac{\nu}{6} & \frac{1-3\nu}{8} \\ \frac{1+\nu}{8} & \frac{3-\nu}{6} & \frac{1-3\nu}{8} & \frac{\nu}{6} & -\frac{1+\nu}{8} & \frac{-3+\nu}{12} & \frac{-1+3\nu}{8} & -\frac{3+\nu}{12} \\ -\frac{3+\nu}{12} & \frac{1-3\nu}{8} & \frac{3-\nu}{6} & -\frac{1+\nu}{8} & \frac{\nu}{6} & \frac{-1+3\nu}{8} & \frac{-3+\nu}{12} & \frac{1+\nu}{8} \\ \frac{-1+3\nu}{8} & \frac{\nu}{6} & -\frac{1+\nu}{8} & \frac{3-\nu}{6} & \frac{1-3\nu}{8} & -\frac{3+\nu}{12} & \frac{1+\nu}{8} & \frac{-3+\nu}{12} \\ \frac{-3+\nu}{12} & -\frac{1+\nu}{8} & \frac{\nu}{6} & \frac{1-3\nu}{8} & \frac{3-\nu}{6} & \frac{1+\nu}{8} & -\frac{3+\nu}{12} & \frac{-1+3\nu}{8} \\ -\frac{1+\nu}{8} & \frac{-3+\nu}{12} & \frac{-1+3\nu}{8} & -\frac{3+\nu}{12} & \frac{1+\nu}{8} & \frac{3-\nu}{6} & \frac{1-3\nu}{8} & \frac{\nu}{6} \\ \frac{\nu}{6} & \frac{-1+3\nu}{8} & \frac{-3+\nu}{12} & \frac{1+\nu}{8} & -\frac{3+\nu}{12} & \frac{1-3\nu}{8} & \frac{3-\nu}{6} & -\frac{1+\nu}{8} \\ \frac{1-3\nu}{8} & -\frac{3+\nu}{12} & \frac{1+\nu}{8} & \frac{-3+\nu}{12} & \frac{-1+3\nu}{8} & \frac{\nu}{6} & -\frac{1+\nu}{8} & \frac{3-\nu}{6} \end{bmatrix}$$

$$= \frac{E}{24(1-\nu^2)}\begin{bmatrix} 12-4\nu & 3+3\nu & -6-2\nu & -3+9\nu & -6+2\nu & -3-3\nu & 4\nu & 3-9\nu \\ 3+3\nu & 12-4\nu & 3-9\nu & 4\nu & -3-3\nu & -6+2\nu & -3+9\nu & -6-2\nu \\ -6-2\nu & 3-9\nu & 12-4\nu & -3-3\nu & 4\nu & -3+9\nu & -6+2\nu & 3+3\nu \\ -3+9\nu & 4\nu & -3-3\nu & 12-4\nu & 3-9\nu & -6-2\nu & 3+3\nu & -6+2\nu \\ -6+2\nu & -3-3\nu & 4\nu & 3-9\nu & 12-4\nu & 3+3\nu & -6-2\nu & -3+9\nu \\ -3-3\nu & -6+2\nu & -3+9\nu & -6-2\nu & 3+3\nu & 12-4\nu & 3-9\nu & 4\nu \\ 4\nu & -3+9\nu & -6+2\nu & 3+3\nu & -6-2\nu & 3-9\nu & 12-4\nu & -3-3\nu \\ 3-9\nu & -6-2\nu & 3+3\nu & -6+2\nu & -3+9\nu & 4\nu & -3-3\nu & 12-4\nu \end{bmatrix}$$

$$= \frac{E}{24(1-\nu^2)}\left(\underbrace{\begin{bmatrix} 12 & 3 & -6 & -3 & -6 & -3 & 0 & 3 \\ 3 & 12 & 3 & 0 & -3 & -6 & -3 & -6 \\ -6 & 3 & 12 & -3 & 0 & -3 & -6 & 3 \\ -3 & 0 & -3 & 12 & 3 & -6 & 3 & -6 \\ -6 & -3 & 0 & 3 & 12 & 3 & -6 & -3 \\ -3 & -6 & -3 & -6 & 3 & 12 & 3 & 0 \\ 0 & -3 & -6 & 3 & -6 & 3 & 12 & -3 \\ 3 & -6 & 3 & -6 & -3 & 0 & -3 & 12 \end{bmatrix}}_{A} + \nu\underbrace{\begin{bmatrix} -4 & 3 & -2 & 9 & 2 & -3 & 4 & -9 \\ 3 & -4 & -9 & 4 & -3 & 2 & 9 & -2 \\ -2 & -9 & -4 & -3 & 4 & 9 & 2 & 3 \\ 9 & 4 & -3 & -4 & -9 & -2 & 3 & 2 \\ 2 & -3 & 4 & -9 & -4 & 3 & -2 & 9 \\ -3 & 2 & 9 & -2 & 3 & -4 & -9 & 4 \\ 4 & 9 & 2 & 3 & -2 & -9 & -4 & -3 \\ -9 & -2 & 3 & 2 & 9 & 4 & -3 & -4 \end{bmatrix}}_{B}\right)$$

$$A_{11} = A_{22} = \begin{bmatrix} 12 & 3 & -6 & -3 \\ 3 & 12 & 3 & 0 \\ -6 & 3 & 12 & -3 \\ -3 & 0 & -3 & 12 \end{bmatrix}$$

$$A_{12} = A_{21} = \begin{bmatrix} -6 & -3 & 0 & 3 \\ -3 & -6 & -3 & -6 \\ 0 & -3 & -6 & 3 \\ 3 & -6 & 3 & -6 \end{bmatrix}$$

$$B_{11} = B_{22} = \begin{bmatrix} -4 & 3 & -2 & 9 \\ 3 & -4 & -9 & 4 \\ -2 & -9 & -4 & -3 \\ 9 & 4 & -3 & -4 \end{bmatrix}$$

$$B_{12} = B_{21} = \begin{bmatrix} 2 & -3 & 4 & -9 \\ -3 & 2 & 9 & -2 \\ 4 & 9 & 2 & 3 \\ -9 & -2 & 3 & 2 \end{bmatrix}$$