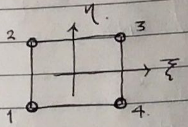
10th May 2021

Strain @ centre of an element.



$\frac{\partial N_i}{\partial \xi}$ & $\frac{\partial N_i}{\partial \eta}$

i	$\partial N_i / \partial \xi$	$\partial N_i / \partial \eta$
1	$-\frac{1}{4}(1-\eta)$	$-\frac{1}{4}(1-\xi)$
2	$-\frac{1}{4}(1+\eta)$	$\frac{1}{4}(1-\xi)$
3	$\frac{1}{4}(1+\eta)$	$\frac{1}{4}(1+\xi)$
4	$\frac{1}{4}(1-\eta)$	$-\frac{1}{4}(1+\xi)$

@ element centre

1	$-\frac{1}{4}$	$-\frac{1}{4}$
2	$-\frac{1}{4}$	$\frac{1}{4}$
3	$\frac{1}{4}$	$\frac{1}{4}$
4	$\frac{1}{4}$	$-\frac{1}{4}$.

$$\frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi}\frac{\partial \xi}{\partial x}$$ if aligned to global coordinates.

$$\frac{\partial \xi}{\partial x} = \frac{2}{h}$$ h : element size

i	$\partial N_i / \partial x$	$\partial N_i / \partial y$
1	$-1/2h$	$-1/2h$
2	$-1/2h$	$1/2h$
3	$1/2h$	$1/2h$
4	$1/2h$	$-1/2h$

<u>strains</u>

$$\varepsilon_{xx} = \frac{1}{2h}\left(u_3 + u_4 - u_1 - u_2\right).$$

$$\varepsilon_{yy} = \frac{1}{2h}\left(-v_1 + v_2 + v_3 - v_4\right)$$

$$\varepsilon_{xy} = \frac{1}{2h}\left(-u_1 - v_1 + u_2 - v_2 + u_3 + v_3 - u_4 + v_4\right)$$

<u>Stresses</u>

$$\sigma = E^* \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

$$E^* = \frac{E}{1-\nu^2}$$ plane σ

$$\sigma_{xx} = \frac{E^*}{2h}\begin{pmatrix} -u_1 - u_2 + u_3 + u_4 \\ + \nu(-v_1 + v_2 + v_3 - v_4) \end{pmatrix}$$

$$\sigma_{yy} = \frac{E^*}{2h}\begin{pmatrix} -v_1 + v_2 + v_3 - v_4 \\ + \nu(-u_1 - u_2 + u_3 + u_4) \end{pmatrix}$$

$$\sigma_{xy} = \frac{E^*(1-\nu)}{4h}\begin{pmatrix} -u_1 + u_2 + u_3 - u_4 \\ -v_1 - v_2 + v_3 + v_4 \end{pmatrix}$$

for plane σ

$$\nu = \frac{\varepsilon^A_{yy}\sigma_{xx} - \varepsilon^A_{xx}\sigma_{yy}}{\varepsilon^A_{yy}\sigma_{yy} - \varepsilon^A_{xx}\sigma_{xx}}$$

$$E = \frac{\sigma_{xx} - \nu\sigma_{yy}}{\varepsilon^A_{xx}}$$

should be σ_1, σ_2 rather than σ_{xx}, σ_{yy}

minimise

$$\| \varepsilon^A_{ij} - \varepsilon^h_{ij} \|$$

function of d $\therefore$ E, ν

with respect to E, ν in each element.

$$\frac{\partial \varepsilon^h_{ij}}{\partial E} = \frac{\partial \varepsilon^h_{ij}}{\partial d}\frac{\partial d}{\partial E}$$

numerical evaluation?

$$d = K^{-1} f$$

constant.

$$\underset{6\times 1}{\frac{\partial \varepsilon^h}{\partial E}} = \underset{6\times n}{\frac{\partial \varepsilon^h}{\partial d}}\ \underset{n\times n^2}{\frac{\partial d}{\partial K}}\ \underset{n^2 \times \cancel{1}}{\boxed{\frac{\partial K}{\partial E}}}\ \underset{n\times 1}{f}$$

knowns. ε^A_{ij}, f

unknowns E, ν per element.

each element is constant in terms of E & ν.

residual $\varepsilon^A_{ij} - \varepsilon^h_{ij} = 0$

per element. $3 \times$ nels

compatibility... can this help

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}$$

co axiality ?

solving for E & ν

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}$$

$$\begin{aligned} \varepsilon_{xx} E - \sigma_{xx} + \nu \sigma_{yy} &= 0 \\ \varepsilon_{yy} E + \nu \sigma_{xx} - \sigma_{yy} &= 0 \\ \gamma_{xy} E - 2(1+\nu)\sigma_{xy} &= 0 \end{aligned}$$

$$\begin{bmatrix} \varepsilon_{xx} & \sigma_{yy} \\ \varepsilon_{yy} & \sigma_{xx} \\ \gamma_{xy} & -2\sigma_{xy} \end{bmatrix} \begin{Bmatrix} E \\ \nu \end{Bmatrix} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ 2\sigma_{xy} \end{Bmatrix}$$

one or four Gauss points
for E & ν determination ?