Available online at www.sciencedirect.com
ScienceDirect
Physica A 380 (2007) 470–480
PHYSICA A
www.elsevier.com/locate/physa

Cellular automaton model for mixed traffic flow with motorcycles

Jian-ping Meng[a,b,c], Shi-qiang Dai[a,*], Li-yun Dong[a], Jie-fang Zhang[b,c]

[a]Shanghai Institute of Applied Mathematics and Mechanics,Shanghai University, Shanghai 200072, China
[b]Key Laboratory of Intelligent Technologies and Systems of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China
[c]Institute of Nonlinear Physics, Zhejiang Normal University,Jinhua 321004, China

Received 5 May 2006; received in revised form 24 January 2007
Available online 6 March 2007

Abstract

A single-lane cellular automaton model is proposed to simulate mixed traffic with motorcycles. By performing numerical simulations under the periodic boundary condition, some density-flow relations and the "lane-changing" behavior of motorcycles are investigated in detail. It is found that the maximum car flow decreases because of the "lane-changing" behavior of motorcycles. The maximum total flow increases first and then decreases with increasing motorcycle density. Moreover, the transition of the total flow from the free flow to the congested flow is smooth in this model. The "lane-changing" rate of motorcycles will decrease to zero finally with the increase of the car density. But its evolutionary trend is considerably complex. Another interesting fact is that, with the increase of the motorcycle density, the "lane-changing" rate increases first and decreases later. This phenomenon is very similar to the findings in other papers on multi-lane car flows. The"lane-changing" is almost of no use in increasing the flow of motorcycles as the motorcycle density is small. But it distinctly causes the increase in the flow of motorcycles when the motorcycle density is sufficiently large, and in this density regime, the flow of motorcycles gradually decreases to the one given with the Nagel–Schreckenberg model for motorcycles with the increase of the car density. The simulation results indicate that it is necessary to set a barrier or a lane for separating the motorcycle flow from the car flow except in some special density regime.

Keywords: Mixed traffic flow; Cellular automaton model; Motorcycles

1. Introduction

In the past few decades, the study of complex systems, such as social, economic, biological and traffic systems, etc. has become one of the most exciting and attracting fields for many physicists. It has been shown that phenomena with great complexity could be observed even in the system composed of very simple basic components interacting with each other. For explaining and reproducing the behavior of these complex systems, a lot of concepts and approaches of physics, especially statistical physics, have been replanted into related areas. The concepts of self-organization and entropy are typical cases in point. On the other hand,

*Corresponding author.
E-mail address: jpmeng@gmail.com (J.-P. Meng).

0378-4371/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2007.02.091

cellular automaton proposed by von Neumann seems to become a universal paradigm for modeling complex systems. It has been used in quite extensive interesting topics, such as the DNA sequence evolution, urban modeling, consensus formation, etc. (see Refs. [1–3] and references therein).

In traffic flow theory, cellular automaton (CA) theory could also find its wide applications (see Refs. [4–6] for review). In 1992, Nagel and Schreckenberg proposed a well-known CA traffic model [7], later named as the Nagel–Schreckenberg model (the NaSch model for short). The model could be used to reproduce some basic phenomena encountered in real traffic, for instance, the start–stop waves appearing in the congested traffic region. From then on, the NaSch model has been extensively studied and applied, and some transportation simulation systems have been developed based on it, e.g., the TRANSMIS [8]. At the same time, for the sake of reproducing more realistic phenomena, several improved versions of the NaSch model or new models were proposed, such as the VDR model which supplementing with a so-called slow-to-start rule, the TOCA-model of Brilon et al., the KKW model based on Kerner's three-phase traffic theory and so on (see Refs. [9,10] and references therein). Inspired by the success in single-lane flow, several lane-changing rules have been proposed for modeling multi-lane traffic [11–18] since multi-lane traffic is more popular. A special phenomenon of the density inversion in the lane usage as well as the fundamental diagram of multi-lane traffic was successfully reproduced.

Obviously, the application of cellular automata in traffic flow is very successful. However, more novel rules in CA models must be presented for more complex situations. Mixed traffic with motorcycles is an important traffic type in some Asian developing countries, for example, in China, Malaysia, Vietnam, etc. According to some surveys (see Refs. [19–21]) motorcycle is one of the most widely used vehicles in these countries. Compared with the developed countries, the difference may be attributed to some factors like the weather, economy, population density and cultural background. Therefore, it is of great importance to uncover mixed traffic flow characteristics for enhancing the traffic performance. However, there have been only few papers concerning this topic in theoretical research. In Refs. [22–24], mixed traffic flow composed of vehicles with different maximum speeds or with different lengths were investigated using the hydrodynamic model or cellular automaton model. Some meaningful results were obtained. But, for mixed traffic flow with motorcycles, there is yet another key characteristic that more than two motorcycles could be ridden side by side on a single-lane road because of their smaller width. It makes the models mentioned above not so appropriate for the simulation. So, in this paper we propose a cellular automaton model to simulate the mixed traffic flow with motorcycles, and as an attempt, we would consider the single-lane model herein.

The paper is organized as follows. In Section 2, our model is outlined. In Section 3, the simulation results are presented and analyzed. A short summary and some conclusions are given in Section 4.

2. Model

Our model will be based on the NaSch model. The NaSch model is defined on a one-dimensional array of L sites under open or periodic boundary conditions. Each site may either be occupied by at most one vehicle or be empty. Suppose that x_n and v_n denote the position and speed of the nth vehicle, respectively, $v_{\max}$ denotes the maximum velocity which a vehicle can reach and $d_n = x_{n+1} - x_n - 1$ is the gap between the nth vehicle and the vehicle in front of it at time t. Then each vehicle can move with an integer velocity $v_i \in \{0, \ldots, v_{\max}\}$ and for an arbitrary configuration, one update of the system consists of the following four consecutive steps, which are performed in parallel for all vehicles: *Step* 1: *Acceleration.* If $v_n < v_{\max}$, the speed of the nth vehicle is increased by one, but v_n remains unaltered if $v_n = v_{\max}$, i.e., $v_n \to \min(v_n + 1, v_{\max})$. *Step* 2: *Deceleration* (*due to other vehicles*). If $d_n < v_n$, the speed of the nth vehicle is reduced to d_n, i.e., $v_n \to \min(v_n, d_n)$. *Step* 3: *Randomization.* If $v_n > 0$, the speed of the nth vehicle is randomly decreased by unity with the probability p but v_n does not change if $v_n = 0$, i.e., $v_n \to \max(v_n - 1, 0)$ with probability p. *Step* 4: *Vehicle movement.* Each vehicle is moved forward according to its new velocity determined in Steps 1–3, i.e., $x_n \to x_n + v_n$.

To establish our model, some additional assumptions are made as follows. Firstly, we must know how many motorcycles could be ridden side by side safely. Unexpectedly, the question is difficult to answer, because there has been no direct report in this aspect in literature. Fortunately, there is a related report on bicycles, so we can have an estimate. According to Ref. [25], the width of a "three-lane" road for bicycles should be 3.5 m. Since motorcycles, especially the modern scooter type used widely now, should have a bit larger width than

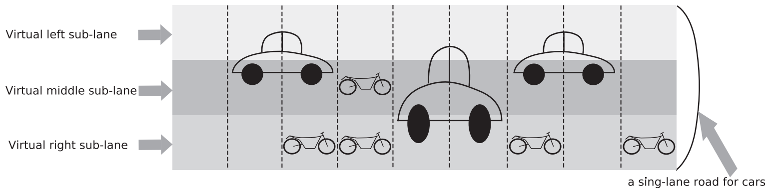

Fig. 1. A sketch for the model.

bicycles, so we suppose that only three motorcycles can be ridden side by side on a standard single-lane road, i.e., one site can be occupied in parallel by three motorcycles at most. Therefore, we can divide a single-lane road for cars into three imaginary ‘‘sub-lanes’’ for motorcycles and we call them the virtual right sub-lane(VRSL), virtual middle sub-lane(VMSL) and virtual left sub-lane(VLSL) (see Fig. 1).

Moreover, further simplification could be made as follows. In many countries, such as in China, it is regulated by traffic laws that motorcycles should be ridden on the right side-lane of a road (Maybe left side-lane in some other countries, however, because of the symmetry, the discussion herein is also applicable). It is affirmed also by some empirical research [26]. Thus, motorcycles are prescribed not to be ridden on the VLSL in our model.

Now we turn to investigate the length of road $L^{(m)}$ occupied by one motorcycle. Since the length of a motorcycle is about 2 m, taking account with the safety-distance, we suppose that each motorcycle occupied about 3.75 m of road in a complete traffic jam, which is the half of the length one car needs. So, in the following simulation, we set the length of one site to be 3.75 m. Thus each motorcycle occupies one site and each car occupies two sites. The similar method has already been employed in the case of mixed car/truck traffic in which cars are ‘‘shorter’’ vehicles and trucks are ‘‘longer’’ vehicles (see TRANSIMS documentation for reference, also see Ref. [27]).

Thus, our model is defined on a one-dimensional array of L sites, but now, each site can be occupied by at most two motorcycles or one half of the car (either its front part or back part) plus one motorcycle. Of course, there might appear three other situations, that is, a site being empty, or occupied by one half of the car (either its front part or back part), or occupied by one motorcycle. Cars could occupy either the whole width of the road, or the VMSL plus VLSL if there is a motorcycle on the VRSL, while motorcycles could be ridden on either the VRSL or VMSL. All the situations are shown in Fig. 1. Naturally, the velocity and position evolution rules of the NaSch model should be adopted for cars. Furthermore, the motorcycle is simply considered as a ‘‘car’’ with a lower maximum velocity when it is ridden on the VRSL or VMSL. So, the NaSch evolution rules of the velocity and position (with a lower maximum velocity) could also be adopted for the motorcycle (named as the NaSch model for motorcycles below). Thus, the update rules for cars and motorcycles are same as those in the NaSch model except the differences of parameters (see Table 1) and the most important ‘‘lane-changing’’[1] rule for motorcycles, which would be discussed detail below.

Since motorcycles could be ridden on the VRSL or VMSL—which is the actual representation of the maneuvering flexibility of motorcycles, some ‘‘lane-changing’’ rules for motorcycles[2] should be proposed. In Refs. [12–17], some lane-changing rules for two-lane car flows were discussed. Here, we would take the asymmetric (hypothetical American criterion) lane-changing rules of Nagel et al for reference, because the situations are quite similar[3] and the rules are more realistic than the other ones (see Ref. [16] and references therein for details).

[1]Because of its maneuvering flexibility, a motorcycle can rush into the car flow for overtaking the motorcycle in front of it and then turn back to the motorcycle flow again. It is quite similar to the lane-changing behavior of cars in two-lane traffic. So, it is also named as ‘‘lane-changing’’ in this paper.

[2]Since a single-lane road for cars is assumed here, there is no lane-changing of cars.

[3]According to different legal constraints, three lane-changing rules (the German criterion rule, the asymmetric rule and the symmetric rule) were proposed for two-lane traffic flow in Ref. [16]. The German criterion rule corresponds to the situation that the right lane has to be used by default and passing has to be on the left lane, the asymmetric rule to the situation that the second constraint is considerably relaxed and the symmetric rule to the situation that two constraints are both considerably relaxed. In the actual mixed traffic flow with motorcycles, the VRSL has to be used by default but it is not explicitly forbidden for motorcycles on the VRSL to overtake cars on the VMSL and VLSL or motorcycles on the VMSL. So, we use the asymmetric rule for reference in this context.

Table 1
Summary of definitions for the variables and parameters used in the present model

Variable or parameter	Description
$x_n^{(m)}$	Position of the nth motorcycle on the VMSL or VRSL
$x_n^{(c)}$	Position of the nth car
$v_{max}^{(m)}$	Maximum velocity of the motorcycle
$v_{max}^{(c)}$	Maximum velocity of the car
$v_n^{(c)}$	Velocity of the nth car
$v_n^{(m)}$	Velocity of the nth motorcycle on the VMSL or VRSL
$p^{(c)}$	Deceleration probability of the car
$p^{(m)}$	Deceleration probability of the motorcycle
d	Number of sites one looks ahead for the incentive criterion
gap_+	Gap on the target lane in front of the motorcycle that wants to change lane
gap_-	Gap on the target lane behind the motorcycle that wants to change lane
$d_n^{(c)}$	Gap between the nth car and the car or motorcycle in front of it
$d_n^{(m)}$	Gap between the nth motorcycle and the motorcycle or car in front of it on the VMSL or VLSL
$\mathrm{gap}_-^{(mc)}$	Gap between the nth motorcycle on the VMSL and the nearest car behind it
v_{VMSL}	Velocity of the car or of the motorcycle on the VMSL within a certain distance one looks ahead d
v_{VRSL}	Velocity of the motorcycle on the VRSL within a certain distance one looks ahead d

First, there must be sufficient space on the target lane, because of the request of the safety. Technically, there must be a gap of size $\mathrm{gap}_- + 1 + \mathrm{gap}_+$. The subscript $+(-)$ stands for the gap on the target lane in front of (behind) the motorcycle that wants to change lane. In this paper, we used $\mathrm{gap}_+ = v$ and $\mathrm{gap}_- = v_{max}^{(m)}$ (if the VRSL is the target lane) or $\mathrm{gap}_- = v_{max}^{(c)}$ (if the VMSL is the target lane), where v is the velocity of the motorcycle that wants to change lane.

Second, the "lane-changing" should reduce drivers' travel time and the behavior should be regulated by the traffic laws. Therefore, the lane changing to the VMSL is triggered by finding a slow front motorcycle on the VRSL and that the VMSL is more attractive. In this context, it means there is a slower motorcycle ahead on the VRSL and the next front car or motorcycle on the VMSL is faster than the motorcycle ahead on the VRSL. The rule can be represented mathematically as follows:

$$v_{\mathrm{VRSL}} \leqslant v_n^{(m)} \quad \text{and} \quad v_{\mathrm{VRSL}} \leqslant v_{\mathrm{VMSL}},$$

where v_{VRSL} refers to the velocity of the motorcycle on the VRSL and v_{VMSL} refers to the velocity of the car or of the motorcycle on the VMSL, within a certain distance d[4] one looks ahead. If neither car nor motorcycle is detected in the range d on the corresponding lane, the corresponding velocity v_{VRSL} or v_{VMSL} is set to be ∞.

The rule for changing back to VRSL is different from Nagel's one. Besides the rule that there is a faster front motorcycle than the considered motorcycle (or no motorcycle at all) on the VRSL or traffic on the VRSL is flowing faster than on the VMSL and VRSL, additional prescription should be given for the safety and efficiency. When a car is blocked by a motorcycle, i.e., $\mathrm{gap}_-^{(mc)} = 0$, the motorcycle must change lane, which is called as car priority rule below. Thus the rule becomes

$$gap_-^{(mc)} = 0 \quad \text{or} \quad v_{\mathrm{VRSL}} \geqslant v_n^{(m)} \quad \text{or} \quad v_{\mathrm{VRSL}} \geqslant v_{\mathrm{VMSL}}.$$

In summary, our model could be divided into two major steps: the lane-changing and the forward movement. Here, the "lane-changing" is implemented as a pure sideways movement. The time motorcycles usually need to change lanes are underestimated and motorcycles do not move forward in the "lane-changing" step. But it is infeasible in reality, only together with the forward movement rule our update rules make physically sense (see Refs. [13,16]). Finally, our model could be outlined as the following five consecutive steps, which are

[4]In the modeling of the two-lane traffic, the distance d is a very important parameter for reproducing the density inversion phenomenon. Obviously, if one looks far ahead, one has a tendency to go to the VMSL already far away from an obstructing vehicle, thus leading to a density inversion at low densities. One can also refer to Ref. [16] for a clearer understanding.

performed in parallel for all cars and motorcycles, and the definitions of the involved variables and parameters are listed in Table 1.

Step 1: Lane-changing (for motorcycles).
From VRSL to VMSL:
if $v_n^{(m)} \leqslant \text{gap}_+$ and $\text{gap}_- \geqslant v_{\max}^{(c)}$ and $v_{\text{VRSL}} \leqslant v_n^{(m)}$ and $v_{\text{VRSL}} \leqslant v_{\text{VMSL}}$,
then the motorcycle changes lane.

From VMSL to VRSL:
if $(v_n^{(m)} \leqslant \text{gap}_+$ and $\text{gap}_- \geqslant v_{\max}^{(m)})$ and
$(\text{gap}_-^{(mc)} = 0$ or $v_{\text{VRSL}} \geqslant v_n^{(m)}$ or $v_{\text{VRSL}} \geqslant v_{\text{VMSL}})$ then the motorcycle changes lane.

Step 2: Acceleration.
$v_n^{(c)} \to \min(v_n^{(c)} + 1, v_{\max}^{(c)})$ for cars,
$v_n^{(m)} \to \min(v_n^{(m)} + 1, v_{\max}^{(m)})$ for motorcycles on the VRSL or VMSL.

Step 3: Deceleration.
$v_n^{(c)} \to \min(v_n^{(c)}, d_n^{(c)})$ for cars,
$v_n^{(m)} \to \min(v_n^{(m)}, d_n^{(m)})$ for motorcycles on the VRSL or VMSL.

Step 4: Randomization.
$v_n^{(c)} \to \max(v_n^{(c)} - 1, 0)$ with the probability $p^{(c)}$ for cars,
$v_n^{(m)} \to \max(v_n^{(m)} - 1, 0)$ with the probability $p^{(m)}$ for motorcycles on the VRSL or VMSL.

Step 5: Car or motorcycle movement.
$x_n^{(c)} \to x_n^{(c)} + v_n^{(c)}$ for cars,
$x_n^{(m)} \to x_n^{(m)} + v_n^{(m)}$ for motorcycles on the VRSL or VMSL.

3. Numerical simulation and discussion

3.1. Procedure of simulation setup

In simulation, a system of 2000 sites is considered under the periodic boundary condition. According to Section 2, the length of each site is set to be 3.75 m, each motorcycle occupies one site and each car occupies two sites, so the system is equivalent to a road of 7.5 km long. Thus the global car density and the global motorcycle density is a conserved quantity, i.e., $\rho^{(c)} = N^{(c)}/L$, $\rho^{(m)} = N^{(m)}/L$, where $N^{(c)}$ and $N^{(m)}$ denotes the numbers of cars and motorcycles, respectively, and $L = 7.5$ km. One iteration time step is taken as 1 s, $v_{\max}^{(c)}$ is set to be 10 *sites/time–step* (i.e., 135 km/h) and $v_{\max}^{(m)}$ is set to be 4 *sites/time–step* (i.e., 54 km/h).

When we started to perform numerical simulation, cars with a given density were initially distributed randomly on the road, while motorcycles with a given density were initially distributed randomly on the VRSL. After a transient period $t_0 (t_0 = 10\,000$ time steps), we recorded the space-averaged velocity of cars and motorcycle, respectively, at each time step in the period of T (1000 time steps) and made the time-average for these velocity values. Finally we obtained the average velocity in a run, i.e.,

$$\bar{v}^{(m)} = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} \left(\frac{1}{N^{(m)}} \sum_{i=1}^{N^{(m)}} v_i^{(m)}(t) \right),$$

$$\bar{v}^{(c)} = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} \left(\frac{1}{N^{(c)}} \sum_{i=1}^{N^{(c)}} v_i^{(c)}(t) \right).$$

In the following diagrams, each point represents an average value of 30 runs.

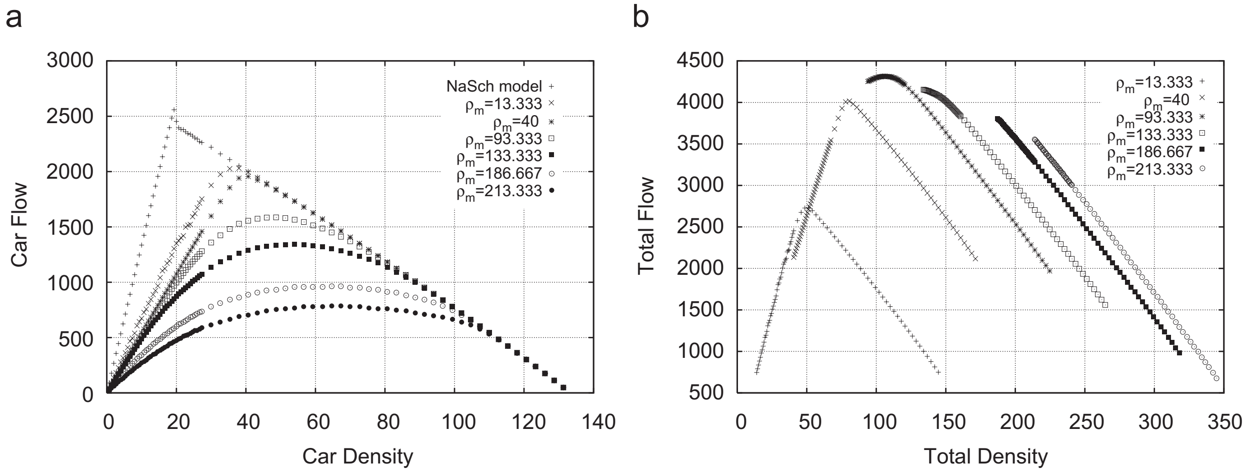

Fig. 2. (a) and (b) are the fundamental diagrams for the car flow and total flow with $p^{(m)} = 0.1$, $p^{(c)} = 0.1$, $d = 6$ and with $\rho^{(m)} = 13.333$ vehicles/km, $\rho^{(m)} = 40$ vehicles/km, $\rho^{(m)} = 93.333$ vehicles/km, $\rho^{(m)} = 133.333$ vehicles/km, $\rho^{(m)} = 186.667$ vehicles/km, $\rho^{(m)} = 213.333$ vehicles/km. For comparison, the flow–density relation for the NaSch model is presented in (a). The unit vehicles/km is used for the density and vehicles/h for the flow.

Considering that the passenger car unit (PCU) of the motorcycle is 1 on a car-based index (i.e., the so called Car Equivalent),[5] we have the following relations:

$$Q^{(c)} = \rho^{(c)}\bar{v}^{(c)},$$
$$Q^{(m)} = \rho^{(m)}\bar{v}^{(m)},$$
$$Q^{(t)} = Q^{(c)} + Q^{(m)} = \rho^{(c)}\bar{v}^{(c)} + \rho^{(m)}\bar{v}^{(m)},$$
$$\rho^{(t)} = \rho^{(c)} + \rho^{(m)},$$

where $Q^{(c)}$ and $Q^{(m)}$ denote the average flows of cars and motorcycles, respectively, and $Q^{(t)}$ and $\rho^{(t)}$ denote the average total flow and total density, respectively. By use of these relations, we could figure out $Q^{(t)}$ and $\rho^{(t)}$ from the simulation data we recorded.

3.2. Flow behavior

One of most important relations for depicting traffic flow is the relation between the density and the flow and the curves of the flow versus the density are called the fundamental diagrams. In Fig. 2, we present the fundamental diagrams for the total flow and the flow of cars at some given $\rho^{(m)}$. First, we investigate the flow of cars. Obviously, because of the ''lane-changing'' behavior of motorcycles, the maximum flow of cars decreases even at a small $\rho^{(m)}$, compared with the one in the NaSch model. The critical density $\rho_c^{(c)}$, at which the maximum flow of cars is reached, increases at the same time. Second, let us examine the total flow. In order to learn more about the flow–density relation for the total flow, the relations at six given $\rho^{(m)}$ are presented. From these figures, one can see that the maximum total flow increases first and decreases later with the increase of $\rho^{(m)}$. Clearly, the total flow function $Q^{(t)} = f(\rho^{(c)}, \rho^{(m)})$ represents a two-dimensional curved surface. For a fixed $\rho^{(m)}$, the free flow occurs in the region $0 \leqslant \rho^{(c)} + \rho^{(m)} \leqslant \rho_c^{(t)}$, where $\rho_c^{(t)}$ representing the critical total density at which the maximum total flow is reached, whereas in the other region congested flow exists. For a fixed $\rho^{(c)}$, the result should be the same. Moreover, we note that the transition of the total flow from the free flow to the congested flow is smooth in the present model.

[5]In a mixed traffic, where different types of vehicles share the same roadway space without any physical segregation, we must convert all vehicles into equivalent the number of PCUs so that we can analyze mixed traffic flow in the same scale. In this paper, we use the standard of China, see Ref. [25].

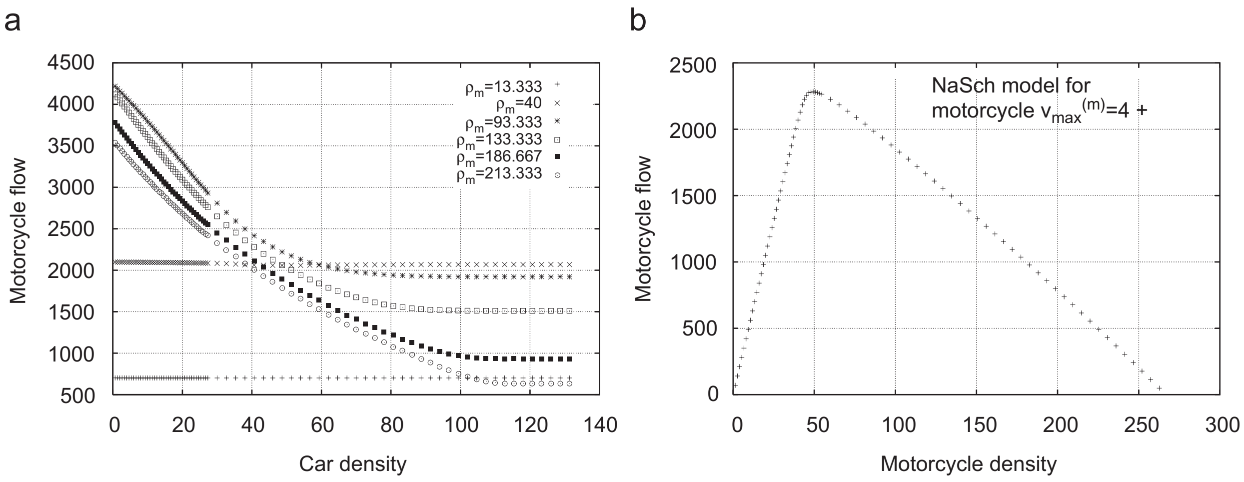

Fig. 3. (a) Diagrams for motorcycle flow versus car density for $p^{(\mathrm{m})}=0.1$, $p^{(\mathrm{c})}=0.1$, $d=6$; (b) The fundamental diagram of the NaSch model for motorcycles. The unit vehicles/km is used for the density and vehicles/h for the flow.

Now we turn to investigate the flow of motorcycles. In Fig. 3(a), we give the diagram for the motorcycle flow versus the car density, and the fundamental diagrams of the NaSch model for motorcycles (single-[virtual sub]lane, no cars) is also shown in Fig. 3(b) for comparison. From the figure, one can see, when the density of cars $\rho^{(\mathrm{c})}$ is small, in the density regime $0 \leqslant \rho^{(\mathrm{m})} \leqslant \rho_c^{(\mathrm{m})}$ ($\rho_c^{(\mathrm{m})}$ denotes a fixed critical density), the flow $Q^{(\mathrm{m})}$ is almost equal to the flow in the NaSch model for motorcycles. Only in the region $\rho^{(\mathrm{m})} \geqslant \rho_c^{(\mathrm{m})}$, the flow $Q^{(\mathrm{m})}$ is larger than the flow in the NaSch model for motorcycles. This phenomenon indicates that rushing into the cars flow is not a reasonable behavior for motorcycles when the density is not so large, since this behavior is more dangerous but seems almost of no use for increasing the efficiency. However, when $\rho^{(\mathrm{m})}$ is sufficiently large, the "lane-changing" behavior increases distinctly the flow of motorcycles. We can see $Q^{(\mathrm{m})} \approx 3028$ vehicles/h when $\rho^{(\mathrm{m})} = 133.333$ vehicles/km and $\rho^{(\mathrm{c})} = 21.333$ vehicles/km. It is twice larger than the one in the NaSch model for motorcycles, which is about 1512 vehicles/h. With the increase of $\rho^{(\mathrm{c})}$, $Q^{(\mathrm{m})}$ gradually decreases to the one given with the NaSch model for motorcycles in the region $\rho^{(\mathrm{m})} \geqslant \rho_c^{(\mathrm{m})}$. The result is not surprising since it becomes more and more difficult for motorcycles to change lanes with the increase of $\rho^{(\mathrm{c})}$.

3.3. Lane-changing behavior

In this section, we investigate the "lane-changing" behavior. The "lane-changing" rate ξ_{r} and ξ_{m} are defined as the average "lane-changing" frequency per motorcycle, in which ξ_{r} denotes the rate of turning to the VRSL and ξ_{m} denotes the rate of turning to the VMSL. Here, we only pay attention to the stationary "lane-changing" rate. Typical results are shown in Fig. 4.

First, we find that $\xi_{\mathrm{r}} \approx \xi_{\mathrm{m}}$. Obviously, it is because the system is running in the stationary state. When $\rho^{(\mathrm{c})}$ is small, ξ_{r} and ξ_{m} is large. With the increase of $\rho^{(\mathrm{c})}$, ξ_{r} and ξ_{m} will decrease to zero finally, since there are so many cars that the "lane-changing" could not be implemented. But an interesting phenomenon is that the "lang-changing" rate increases first and decreases later in some motorcycle density regime with the increase of $\rho^{(\mathrm{c})}$. In some more special regime, the trend can even be decrease–increase–decrease. The phenomenon may be interpreted as follows. In this special motorcycle density regime, with the increase of $\rho^{(\mathrm{c})}$, more and more motorcycles must be distributed on the VRSL, while $\rho^{(\mathrm{c})}$ is small, they can be ridden on the VMSL, the traffic condition of the VRSL becomes even worse than that of the VMSL at a certain car density. Thus, the desire of "lane-changing" will become stronger though it is more and more difficult to implement the "lane-changing" behavior, and the "lane-changing" rate will increase with the increase of the car density until the traffic condition of the VMSL is worse than the one of the VRSL again. In some more special density regime, the traffic condition on the VRSL is worse than that on the VMSL when the car density begin to increase.

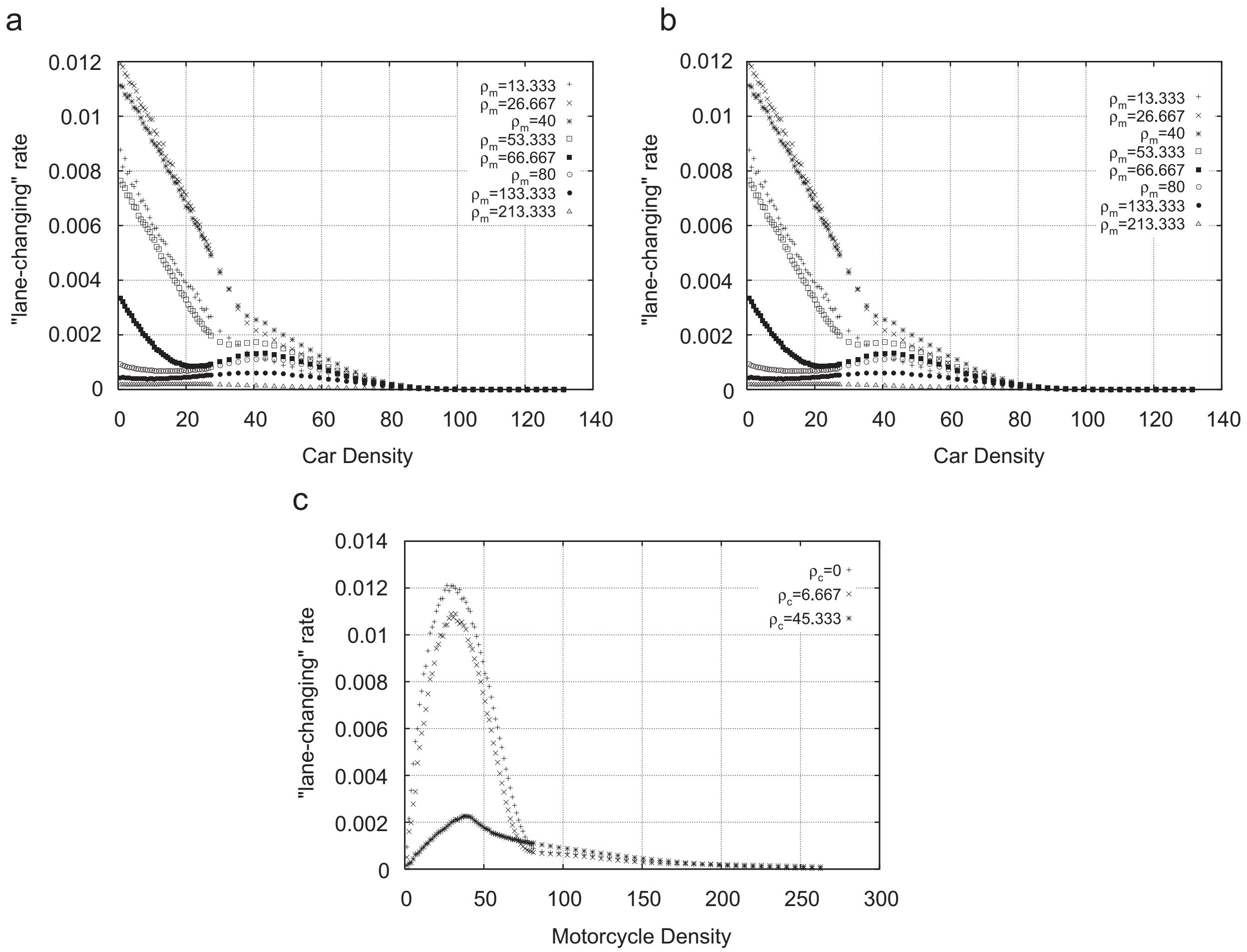

Fig. 4. "Lane-change" rate as a function of car density (a,b) and motorcycle density (c) in case of (a) turning to the VRSL, (b)turning to the VMSL and (c) turning to the VRSL for $p^{(m)} = 0.1$, $p^{(c)} = 0.1$, $d = 6$. The figures show the data of stationary "lane-changing" rate. The unit vehicles/km is used for the density.

The above fact causes the interesting phenomenon we have found. On the other hand, when $\rho^{(m)}$ is sufficiently small, the traffic condition of the VRSL keeps good all the time; when $\rho^{(m)}$ is sufficiently large, the traffic condition of the VMSL is not better than the one of the VRSL, and with the increase of $\rho^{(c)}$, the condition will become worse. So in these two density regimes, the "lane-changing" rate keeps decreasing as the car density increases.

Another interesting fact is that ξ_r and ξ_m increases first and decreases later with the increase of $\rho^{(m)}$. When $\rho^{(m)}$ is sufficiently large, ξ_r and ξ_m decreases in contrary to the expectation, though $\rho^{(c)}$ is small. Fig. 4(c) shows that this phenomenon is very similar to the findings in previous work on multi-lane car flows (e.g., Fig. 6 in Ref. [13]). But, we can find that the density at which the maximum lane changing rate occurs is much higher than the one in the default two lane NaSch model presented in Fig. 6 of Ref. [13]. It is about 32 vehicles/km, while the one presented in Ref. [13] is only about 8 vehicles/km. It may be due to the fact that maximum gap_+ of motorcycles is smaller than that of cars, since $v_{\max}^{(m)}$ is smaller.

3.4. Impact of car priority rule on the car flow

In the Section 2, the car priority rule has been introduced into our model for the safety and efficiency. Now we are interested in the impact of this special rule on the car flow. For this purpose, the fundamental diagrams

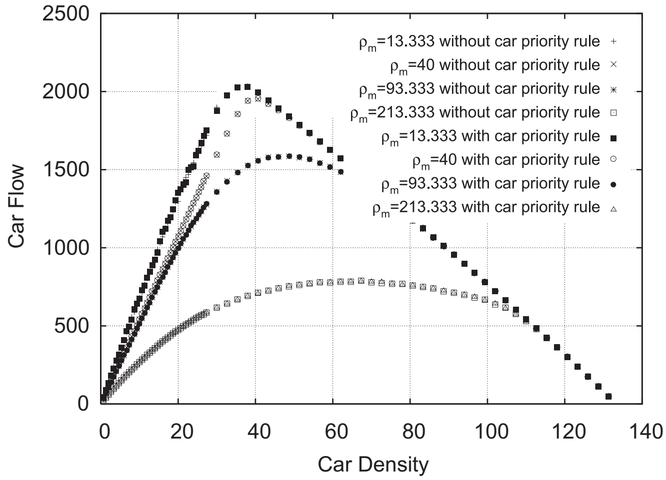

Fig. 5. The fundamental diagrams of the model with and without car priority rule. The unit vehicles/km is used for the density and vehicles/h for the flow.

of the model with and without car priority rule, where $\rho_m = 13.333, 40, 93.333$ and 213.333 (vehicles/km), are depicted in Fig. 5. From the figure, we find that the rule has no impact on the car flow. This result might be caused by the periodic boundary conditions and the "platoon" effect, i.e., slower motorcycles are followed by the faster cars and the average velocity reduces to the free-flow velocity of the motorcycles. So, local car priority has no impact on the global flow. But, from the realistic point of view, this rule should be necessary and it may work under the open boundary conditions. We will study this problem further in the future research.

Finally, for a short summary, we could point out that, from the simulation results, mixed traffic may increase the flow only in the situation that $\rho^{(m)}$ is large while $\rho^{(c)}$ is small (the region of $\rho^{(m)}$ depends on the value $\rho^{(c)}$), compared with the total flow of the separated car flow and motorcycle flow. In other cases, it is necessary to set a physical barrier or a special lane for separating the motorcycle flow from the car flow, since it is safer and more efficient.

4. Summary and conclusion

In summary, we have proposed a single-lane cellular automaton model to simulate the mixed traffic with motorcycles. By performing numerical simulations under the periodic boundary condition, we have investigated some density–flow relations and the "lane-changing" behavior of motorcycles in detail. We found that the maximum car flow decreases because of the "lane-changing" of motorcycles. The maximum total flow increases first and decreases later with the increase of the motorcycle density. Moreover, we note that the transition of the total flow from the free flow to the congested flow is smooth in the presented model. The "lane-changing" rate of motorcycles will decrease to zero finally with the increase of the car density. But its evolutionary trend is complex, as we have pointed out in Section 3.3. Another interesting fact is that with the increase of the motorcycle density, the "lane-changing rate" increases first and decreases later. This phenomenon is very similar to the findings in previous work on multi-lane car flows. The "lane-changing" seems almost of no use for increasing the motorcycle flow when the motorcycle density is small. But, it increases the flow distinctly when the motorcycle density is high enough, and in this density regime, the motorcycle flow gradually decreases to that in the NaSch model for motorcycles with the increase of the car density. Finally, based on the results of the simulation, we pointed out that it is necessary to set a physical barrier or a special lane for separating the motorcycle flow from the car flow except in some special density regime.

Although some meaningful results have been obtained in the present paper, it is only a preliminary attempt. There are many open problems yet, such as, the applicability of our model to more complicated mixed traffic, the limitations of the model and the simulation of mixed traffic with cars, motorcycles and bicycles, etc. In our future work, we would study these problems further.

Acknowledgments

This work was supported by the National Basic Research Program of China (Grant no. 2006CB705500), the National Natural Science Foundation of China (Grant no. 10532060), the Special Research Fund for the Doctoral Program in Higher Education of China (Grant no. SRFDP 20040280014) and the Scientific Research Project of Department of Education of Zhejiang Province (Grant no. 20050306).

References

[1] G. Sirakoulisa, I. Karafyllidisb, C. Mizasa, V. Mardirisa, A. Thanailakisb, P. Tsalidesb, A cellular automaton model for the study of dna sequence evolution, Comput. Biol. Med. 33 (2003) 439–453.
[2] L. Li, Y.S. Sato, H.H. Zhu, Simulating spatial urban expansion based on a physical process, Landscape Urban Plann. 64 (2003) 67–76.
[3] M. Tanaka-Yamawaki, S. Kitamikado, T. Fukuda, Consensus formation and the cellular automata, Robotics Autonomous Syst. 19 (1996) 15–22.
[4] D. Chowdhury, L. Santen, A. Schadschneider, Statistical physics of vehicular traffic and some realted systems, Phys. Rep 329 (4–6) (2000) 199–329.
[5] S.Q. Dai, S.W. Feng, G.Q. Gu, Dynamics of traffic flow: its content, methodology and intent, Ziran Zazhi 19 (4) (1997) 196–201 (in Chinese).
[6] D. Helbing, Traffic and related self-driven many-particles systems, Rev. Mod. Phys. 73 (4) (2001) 1067–1141.
[7] K. Nagel, M. Schreckenberg, A cellular automaton model for freeway traffic, J. Phys. I France 2 (12) (1992) 2221–2229.
[8] K. Nagel, M. Rickert, Parallel implementation of the transims micro-simulation, Parallel Comput. 27 (2001) 1611–1639.
[9] W. Knospe, L. Santen, A. Schadschneider, M. Schreckenberg, Empirical test for cellular automaton models of traffic flow, Phys. Rev. E 70 (1) (2004) 016115–016139.
[10] B. Kerner, S. Klenov, A microscopic model for phase transitions in traffic flow, J. Phys. A 35 (2002) L31–L43.
[11] W.Y. Chen, D.W. Huang, W.N. Huang, W.L. Hwang, Traffic flow on a 3-lane highway, Int. J. Mod. Phys. B 18 (31–32) (2004) 4161–4171.
[12] D. Chowdhury, D. Wolf, M. Schreckenberg, Particle hopping models for two-lane traffic with two kinds of vehicles: effects of lane-changing rules, Physica A 235 (3–4) (1997) 417–439.
[13] M. Rickert, K. Nagel, M. Schreckenberg, A. Latour, Two lane traffic simulations using cellular automata, Physica A 231 (1996) 534–550.
[14] W. Knospe, L. Santen, A. Schadschneider, M. Schreckenberg, A realistic two-lane traffic model for highway traffic, J. Phys. A 35 (2002) 3369–3388.
[15] P. Wagner, K. Nagel, D.E. Wolf, Realistic multi-lane traffic rules for cellular automata, Physica A 234 (1997) 687–698.
[16] K. Nagel, D.E. Wolf, P. Wagner, P. Simon, Two-lane traffic rules for cellular automata: a systematic approach, Phys. Rev. E 58 (2) (1998) 1425–1437.
[17] B. Jia, R. Jiang, Q.S. Wu, A realistic two-lane cellular automaton model for traffic flow, Int. J. Mod. Phys. C 15 (3) (2004) 381–392.
[18] A. Ebersbach, J. Schneider, Two-lane traffic with places of obstruction to traffic, Int. J. Mod. Phys. C 15 (4) (2004) 535–544.
[19] S. C. C. T. P. Institute, Shanghai city comprehensive transportation annual report, Technical Report, Shanghai City Comprehensive Transportation Planning Institute, 2005 (in Chinese).
[20] T.P. Hsu, F.M.S. Ahmad, X.D. Nguyen, A comparison study on motorcycle traffic development in some asian countries—case of Taiwan, Malaysia and Vietnam, Technical Report, Institute of Civil Engineering National Taiwan University, 2003.
[21] N. Matsuhashi, T. Hyodo, Y. Takahashi, Image processing analysis of motorcycle oriented mixed traffic flow in Vietnam, in: Proceedings of the Eastern Asia Society for Transportation Studies, vol. 5, 2005, pp. 929–944.
[22] H. Ez-Zahraouya, K. Jetto, A. Benyoussef, The effect of mixture lengths of vehicles on the traffic flow behaviour in one-dimensional cellular automaton, Eur. Phys. J. B 40 (2004) 111–117.
[23] A. Ebersbach, J. Schneider, I. Morgenstern, R. Hammwöhner, The influence of trucks on traffic flow an investigation on the Nagel–Schreckenberg model, Int. J. Mod. Phys. C 11 (4) (2000) 837–842.
[24] P. Zhang, R.X. Liu, S.C. Wong, High-resolution numerical approximation of traffic flow problems with variable lanes and free-flow velocities, Phys. Rev. E 71 (5) (2005) 056704.

[25] F. T. Ren, X. M. Liu, J. Rong, Traffic Engineering Science, China Communications Press, 2003 (in Chinese).
[26] L. Ling, W.Q. Li, W. Wang, L.J. Xu, Analysis of traffic characteristics of mixed traffic flow of automobile and motorocycle in signalized intersection, J. Highw. Transp. Res. Dev. 21 (5) (2004) 0113–0116 (in Chinese).
[27] K. Nagel, P. Stretz, M. Pieck, S. Leckey, R. Donnelly, C.L. Barrett, TRANSIMS traffic flow characteristics, Los Alamos Unclassified Report (LA-UR) 97-3530, 1997.