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Option pricing model with sentiment

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Abstract In this paper, we develop a closed-form option pricing model with the stock sentiment and option sentiment. First, the model shows that the price of call option is amplified by bullish stock sentiment, and is reduced by stock bearish sentiment, and the price of put option is in the opposite situation. Second, the price of call option is more sensitive to bullish stock sentiment; the price of put option is more sensitive to bearish stock sentiment. Third, the price of call option increases substantially with respect to the stock sentiment and the option sentiment. The price of put option decreases substantially with respect to the stock sentiment, increases substantially with respect to the option sentiment. Fourth, our models also reveal that the option volatility smile is steeper (flatter) when the stock sentiment becomes more bearish (bullish). Finally, stock sentiment and option sentiment lead to the option price deviating from the rational price. The model could offer a partial explanation of some option anomalies: option price bubbles and option volatility smile.

Keywords Stock sentiment · Option sentiment · Option pricing model · Option price bubbles · Option volatility smile

JEL Classification G12

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1 Introduction

> When investors move from a risk-neutral world to a risk-averse world, two things happen: the expected growth rate in the stock price changes and the discount rate that must be used for any payoffs from the derivative changes. —— John C. Hull (2008)

In traditional option pricing researches, the investors are assumed to be rational and risk neutral, both the expected growth rate of underlying stock and the discount rate are equal to the riskless rate, for example Black and Scholes (1973), Bick (1987), He and Leland (1993), Ricardo (2002), Faria and Correia-da-Silva (2014) and Guo and Yuan (2014). However, a lot of literatures shows the investors are irrationality and risk-averse in option market. Some literatures present evidence of irrationality of investors in option markets, for example Poteshman (2001), Poteshman and Serbin (2003), Bauer et al. (2009), Mahani and Poteshman (2008) and Siddiqi (2011). Some literatures present evidence of risk-averse of investors in option markets, for example Jackwerth and Rubinstein (1996), Jackwerth (2000).

In this paper, we explore a different angle of analysis: sentiment factor which is the nature of investors involved in the option pricing model and we derive a closed-form solution for the price of a European call and put option.

Traditional hypothesis gives no role to the investor sentiment, suggests that rational investment decision necessarily brings prices closer to fundamentals (Friedman 1953). But this argument is not adopted by behavioral finance, which argues that the asset prices may be impacted by sentiment factor. See, for example, Delong et al. (1990) and Lee et al. (1998) find noise traders could drive prices away from the equilibrium price exacerbating mispricing in an environment where limits to arbitrage exist. A lot of literatures have shown that stock sentiment has a systematic impaction on expected stock return, such as Brown and Cliff (2004); Kumar and Lee (2006); Baker and Wurgler (2006); Lemmon and Portniaguina (2006); Yang and Zhang (2013) and Yang and Zhou (2014).

Traditional option pricing model, which suggests that investors are risk neutral, assumes the expected return on all investment assets is the risk-free rate. However, some economics and neuroscience literatures show that discount rate could be affected by subjective opinion, psychology and sentiment factors. Thaler (1981) estimates individual discount rates from survey evidence and find that discount rate are found to be much smaller for losses than for gains. Warner and Pleeter (2001) estimate of discount rates range from 0 to over 30 % and which are influenced by education, age, race, sex, number of dependents, ability test score, the size of payment and other personal characteristics. Kable and Glimcher (2007) study the subject's behavioral discount rate by functional magnetic resonance, show that the subjective value of potential rewards is explicitly represented in the human brain. Lawrence et al. (2007) find that investor sentiment influences the expected discount rate. For an investor with high sentiment for a future performance, the expected discount rate will be low. Yang and Zhou (2014) present the concept of sentiment discount rate, and find that the higher the investor sentiment is, the lower the sentiment discount rate is. The large numbers of literatures show that investor's individual discount rate is also affected by sentiment factors.

Compared with the other psychology bias factors, sentiment factors which are quantitatively measured can be a better research angle. Some empirical results show that sentiment factors from option market and stock market both have important and systematic effects on option pricing. Stock sentiment is the aggregate error in investor beliefs to future stock price distribution. Han (2008) finds that the price of option can be affected by the stock sentiment. A variety of proxies for stock sentiment are found to be significantly related to index option return. Three sentiment proxies are used in Han (2008)'s empirical tests: (1) Investors Intelligence's weekly surveys, (2) trading activity in the S&P 500 futures, (3) Sharpe's (2002) valuation errors of the S&P 500 index. Some researchers found that the price of option can be affected by the option sentiment. Option sentiment is the aggregate error in investor beliefs to future option price distribution. Schmitz et al. (2007) derive a sentiment measure from warrants and find that sentiment has a positive impact on future returns. Sheu and Wei (2011) use future volatility, alternative volatility measures, ARMS index, market turnover, options volatility index, put-call trading volume and open interest ratios as option sentiment proxies to study how such option sentiment can have an impact on option price and further influence variations in returns. Nevertheless, the related literatures only focus on empirical research that sentiment factors affect option prices, they have a gap on the theoretical model for option pricing with stock sentiment and option sentiment.

This paper attempts to extend the analytical tractability of Black–Scholes analysis for the classical geometric Brownian motion to alternative models with stock sentiment and option sentiment. For more hedging and arbitrage demand, institutional investors may wish to hold a higher portion of option contracts. Small investors' sentiments are not included in Han (2008) study as they are not important participants in the index options market. However, small investors are important participants in the stock market. Stock investors are different from option investors of which there is relatively a higher portion of institutional investors. Thus, these two sets of investors may be subject to the effect of sentiment differently. So we incorporate stock sentiment and option sentiment into option pricing model.

With the differences from the previous literatures on option pricing model, our model can elaborate response mechanism that stock sentiment and option sentiment affect option mispricing. To the extent such mispricing exists, overpricing of call option should be more prevalent when stock sentiment and option sentiment are high at the same time, and underpricing of call option should be more prevalent when stock sentiment and option sentiment are low at the same time. This is related to the finding of Yang and Gao (2014) that stock index futures sentiment and stock sentiment are both significant to stock index futures returns.

To capture sentiment factors effect in option market, the model developed in this paper has many interesting and important implications for the behavior of option prices: (1) the call option price is amplified by bullish stock sentiment, and is reduced by bearish stock sentiment, and the put option price is in the opposite situation; (2) our models show that call option is more sensitive to bullish stock sentiment; put option is more sensitive to bearish stock sentiment; (3) the price of call option increases substantially with respect to the stock sentiment and the option sentiment, the price of put option decreases substantially with respect to the stock sentiment, increases substantially with respect to the option sentiment.

Our model has some important contributions different from other option pricing models: (1) we present an option pricing model with stock sentiment and option sentiment; (2) our model can explain how stock sentiment and option sentiment lead to the option price deviating from the rational price; (3) the model can interpret option volatility smile.

The rest of this paper is organized as follows. Section 2 will establish the sentiment European option pricing model. Section 3 demonstrates the relative simulation analysis of the option price, and interprets option bubbles and option volatility smile. In Sect. 4, concluding remarks are made.

2 Option pricing model with investor sentiment

2.1 The economy

Our goal is to set up a continuous option pricing model with stock sentiment and option sentimentfor. In this paper, we adopt the framework of option price model with lognormal distributed underlying asset (Black and Scholes 1973; Ricardo 2002).

Consider a continuous-time economy with an infinite horizon. There are two tradable assets in the economy: an option and a stock. The stock is an underlying asset. There is one class of investors whose decisions are influenced by the option sentiment and stock sentiment. Rational investor is that his option sentiment and stock sentiment are both zero.

Both options are written on the same underlying asset with current price U_t. The expected price of the underlying asset at T is U_T^r, if investor is rational. C_t^r and P_t^r are the rational price of a European call and put option at t time, respectively, both with exercise price X and time to maturity T. C_T^r and P_T^r are the price of a European call and put option at T, if investor is rational. The random terminal value of the rational call option is $C_T^r = \max\left[0, U_T^r - X\right]$, the random terminal value of the rational put option is $P_T^r = \max\left[X - U_T^r, 0\right]$.

2.2 Stock sentiment

If investors are fully rational, the classical hypothesis by Black and Scholes (1973) is that the natural logarithm of stock price obeys normal distribution.

$$\ln U_T^r \sim \varphi\left[\ln U_t + \left(\mu_u - \frac{\sigma_u^2}{2}\right)(T-t),\ \sigma_u^2(T-t)\right].$$

The mean (m^r) and standard deviation (v^r) of $\ln U_T^r$ are:

$$\begin{aligned} m^r &= E(\ln U_T^r) = \ln U_t + \left(\mu_u - \frac{\sigma_u^2}{2}\right)(T-t), \\ v^r &= \sqrt{Var(\mathrm{lnU}_T^r)} = \sigma_u\sqrt{T-t}. \end{aligned}$$

However, Lakonishok et al. (2007) show that a large fraction of individuals' option activity is motivated by speculation on the direction of future stock price movements. Moreover, a large number of literatures show that investors are not fully rational in stock market, and stock price is governed not just by fundamentals, but by stock sentiment. Kumar and Lee (2006) showed that stock sentiment has a significant incremental ability to explain return co-movements. Cochrane (2001) also implies that stock sentiment is an important conditioning variable for determining stock returns. Yang and Zhang (2013) and Yang and Li (2014) show that the higher the stock sentiment is, the higher the stock returns are.

According to the empirical studies and theoretical results, higher stock sentiment causes higher stock sentiment bias. So we assume that the stock sentiment $f(S_u) = a_u S_u$, where S_u is the stock sentiment, $\alpha_u$is the constant coefficient of stock sentiment and $\alpha_u > 0$.

The stock sentiment function $f(\cdot)$ is excess growth about the mean of stock price, and satisfies with the properties as follows: (1) $f(\cdot) > 0$ when stock sentiment is optimistic ($S_u > 0$); (2) $f(\cdot) < 0$ when stock sentiment is pessimistic ($S_u < 0$); (3) $f(\cdot) = 0$ when stock sentiment is rational ($S_u = 0$).

The expected price of the underlying asset at time T is U_T, if investor is sentimental. We assume that stock price obeys the following normal distribution:

$$\ln U_T \sim \varphi\left[\ln U_t + f(S_u) + \left(\mu_u - \frac{\sigma_u^2}{2}\right)(T-t),\ \sigma_u^2(T-t)\right].$$

The mean (m) and standard deviation (v) of $\ln U_T$ are:

$$m = E(\ln U_T) = \ln U_t + f(S_u) + \left(\mu_u - \frac{\sigma_u^2}{2}\right)(T-t) = m^r + f(S_u),$$
$$v = \sqrt{Var(\mathrm{lnU}_T)} = \sigma_u\sqrt{T-t}.$$

2.3 Option sentiment

In Black and Scholes (1973) and Ricardo (2002)'s models, the price of call option at t is the present value of its expected terminal value, discounted at the risk-free rate:

$$C_t^r = e^{-r_f(T-t)}E(C_T^r).$$

According to put–call parity relationship:

$$P_t^r = e^{-r_f(T-t)}E(P_T^r).$$

The price of option is a function of the underlying stock's price and time. By Ito's lemma, it follows that a portfolio of the stock and the option can be constructed so that the Wiener process is eliminated. The assumptions imply that the portfolio must instantaneously earn the same rate of return. Where investors are risk neutral, the expected return on all investmentassets is the risk-free rate of interest r_f.

However, some literatures show that discount rate could be affected by subjective opinion in Warner and Pleeter (2001) and Kable and Glimcher (2007). Lawrence et al. (2007) and Yang and Zhou (2014) find that the higher the investor sentiment is, the lower the expected discount rate is.

According to the Lawrence et al. (2007) and Yang and Zhou (2014), we assume that the sentiment discount rate for call option is $\mu_c^* = \mu_c h(S_c)$, $h(S_c) = e^{\alpha_c S_c}$. μ_c is expected return rate of call option. S_c is the call option sentiment. a_c is the constant coefficient of call option sentiment. On the other hand, the sentiment discount rate for put option is $\mu_p^* = \mu_p h(s_p)$, $h(S_p) = e^{\alpha_p S_p}$. μ_p is expected return rate of put option. S_p is the put option sentiment. a_p is the constant coefficient of put option sentiment. We assumethe higher the option sentiment is, the lower the sentiment discount rate is, so $\alpha_c < 0$, $\alpha_p < 0$. The option sentiment function $h(\cdot)$ is sentiment discount rate of option, andsatisfies properties as follows: (1) $h(\cdot) < 1$ when option sentiment is optimistic ($S_c > 0$ or $S_p > 0$); (2) $h(\cdot) > 1$ when option sentiment is pessimistic($S_c < 0$ or $S_p < 0$); (3) $h(\cdot) = 1$ when option sentiment is rational ($S_c = 0$ or $S_p = 0$).

C_t and P_t are the price of a European call and put option at t, if investor is sentimental. C_T and P_T are the price of a call and put option at T, if investor is sentimental. The random terminal value of the call option is $C_T = \max[0, U_T - X]$, the random terminal value of the put option is $P_T = \max[X - U_T, 0]$, if investor is sentimental. We use the sentiment discount rate of option instead of the risk-free rate. The prices of call and put option at t time are the present value of its expected terminal value discounted at the sentimentdiscount rate:

$$
\begin{aligned}
C_t &= e^{-\mu_c^*(T-t)} E(C_T), \\
P_t &= e^{-\mu_p^*(T-t)} E(P_T).
\end{aligned}
$$

2.4 Put–call parity relationship with sentiment

The random price of the underlying asset at T is U_T, which is affected by stock sentiment, with distribution function $F\left(u'\right)$ and $u' \in [0, \infty)$.The random terminal value of the call and put option are C_T, P_T. The expected terminal prices are given by the expressions:

$$E(C_T) = \int_X^\infty (u' - X) dF(u'), \tag{2.1}$$

$$E(P_T) = \int_0^X (X - u') dF(u'). \tag{2.2}$$

Direct integration of Eqs. (2.1) and (2.2) yields

$$E(C_T) = \int_X^\infty u' dF(u') - X[1 - F(X)], \tag{2.3}$$

$$E(P_T) = XF(X) - \int_0^X u' dF(u'). \tag{2.4}$$

To begin, the integral in Eqs. (2.3) and (2.4) may be split as shown below,

$$E(C_T) = \int_0^{\infty} u' dF(u') - \int_0^{X} u' dF(u') - X[1 - F(X)]. \tag{2.5}$$

By the definition, the first integral in Eq. (2.5) is the expected terminal price of the underlying asset $E(U_T)$. Next, evaluate the second integral by parts and simplify:

$$\int_0^{X} u' dF(u') = XF(X) - \int_0^{X} F(u')du', \tag{2.6}$$

Take Eq. (2.6) into Eq. (2.5), we obtain the general expression for the expected terminal price of any European call option:

$$E(C_T) = E(U_T) + \int_0^{X} F(u')du' - X. \tag{2.7}$$

$$\begin{aligned} E(P_T) &= XF(X) - \int_0^{X} u' dF(u') \\ &= XF(X) - \left[XF(X) - \int_0^{X} F(u')du' \right] \\ &= \int_0^{X} F(u')du'. \end{aligned} \tag{2.8}$$

Notice that the integral in Eq. (2.7) is the expected terminal price of a put option $E(P_T)$. We take Eq. (2.8) into Eq. (2.7), we can get put–call parity relationship at T,

$$E(C_T) = E(U_T) + E(P_T) - X. \tag{2.9}$$

2.5 Option pricing formula with sentiment

According to the assumption, $E(U_T) = U_t e^{[\mu_u (T-t) + f(S_u)]} = U_t e^{\mu_u^* (T-t)}$, μ_u^* is the sentiment stock drift rate, $\mu_u^* = \mu_u + \frac{f(S_u)}{T-t}$. μ_u^* obeys the following properties: (1) $f(S_u) > 0$, $\mu_u^* > \mu_u$, if $S_u > 0$; (2) $f(S_u) < 0$, $\mu_u^* < \mu_u$, if $S_u < 0$; (3) $f(S_u) = 0$, $\mu_u^* = \mu_u$, if $S_u = 0$.

Therefore, under the assumption, the expected call or put option prices at T are given by the expressions: (For the derivation see Appendix).

$$E(C_T) = U_t e^{\mu_u^* (T-t)} N(d_1) - XN(d_2), \tag{2.10}$$

$$E(P_T) = XN(-d_2) - U_t e^{\mu_u^* (T-t)} N(d_1), \tag{2.11}$$

$$d_1 = \frac{\ln(U_t/X) + \left(\mu_u^* + \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u \sqrt{T-t}},$$

$$d_2 = \frac{\ln(U_t/X) + \left(\mu_u^* - \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u \sqrt{T-t}} = d_1 - \sigma_u \sqrt{T-t}.$$

Proposition *Consider the value of continuously monitored call and put options on a given asset U_t with strike X and maturity T. In the economy where investors are sentimental, the call and put options present value formulas are given by:*

$$C_t = U_t e^{(\mu_u^* - \mu_c^*)(T-t)} N(d_1) - X e^{-\mu_c^*(T-t)} N(d_2), \tag{2.12}$$

$$P_t = X e^{-\mu_p^*(T-t)} N(-d_2) - U_t e^{(\mu_u^* - \mu_p^*)(T-t)} N(-d_1). \tag{2.13}$$

In Eqs. (2.12) *and* (2.13)*, it shows that stock sentiment and option sentiment have a systematicimpact on the call and put option price.*

3 Simulation analysis of the model

3.1 The price of option and sentiment

Some model parameters are chosen as follows: $\mu_u = 5\,\%$, $\mu_c = 5\,\%$, $\mu_p = 5\,\%$, $\sigma_u = 17\,\%$, $T = 1$, $X = 10$, $U_t = 10$, $S_u \in [-5, 5]$, $S_c \in [-5, 5]$, $S_p \in [-5, 5]$, $h(S_c) = e^{-0.1S_c}$, $h(S_p) = e^{-0.1S_p}$, $f(S_u) = 0.1S_u$. We use principal component analysis to form a composite stock sentiment index and option sentiment index. The stock sentiment index is based on the common variation in six underlying proxies by Baker and Wurgler (2006). There are seven option sentiment proxies by Sheu and Wei (2011).We compute stock sentiment and option sentiment by weekly time-series data. In order to facilitate the calculation and analysis, stock sentiment and option sentiment are measured prior to the last trading day of each week. Next, we simulate the relationship between the price of option and two sentiments.

Figure 1 plots the price of call option against stock sentiment. It shows that the price of call option increases substantially with respect to stock sentiment. With stock sentiment becomes more bullish, the call option price curve becomes steeper. For example, in the case of $S_u = -2$, the price of call option is about 0.4. In the case of $S_u = 2$, the price of call option is about is about 3.5. In the case of $S_u = 0$, the price of call option is about 1.7, this is BS model's call option price. The BS model'scall option price is our model special case when stock sentiment is zero.

Figure 2 plots the price of put option against stock sentiment. It shows that the price of put option decreases substantially with respect to stock sentiment. With stock sentiment becomes more bullish, the put option price curve becomes gentler. For example, in the case of $S_u = -2$, the price of call option is about 1.2. In the case of $S_u = 2$, the price of call option is about 0.1. In the case of $S_u = 0$, the price of call option is about 0.25, this is also BS model's put option's price. The BS model's put option price is our model special case when stock sentiment is zero. Form Figs. 1 and 2, it shows that call option is more sensitive to bullish stock sentiment, put option is more sensitive to bearish stock sentiment.

Figure 3 plots the price of call option against stock sentiment and the date of expiry. It shows that the price of call option increases substantially with respect to stock sentiment. Moreover, the price of call option also increases substantially with respect to the time of expiry. For example, the call option price changes from 0 to 8,

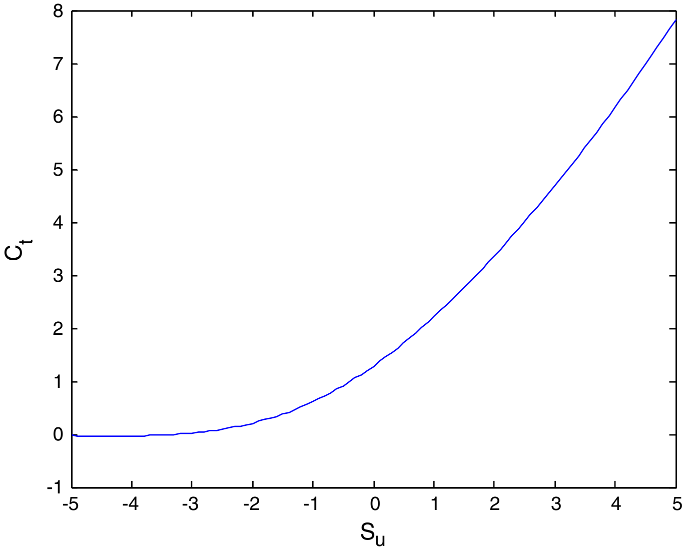

Fig. 1 The price of call option and stock sentiment

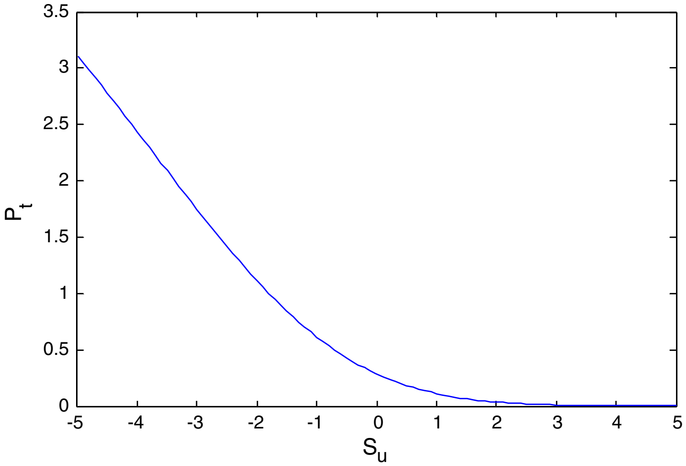

Fig. 2 The price of put option and stock sentiment

when the stock sentiment changes from -5 to 5 and the time of expiry changes from 0 to 1.

Figure 4 plots the price of put option against stock sentiment and the date of expiry. It shows that the price of put option decreases substantially with respect to stock sentiment. Moreover, the price of put option also decreases substantially with respect to the date of expiry. For example, the put option price changes from 0 to 4.5, when the stock sentiment changes from -5 to 5 and the time of expiry changes from 0 to 1.

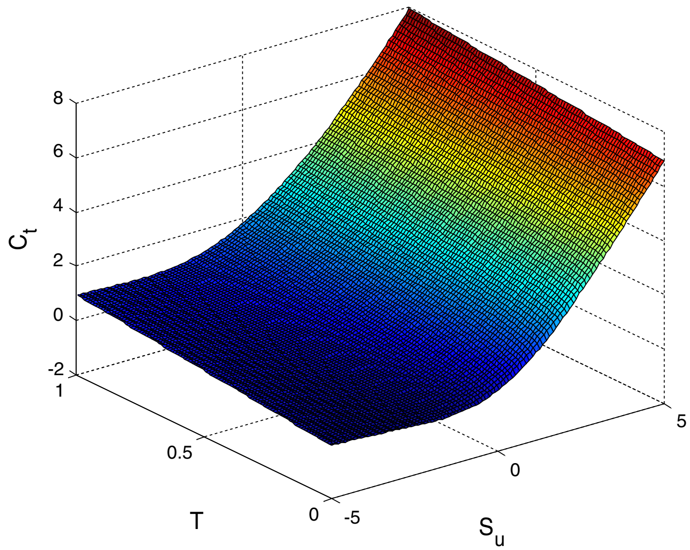

Fig. 3 The price of call option with the date of expiry

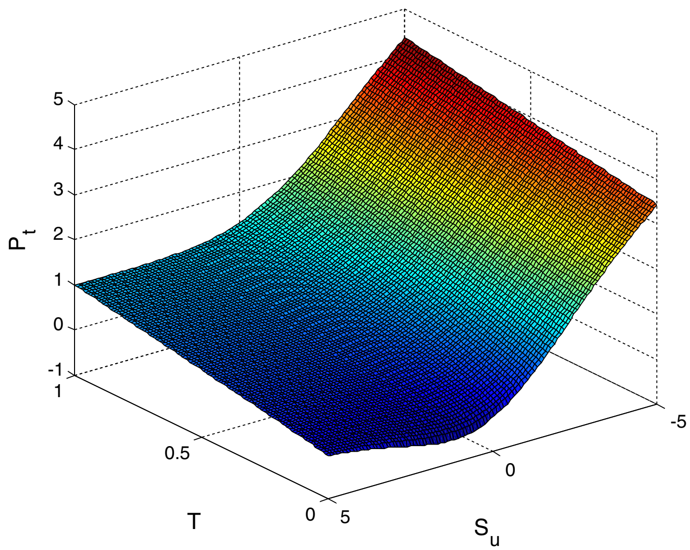

Fig. 4 The price of put option with the date of expiry

Figure 5 plots the price of call option against stock sentiment and option sentiment. It shows that the price of call option increases substantially with respect to stock sentiment. Moreover, the price of call option also increases substantially with call option sentiment.For example, the call option price changes from 0 to 13, when the stock sentiment changes from −5 to 5 and the call option sentiment changes from −5 to 5.

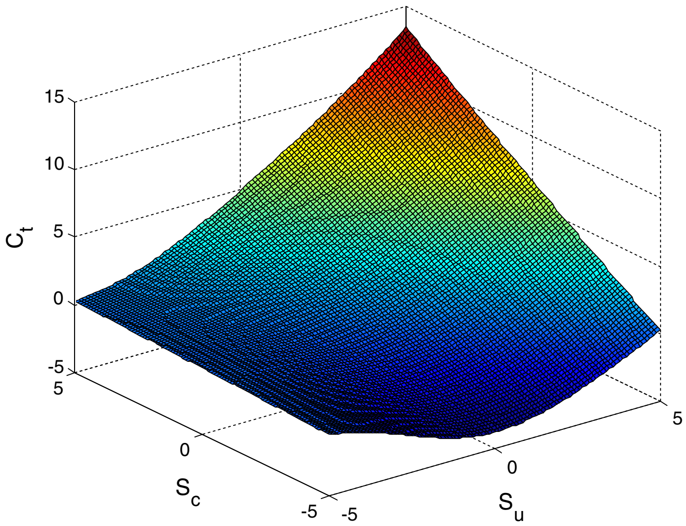

Fig. 5 The price of call option with two sentiments

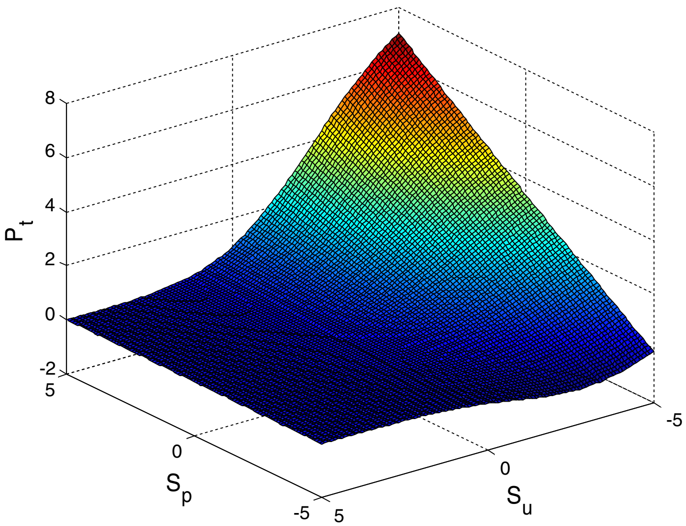

Fig. 6 The price of put option with two sentiments

Figure 6 plots the price of put option against stock and option sentiment. It shows that the price of put option decreases substantially with respect to stock sentiment. However, the price of put option increases substantially with respect to put option sentiment. For example, the call option price changes from 0 to 7, when the stock sentiment changes from 5 to -5 and the put option sentiment changes from -5 to 5.

Table 1 The option impliedvolatility and stock sentiment

μ (%)	T	C	X	$\sigma\,\lvert S_u = 5$ (%)	$\sigma\,\lvert S_u = 2$ (%)	$\sigma\,\lvert S_u = 0$ (%)	$\sigma\,\lvert S_u = -2$ (%)	$\sigma\,\lvert S_u = -5$ (%)
5	1	1	8	0.003	0.003	0.003	0.003	0.003
5	1	1	9	0.003	0.003	0.003	0.003	21.756
5	1	1	10	0.003	0.003	5.984	18.796	33.731
5	1	1	11	0.003	16.354	23.532	29.947	42.331
5	1	1	12	14.291	26.834	32.425	37.961	49.277
5	0.25	1	8	0.003	0.003	0.003	0.003	0.003
5	0.25	1	9	0.003	0.003	0.003	0.003	53.012
5	0.25	1	10	0.003	0.003	31.540	47.360	74.545
5	0.25	1	11	0.003	42.734	55.222	67.111	90.817
5	0.25	1	12	38.889	60.995	71.555	82.169	104.172
5	1	2	8	0.003	0.003	0.003	0.003	35.788
5	1	2	9	0.003	0.003	15.286	30.942	51.144
5	1	2	10	0.003	27.005	36.636	45.236	61.832
5	1	2	11	23.734	40.421	47.842	55.191	70.273
5	1	2	12	36.417	49.747	56.357	63.101	77.298
4	1	1	8	0.003	0.003	0.003	0.003	0.003
4	1	1	9	0.003	0.003	0.003	0.003	23.105
4	1	1	10	0.003	0.003	10.178	20.193	34.702
4	1	1	11	0.003	17.807	24.667	30.942	43.167
4	1	1	12	15.811	27.841	33.340	38.809	50.034

3.2 Sentiment and options volatility smile

A large number of literatures study the slope of the index option. Jackwerth and Rubinstein (1996) find a pronounced "options volatility smile" effect in S&P 500. Bakshi et al. (2003) show risk-neutral skewness is closely related to the option volatility smile in individual equity options. Bollen and Whaley (2004) find net buying pressure causes the slope of the index option volatility smile changing dramatically from month to month. Han (2008) examines whether investor sentiment about the stock market affects option volatility smile ofthe S&P 500 options. The empirical result shows the index option volatility smile becomes steeper (flatter) when investor sentiment is more bearish (bullish). In this section, we test the slope of the option volatilitysmile with stock sentiment in theoretical model. We find more pessimistic stock sentiment is expected to be associated with a flatter option volatility smile. It has the same conclusion with Han (2008).

To illustrate that the model can produceoption volatility smile, we simulate the change of impliedvolatility with the change of stock sentiment and other control variables. Table 1 shows that the stock sentiment is significantly and negatively related to the slope of the option volatility smile. More optimisticstock sentiment is associated with a flatter option volatility smile, and more pessimistic stock sentiment is associated with a steeper option volatility smile. Moreover, we test the whether the conclusion is robustwhen other control variables (the drift rate, the date of expiry and call option price) are different. We find the conclusion is robust.

Why stock sentiment affects the slope of the option volatilitysmile? On the one hand, more pessimistic sentiment about the underlying stockmarket would increase hedging demand and pay offwhen the stock price is low. On the other hand, more pessimistic stock sentiment would increase the prices of out-of-money put optionspro-portionally more than the prices of at-the-money options (Han 2008). In addition, more pessimistic stock sentimentcause high market volatility, there are more differences in investor beliefs which could have disproportionate effects on option prices.Thus, more pessimistic is expected to be associated with a steeper option volatilitysmile.

3.3 Sentiment and option bubbles

Some empirical literatures show that the option prices are usually mispriced or not efficiently priced (Jackwerth 2000; Ait-Sahalia et al. 2001; Constantinides et al. 2009). In this section, we analyze how stock sentiment and option sentiment could cause mispricing and option bubbles. To be more specific, we first define the option bubbles $C_{bubbles} = C_t - C_t^r$, which we take to be C_t, the equilibrium price,

$$
\begin{aligned}
C_t &= U_t e^{(\mu_u^* - \mu_c^*)(T-t)} N(d_1) - X e^{-\mu_c^*(T-t)} N(d_2),\\
d_1^* &= \frac{\ln(U_t/X) + \left(\mu_u^* + \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u\sqrt{T-t}},\\
d_2^* &= \frac{\ln(U_t/X) + \left(\mu_u^* - \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u\sqrt{T-t}} = d_1^* - \sigma_u\sqrt{T-t}.
\end{aligned}
$$

minus C_t^r, the price of the asset with no aggregate sentiment ($S_u = 0$, $S_c = 0$).

$$
\begin{aligned}
C_t^r &= U_t e^{(\mu_u - \mu_c)(T-t)} N(d_1) - X e^{-\mu_c(T-t)} N(d_2),\\
d_1 &= \frac{\ln(U_t/X) + \left(\mu_u + \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u\sqrt{T-t}},\\
d_2 &= \frac{\ln(U_t/X) + \left(\mu_u - \frac{\sigma_u^2}{2}\right)(T-t)}{\sigma_u\sqrt{T-t}} = d_1 - \sigma_u\sqrt{T-t}.
\end{aligned}
$$

We test how stock sentiment and option sentiment cause mispricing. Wefind the model could offer a partial explanation for the financial anomaly of option bubbles. Figure 7 shows that the mispricing increases substantially with respect to the stock sentiment and option sentiment. For example, the mispricing increases from −10 to 10 when the stock sentiment increases from −5 to 5 and the call option sentiment increases from −5 to 5. When investors become more optimism (pessimistic) about the underlying asset fundamental and the future price of option, the investor optimism

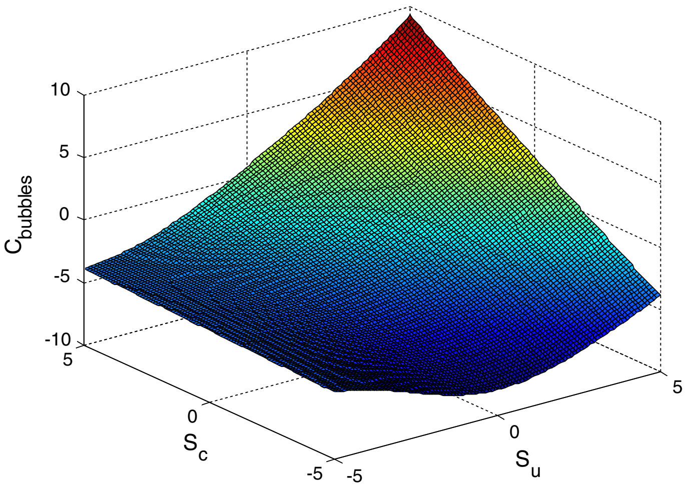

Fig. 7 The call option bubbles with two kinds of sentiment

(pessimistic) expectations will drive the option price up (down). According to the sentiment factors, the financial anomaly of option price bubbles (Scheinkman and Wei 2003) can be interpretated by our model.

4 Conclusions

The option mispricing and option volatility smile cannot be well explained by the BS option model (1973) and Ricardo's option model (2002). Their framework needs to be extended to imperfect rational market. Some empirical literatures show that the option pricing can be affected by stock sentiment and option sentiment. Nevertheless, sentiment-based option pricing model is still in the exploratory stage. So we set up sentiment-based option pricing model to fill the gap.

Our model shows that the stock sentiment and the option sentiment have a significant impact on the option price. First, the call option price is amplified by bullish stock sentiment, and is reduced by stock bearish sentiment, and the put option price is in the opposite situation. Second, our models show that call option is more sensitive to bullish stocksentiment; put option is more sensitive to bearishstock sentiment. Third, the price of call option increases substantially with respect to the stock sentiment and the option sentiment. The price of put option decreases substantially with respect to the stock sentiment, increases substantially with respect to the option sentiment. Fourth, our models also reveal that the option volatility smile is steeper (flatter) when the stock market sentiment becomes more bullish (bearish). Finally, we test how stock sentiment and option sentiment amplify mispricing and cause option bubbles.

Our model could have many practical applications. This model can help investors understand option market is more irrational and remind the investors of the sentiment factor when they trade the options. For financial derivative product designers, the model could give a reasonable price to reduce market risk. Our findings could raise some interesting issues for future research; we could analyze the heterogeneous of sentiment factors in option market and understand how heterogeneous sentiment affects the valuation of options.

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Appendix

Proof of Eqs. (2.10) and (2.11)

The probability density function of U_T is $g(U_T)$, X is strike price

$$E[\max(U_T - X, 0)] = \int_X^{\infty} (U_T - X) g(U_T) dU_T. \tag{5.1}$$

If $\ln U_T$ follows Gaussian distributions, according to the properties of the lognormal distribution, mean (m) and standard deviation (v) are

$$\begin{aligned} m &= E(\ln U_T) = \ln U_t + \left(\mu_u^* - \frac{\sigma_u^2}{2}\right)(T-t), \\ v &= \sqrt{Var(\mathrm{lnU}_T)} = \sigma_u (T-t). \end{aligned}$$

For standardization, we define new variables:

$$W = \frac{\ln U_T - m}{v}. \tag{5.2}$$

The new variables W subject to normal distribution, its mean value is 0, the standard deviation is 1, and the probability density function $h\,(W)$ is

$$h(W) = \frac{1}{\sqrt{2\pi}} e^{-W^2/2}. \tag{5.3}$$

By (5.3) and (5.1) we get

$$\begin{aligned} E[\max(U_T - X, 0)] &= \int_{(\ln X - m)/v}^{\infty} (e^{vW+m} - X) h(W) dW \\ &= \int_{(\ln X - m)/v}^{\infty} e^{vW+m} h(W) dW - X \int_{(\ln X - m)/v}^{\infty} h(W) dW. \end{aligned} \tag{5.4}$$

where,

$$e^{\mathrm{v}W+m} h(W) = \frac{1}{\sqrt{2\pi}} e^{(-W^2+2\mathrm{v}W+2m)/2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{[-(W-\mathrm{v})^2+2m+\mathrm{v}^2]/2} = \frac{e^{m+\mathrm{v}^2/2}}{\sqrt{2\pi}} e^{[-(W-\mathrm{v})^2]/2}$$
$$= e^{m+\mathrm{v}^2/2} h(W-\mathrm{v}). \tag{5.5}$$
$$E[\max(U_T - X, 0)] = e^{m+v^2/2} \int_{(\ln X-m)/v}^{\infty} h(W-v)dW - X \int_{(\ln X-m)/v}^{\infty} h(W)dW.$$

We use the $N(x)$ represents the cumulative probability distribution function of the standard normal distribution function, then the first integral of formula (5.5) is equal to

$$1 - N\left[\frac{(\ln X - m)}{v} - v\right] = N\left[\frac{-\ln X + m}{v} + v\right].$$

Then, the second integral of formula (5.5) is equal to $N\left[\frac{\ln[E(U_T)/X]-v^2/2}{v}\right]$. Formula (5.5) becomes

$$E[\max(U_T - X, 0)] = e^{m+v^2/2} N(d_1) - XN(d_2), \tag{5.6}$$

$$d_1 = \frac{\ln[E(U_T)/X] + v^2/2}{v}, d_2 = \frac{\ln[E(U_T)/X] - v^2/2}{v}.$$

We take $\ln U_T$'s mean(m) and Standard deviation (v) into Formula (5.6), we can obtain Eq. (2.10)

$$\begin{aligned} E(C_T) &= E[\max(U_T - X, 0)] \\ &= e^{m+v^2/2} N(d_1) - XN(d_2) \\ &= U_t e^{\mu_u^*(T-t)} N(d_1) - XN(d_2). \end{aligned} \tag{5.7}$$

By Eq. (9) and (5.7), we get Eq. (2.11)

$$\begin{aligned} E(P_T) &= E(C_T) - E(U_T) + X \\ &= U_t e^{\mu_u^*(T-t)} N(d_1) - XN(d_2) - E(U_T) + X \\ &= U_t e^{\mu_u^*(T-t)} [N(d_1) - 1] + X[1 - N(d_2)] \\ &= XN(-d_2) - U_t e^{\mu_u^*(T-t)} N(-d_1). \end{aligned} \tag{5.8}$$

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