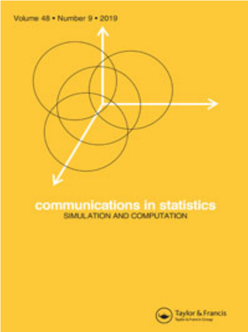

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A comparative study of series arima/mlp hybrid models for stock price forecasting

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ABSTRACT

Series hybrid models are one of the most widely-used hybrid models that in which a time series is assumed to be composed of two linear and nonlinear components. In this paper, the performance of two types of these hybrid models is evaluated for predicting stock prices in order to introduce the more reliable series hybrid model. For this purpose, ARIMA and MLPs are elected for constructing series hybrid models. Empirical results for forecasting three benchmark data sets indicate that despite of more popularity of the conventional ARIMA-ANN model, the ANN-ARIMA hybrid model can overall achieved more accurate results.

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Auto-regressive integrated moving average (ARIMA); Hybrid linear/nonlinear models; Multi-Layer Perceptrons (MLPs); Series and parallel structures; Time series forecasting

MATHEMATICS SUBJECT CLASSIFICATION

1. Introduction

Among many factors that are involved in making decision about selecting a right forecasting model, accuracy is known as a most effective criteria. Thus, improving forecasting accuracy has become vital for decision makers and managers in various fields of science especially time series forecasting. Many researchers believe that combining different models or using hybrid models can be an effective solution to improve forecasting accuracy and to overcome limitations of single models. Theoretical, as well as, empirical evidences in the literature suggest that by combining inhomogeneous models, the hybrid models will have lower generalization variance or error. On the other hand, the main aim of combined models is to reduce the risk of using an inappropriate model by using several models in structure of hybrid models simultaneously. Typically, the theory of combination evolves from this fact that the underlying process of data generation cannot easily be determined and either one individual model cannot identify the true data generation process. In a other word, a single model may not be totally sufficient to identify all the characteristics of the time series.

In recent decades, combination techniques of different forecasting models have attracted the attention of many researchers in many areas especially in financial time series forecasting. These combination models can generally be categorized in to two main classes: series and parallel models. In parallel combination models, hybrid forecasts are calculated based on a combination of forecasting results obtained by single models. While, in series combination models, a time series is considered to be composed of two main part. Consequently, in the

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first stage, the first model is used to analyze one of the components of time series and then by using the obtained values from the first stage, the second part is modeled. Series combination models, especially linear/nonlinear hybrid models, are one of the most common hybrid techniques that used widely used for time series forecasting in various fields.

The literature of series linear/nonlinear combination models for time series forecasting has dramatically expanded since the early work of (Zhang 2003). Chen and Wang (2007) constructed a series combination model incorporating seasonal autoregressive integrated moving average (SARIMA) and support vector machine(SVM) for seasonal time series forecasting. Diaz-Robles et al. (2008) presented a series hybrid model by using autoregressive integrated moving average (ARIMA) with explanatory variables(ARIMAX) and multilayer perceptron neural network (MLPNN) in order to forecast the particulate matter in urban areas. Pham and Yang (2010) proposed a series hybrid model integrating autoregressive moving average (ARMA) with generalized auto-regressive conditional heteroskedasticity (GARCH) models to obtain accurate prediction in estimation and forecasting of machine health condition. Lee and Tong (2011) proposed a series hybrid model based on ARIMA and genetic programming (GP) for financial time series forecasting. Nie et al. (2012) described a series combining methodology using support vector machines (SVMs) and ARIMA models for short-term load forecasting. Monfared and Enke (2014) presented a series approach that integrated artificial neural networks (ANNs) and GARCH models for volatility forecasting. Sun et al. (2015) based on the series combination methodology proposed a hybrid model by using ARIMA and different kinds of GARCH models to forecast solar radiation.

The literature review indicates that ARIMA and ANN models are the most popular and widely-used linear and nonlinear models, which are applied for constructing series linear/nonlinear combination models, respectively. The reason of this popularity is unique advantages of these two types of models in modeling linear and nonlinear structures. The auto-regressive integrated moving average model is one of the most popular linear time series models that have enjoyed useful applications. The popularity of the ARIMA model is due to its statistical properties as well as to the well-known Box-Jenkins (Box and Jenkins 1976) methodology in the model building process. The ARIMA model assumes that there is a linear correlation between the values of a time series and by analyzing historical data, extrapolate the linear relationships in the data, so this statistical model is suitable for linear problems. Due to the ability of ARIMA models in linear modeling, this model is frequently used for building series hybrid models.

Artificial neural networks are also one of the most important and widely used types of nonparametric nonlinear time series models, which have been proposed and examined for time series forecasting (Rumelhart and McClelland 1986). The highlighted feature of neural networks is their nonlinear pattern recognition without any information about the relationships exists in the data. In artificial neural networks, no longer need to specify the form of the particular model. Moreover, the model is adaptively formed based on the features presented from the data. Because of the capacity of these models in nonlinear modeling, they are also frequently used in several studies as a part of series hybrid models. Of course, it must be noted that the real word problems are rarely pure linear or nonlinear, thus the ARIMA and ANN models based on their unique features in linear and nonlinear modeling, are not comprehensive to capture both linear and nonlinear patterns simultaneously. Thus, it seems that combing these models together can be an effective way to forecast real world systems such as financial markets.

In recent years, several series combination models have been developed in the literature, incorporating autoregressive integrated moving average and artificial neural networks and have been applied to time series forecasting with good performance. Generally, by changing the sequence of using ARIMA and ANN models in the series combination methodology, two possible hybrid models e.g. ANN-ARIMA and ARIMA-ANN can be presented. The vast majority of researchers prefer to use the ARIMA-ANN models for time series forecasting. Aladag, Egrioglu, and Kadilar (2009) constructed an ARIMA-ANN model incorporating Elman's recurrent neural networks and ARIMA models. Areekul et al. (2010) used the series combination of ARIMA and multilayer perceptrons for short-term price forecasting. Shafie-khah, Moghaddam, and Sheikh-El-Eslami (2011) proposed an ARIMA-ANN hybrid model based on the ARIMA and the radial basis function neural network (RBFNs) and used it to forecast electricity price. Khashei, Bijari, and Raissi Ardali (2012) developed a series hybrid model for time series forecasting, combining the ARIMA and probabilistic neural networks (PNNs). Wang et al. (2013) used the hybrid ARIMA and multi-layer perceptrons for time series forecasting. Chaâbane (2014) presented a hybrid series FARIMA-MLP model for electricity price prediction. Gairaa et al. (2016) proposed an ARIMA-ANN model to estimate daily global solar radiation. Despite of applying ARIMA-ANN model in numerous studies, ANN-ARIMA model is only used in two papers. Zeng et al. (2008) used ANN-ARIMA model to predict short-term traffic flow. Sallehuddin, Shamsuddin, and Hashim (2008) based on the Grey relational artificial neural network presented an ANN-ARIMA model for time series forecasting.

In this paper, the performance of these two types of series hybrid models for financial time series forecasting are compared together and also with their components. The main aim of this paper is to determine the relative predictive capabilities of the ARIMA-ANN and ANN-ARIMA models. On the other hand, this paper aims to conclude that which sequence of ARIMA and ANN is better for constructing series hybrid models for financial time series forecasting. Three well-known benchmark data sets including the closing of the Shenzhen Integrated Index (SZII), opening of the Dow Jones Industrial Average Index (DJIAI) and the closing of Nikkei 225(N225) index are elected for this purpose. The rest of the paper is organized as follow: In the Section 2, basic concepts and modeling procedures of ARIMA and ANN models for time series forecasting is briefly introduced. In Section 3, the series hybrid methodology and basic concepts of the ARIMA-ANN and the ANN-ARIMA models are descripted. The description of used benchmark data sets and obtained results of both hybrid models are presented in Section 4. In Section 5, the performance of models in forecasting benchmark data sets are compared together. Section 6 contains the concluding remarks.

2. The autoregressive integrated moving average and multilayer perceptrons

Individual approaches for time series forecasting can be categorized as follows: statistical classic and intelligent models. The procedure modelling of statistical models is based on the analyzing past value of components of time series. While in intelligent models regardless of the form of the relationship between input and outputs, the relationships between input and output nodes are calculated by analyzing the features of the data. The main advantage of these types of individual models is nonlinear processing. ARIMA and ANN models are two of the most important and widely used statistical and intelligent models respectively, which are used many times for constructing hybrid models. Therefore, in this section, basic concepts of modeling procedures of the two autoregressive integrated moving average (ARIMA) and multilayer perceptron neural network (MLPNN) models for time series forecasting is briefly introduced.

2.1. Autoregressive integrated moving average (ARIMA) models

ARIMA model is a procedure of forecasting future values of time series that same as statistical models by using historical data generate forecasting value of variables. ARIMA model consists of two main parts, Auto Regressive (AR) and Moving Average (MA) which are combined together and build ARIMA models. The formula of ARIMA model for time series forecasting is shown in Eq. (1).

$$\phi(B)\nabla^{d}(y_t - \mu) = \theta(B)\,a_t \tag{1}$$

Where, y_t is the actual value in time t, a_t is the white noise which assumed to be independent and identically distributed with a mean of zero and constant variance of σ^2. $\phi(B) = 1 - \sum_{i=1}^{p} \varphi_i B^i$, $\theta(B) = 1 - \sum_{j=1}^{q} \theta_j B^j$ are polynomials in B of degree p and q, $\phi_i \quad (i = 1, 2, \ldots, p)$ and $\theta_j \quad (j = 1, 2, \ldots, q)$ are model parameters, $\nabla = (1 - B)$, B is the backward shift operator, p and q are integers and often referred to as orders of the model, and d is an integer and often referred to as order of differencing. The modeling procedure of ARIMA models based on the Box–Jenkins (Zhang, Patuwo, and Hu 1998) methodology always contain three iterative steps, including model identification, parameter estimation and diagnostic checking. These steps are described as follows in detail.

1) Identification: In this step we are searching for the actual values of p (the number of auto regressive), d (the number of differencing) and q (the number of moving average). For this purpose Box and Jenkins (Zhang, Patuwo, and Hu 1998) proposed the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the sample data as the basic tools to identify the order of the autoregressive integrated moving average models.
2) Estimation: After choosing a specify ARIMA (p, d, q), the parameters which were identified in the previous phase, should be estimated by the ordinary least squared (OLS) method.
3) Diagnosis checking: The last step in model building is the diagnostic checking of model adequacy. This is basically done to check whether the selected model is adequate to model and predict time series well or not. Because another model may be exist to present a better modeling of historical data. Therefore, for recognizing final structure by checking several diagnostic statistics and plots of the residuals, the best structure is selected.

If the model is not adequate, a new structure of ARIMA model will be identified, and the three previous steps should be repeated until the best structure is found.

2.2. Multilayer perceptron neural networks (MLPS) for forecasting time series

One of the most frequently used intelligent models that has been successfully applied in many fields especially, time series modelling and forecasting is ANN model. The main reason of this popularity is that ANN models extrapolate the underlying data generation without any assumption of the model form. Besides another highlighted features of neural networks is that they are universal approximators that can approximate a large class of function accurately. There are various ANN model architectures available in the literature. Although neural networks have a similar structure, but based on the how to design, distinction between different types of neural networks have been created. Single hidden layer feed forward network (also known as multilayer perceptrons) is the most widely used neural network model architectures for time series modeling and forecasting. In this paper, this type of neural networks is used for all nonlinear modeling. The MLP network consists of three layers including input, hidden and output layers.

Input layer phase: For the time series problems, a MLP is fitted with past lagged value of actual data $(y_{t-1}, \ldots, y_{t-p})$ as an input vector. Therefore, input layer is composed of p nodes that are connected to the hidden layer.

Hidden layer phase: The hidden layer is an interface between input and output layers. The MLPs model which are designed in this paper have a single hidden layer with q nodes. In this step one of the important tasks is determining the type of activation function (g) which is identifying the relationship between input and output layer. Neural networks support a wide range of activation function such as linear, quadratic, tanh and logistic. The logistic function is often used as the hidden layer transfer function that are shown in Eq. (2).

$$g(x) = \frac{1}{1 + \exp(-x)} \tag{2}$$

Output layer phase: In this step, by selecting an activation transfer function and the appropriate number of nodes, the output of neural network is used to predict the future values of time series. In this paper output layer by designed neural networks contains one node because the one –step-ahead forecasting is considered. Also, the linear function as a non-linear activation function is introduced for the output layer. The formula of relationship between input and output layer is presented in Eq. (3).

$$y_t = w_0 + \sum_{j=1}^{q} w_j . g\left[w_{0,j} + \sum_{i=1}^{p} w_{i,j} . y_{t-i} \right] + \varepsilon_t \tag{3}$$

where, $w_{i,j} (i = 0, 1 \ldots, p, \; j = 1, 2, \ldots, q)$ and $w_j (j = 0, 1, 2 \ldots, q)$ are referred as an connection weights.

It should be noted that deciding the number of neurons in hidden layer *(q)* and the number of lagged observations, (p) and the dimension of the input vector in input layer are vital parts of neural network architectures, but no methodical rule exists in order to selecting theses parameters and the only possible way to choose an optimal number of p and q is trial and error (Khashei and Bijari, 2010).

3. The series combination of ARIMA and MLP models

In series linear/nonlinear combination models, a time series is considered to be composed of a linear autocorrelation structure and a nonlinear component as follows:

$$y = Sum\,(L, N) \tag{4}$$

where, L denotes the linear and N denotes the nonlinear part. These two components have to be estimated from the data. Thus, in the first stage of these models, linear (nonlinear) component is first modeled by ARIMA (MLP) model. Then, the nonlinear (linear) part is modeled by MLP (ARIMA) model using residuals of the first stage. The main idea of series hybrid models comes from this fact that if a time series is modeled by a linear model such as ARIMA, then residuals of the linear model will only contain nonlinear structure. Therefore, the nonlinear part of time series can be modeled by residuals. In the similar fashion, if a time series is modeled by a nonlinear model such as MLP, then residuals of the nonlinear model will only contain linear structure. Therefore, the linear part of time series can be modeled by residuals. In this way, series combination models exploit unique attributes and strengths of the ARIMA as well as MLP model in determining different patterns. Thus, it could be advantageous to model linear and nonlinear patterns separately by using different models and then combine the forecast to improve the overall forecasting performance. In addition, since it is difficult to completely know the characteristics of the data in real problem, this hybrid methodology that model different parts of time series sequentially by using linear and nonlinear modeling capabilities of each individual model, can be a good strategy for improving forecasting accuracy in practical uses. In the next sections, two possible hybrid models based on the sequence of using ARIMA and MLP models in series combination (e.g. ARIMA- MLP and MLP-ARIMA) are presented.

3.1. The ARIMA-MLP model

According to the procedure of the series models, in the ARIMA-MLP model, the ARIMA is used in the first permutation to model the linear component. Let e_t denote the residual of the ARIMA model at time t, then:

$$e_t = y_t - \hat{L}_t \tag{5}$$

where, the $\hat{L}_t$ is the output of ARIMA model at time t. The ARIMA model left the nonlinear patterns in its residual. Thus, in the second stage, by modeling residuals using MLPs, nonlinear relationships can be discovered. With n input nodes, the MLP model for the residuals will be:

$$e_t = f(e_{t-1}, e_{t-2}, \ldots, e_{t-n}) + \varepsilon_t \quad \Rightarrow \quad \hat{N}'_t = \hat{e}_t = f(e_{t-1}, e_{t-2}, \ldots, e_{t-n}) \tag{6}$$

where f is a nonlinear function determined by the MLP, $\hat{N}'_t$ is the forecasting value at time t from the MLP model on the residual data, and ε_t is the random error. The framework of ARIMA-MLP model is displayed in Fig. 1(a). Note that if the model f is an inappropriate one, the error term is not necessarily random. Therefore, the correct identification is critical.

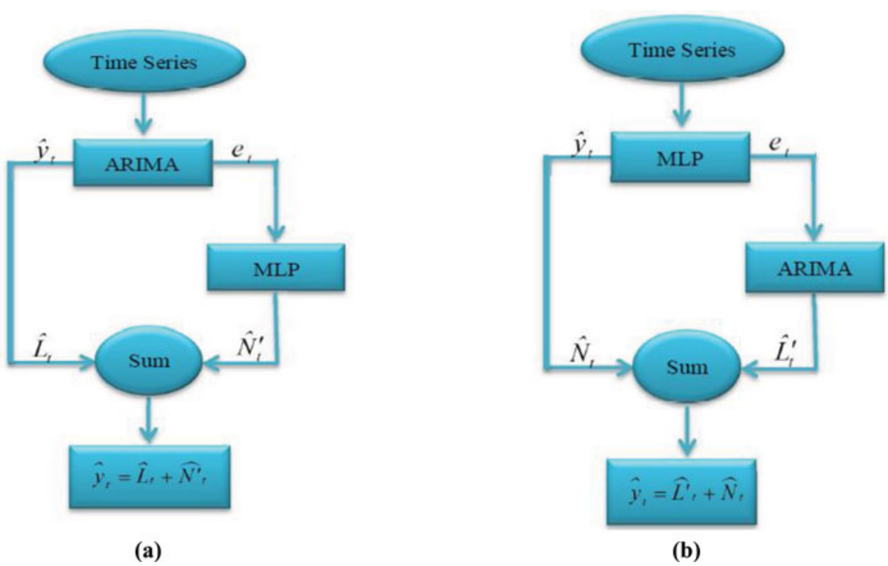

Figure 1. Framework of (a) the ARIMA-MLP model and (b) the MLP-ARIMA model.

In this way, the combined forecast will be as follows:

$$\hat{y}_t = \hat{L}_t + \hat{N}'_t \tag{7}$$

3.2. The MLP-ARIMA model

Similar to the ARIMA-MLP model, the MLP-ARIMA also has two main stages. MLP model is used firstly in order to model the nonlinear part of time series. Let e'_t denote the residual of MLP model at time t, then:

$$e'_t = y_t - \hat{N}_t \tag{8}$$

where, $\hat{N}_t$ is the output of the MLP model at t where the inputs are the original data. According to limitation of neural networks in linear processing, its residuals contain linear patterns. Thus, ARIMA model is used to model remained linear relationships in the residuals of the MLP model in the next step. In this way, the ARIMA model with m lags for the residuals will be:

$$e'_t = f(e'_{t-1}, e'_{t-2}, \ldots, e'_{t-m}) + \varepsilon_t \quad \Rightarrow \quad \hat{L}'_t = \hat{e}'_t = f(e'_{t-1}, e'_{t-2}, \ldots, e'_{t-m}) \tag{9}$$

where, f is a linear function determined by the ARIMA, $\hat{L}'_t$ is the forecasting value for time t from the ARIMA model on the residual data, and ε_t is the random error. The framework of MLP-ARIMA model is displayed in Fig. 1(b). In this way, the combined forecast will be as follows:

$$\hat{y}_t = \hat{L}' + \hat{N}_t \tag{10}$$

4. Appling series linear/nonlinear hybrid models for stock price forecasting

In this section, two abovementioned series linear/nonlinear hybrid models are applied for stock price forecasting. Three benchmark data sets including the closing of the Shenzhen Integrated Index (SZII), opening of the Dow Jones Industrial Average Index (DJIAI) and the closing Nikkei 225 (N225) index are chosen for this purpose. The description of data sets, the procedure of hybrid models and designed models for each case are briefly presented in the next three subsections.

4.1. Dow Jones Industrial Average Index (DJIAI) data set

The Dow Jones Industrial Average Index data set contains stock opening prices from the January 1991 to the December 2010 and totally has 240 monthly values. The plot of the DJIAI data set is show in the Fig. 2. According to the literature and previous works (Wang et al. 2012), the first 180 values (75% of the sample, from January 1991 to December 2005) are used as training sample and the remaining 60 values are applied as test sample.

4.1.1. The ARIMA-MLP model

Stage I: (*Linear modeling*): In the first stage of the ARIMA-MLP model, using *Eviews* software, the best-fitted model is $ARIMA(1, 2, 0)$.

Stage II: (*Nonlinear modeling*): In order to analyze the obtained residuals from the previous stage and based on the concepts of MLP models, in *MATLAB* software, the best fitted model

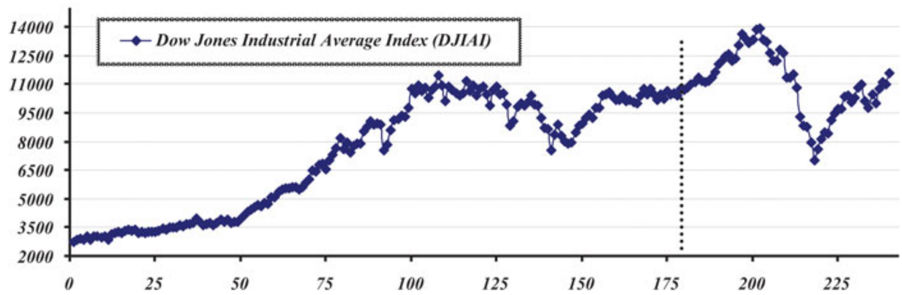

Figure 2. The monthly DJIAI stock opening prices from January 1991 to December 2010.

composed of Five inputs, three hidden and one output neurons (in abbreviated form $N^{(5,3,1)}$), is designed.

Stage III: (*Combination*): In the last stage, obtained results from stage I and II are combined together. The estimated values of the ARIMA-MLP model against actual values for all data and test data are plotted in Fig. 3 and Fig. 4; respectively.

4.1.2. The MLP-ARIMA model

Stage I: (*Nonlinear modeling*): In the first stage of the MLP-ARIMA model, in order to capture nonlinear patterns of time series a multilayer perceptron with three inputs, three hidden and one output neurons (in abbreviated form $N^{(3,3,1)}$), is designed.

Stage II: (*Linear modeling*): In the second stage of the MLP-ARIMA model, obtained residuals from previous stage are treated as the linear model. Thus, considering lags of the MLP residuals as input variables of the ARIMA model, the best fitted model is $ARIMA(3, 0, 3)$.

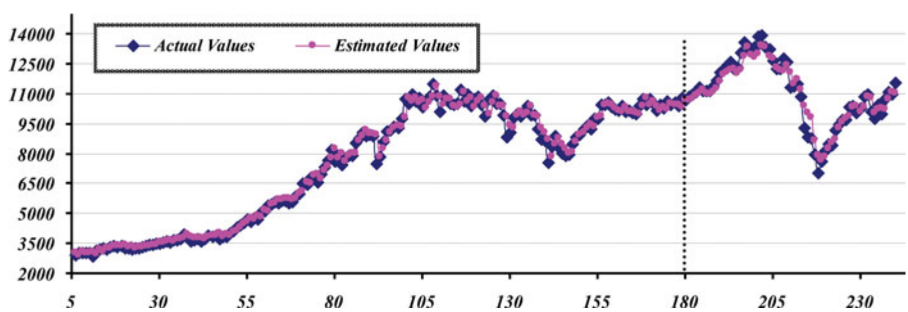

Figure 3. Estimated values of the ARIMA-MLP model for DJIAI (training and test data set).

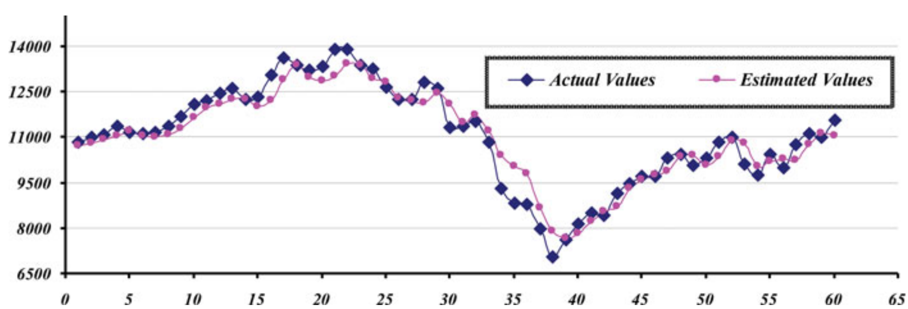

Figure 4. Estimated values of the ARIMA-MLP model for DJIAI (test data set).

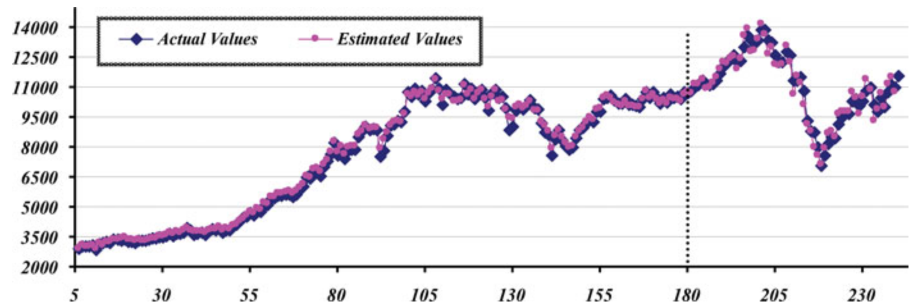

Figure 5. Estimated values of the MLP-ARIMA model for DJIAI (training and test data set).

Stage III: (*Combination*): In the last stage, obtained results from stage I and II are combined together. The estimated values of the MLP-ARIMA model against actual values for all data and test data are plotted in Fig. 5 and Fig. 6; respectively.

4.2. The Shenzhen Integrated Index (SZII) data set

The Shenzhen Integrated Index (SZII) data set covers from January 1993 to December 2010 and totally has 216 monthly observations. The plot of the SZII data set is show in the Fig. 7. According to previous works (Wang et al. 2012), the first 168 observations (77% of the sample, from January 1993 to December 2006) are used as training sample and the remaining 48 observations are applied as test sample.

Similar to the previous section, the best fitted models in the ARIMA-MLP model are $ARIMA(1, 0, 0)$ and a multilayer perceptron with four input, four hidden and one output neurons ($N^{(4,4,1)}$); respectively. And in MLP-ARIMA model are a multilayer perceptron with

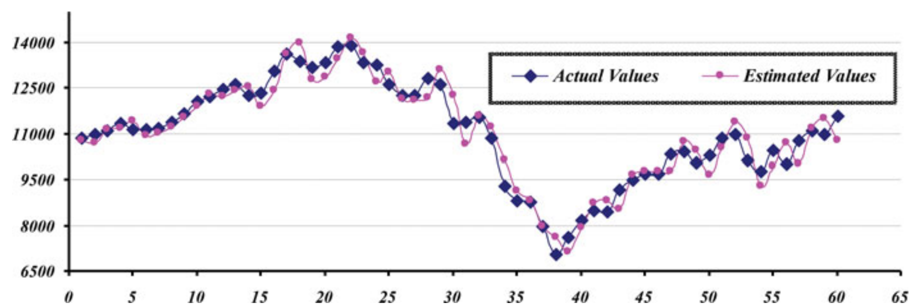

Figure 6. Estimated values of the MLP-ARIMA model for DJIAI (test data set).

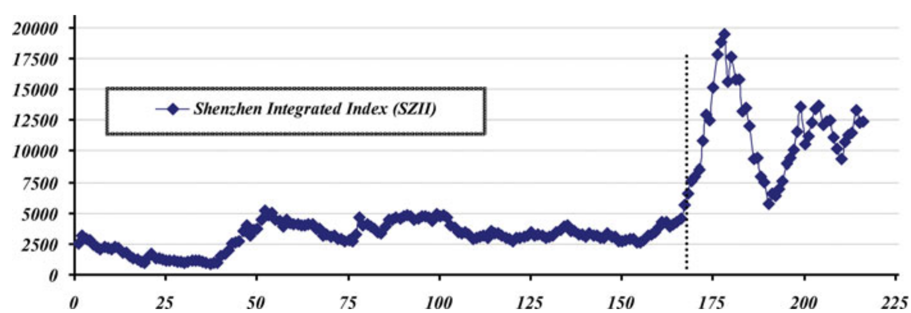

Figure 7. The monthly SZII stock closing prices from January 1993 to December 2010.

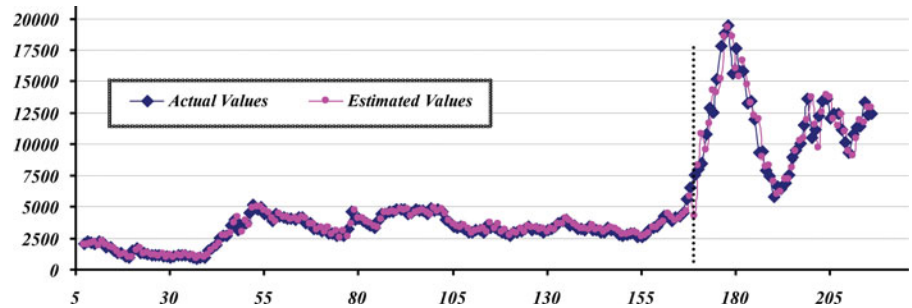

Figure 8. Estimated values of the ARIMA-MLP model for SZII (training and test data set).

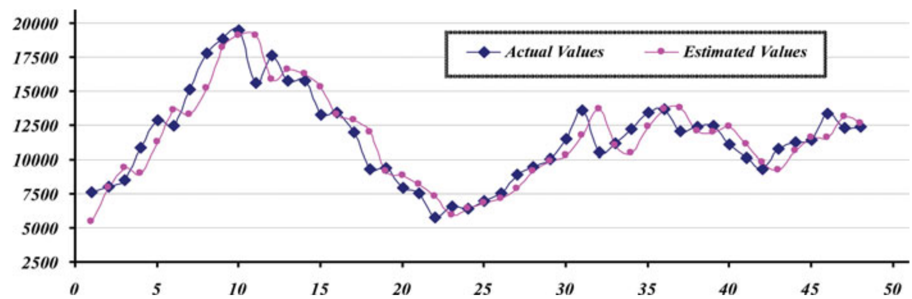

Figure 9. Estimated values of the ARIMA-MLP model for SZII (test data set).

three input, two hidden and one output neurons ($N^{(3,2,1)}$) and $ARIMA(2, 0, 2)$; respectively. The estimated values of the ARIMA-MLP and MLP-ARIMA models against actual values for all data and test data are plotted in Figs. 8 to 11; respectively.

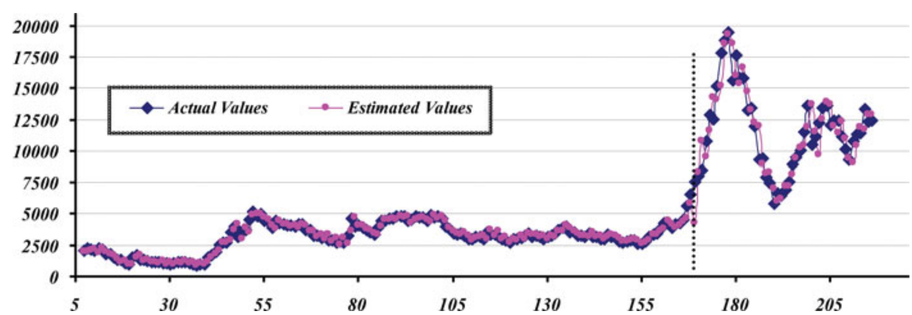

Figure 10. Estimated values of the MLP-ARIMA model for SZII (training and test data set).

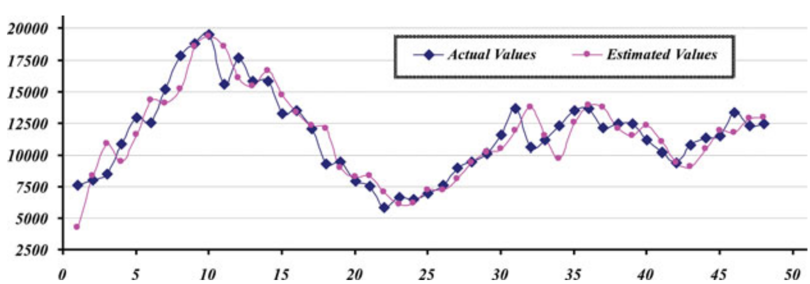

Figure 11. Estimated values of the MLP-ARIMA model for SZII (test data set).

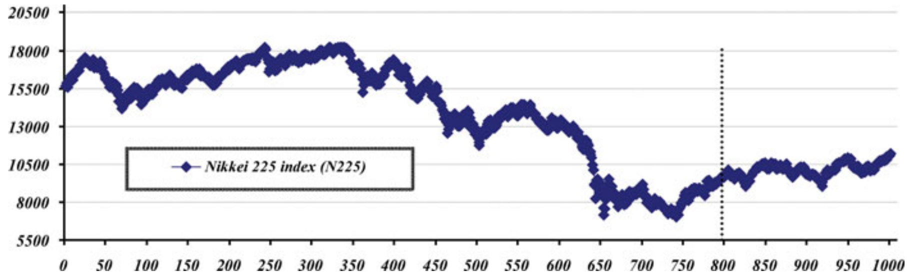

Figure 12. The daily N225 stock closing prices from 2006/3/3 to 2010/4/1.

4.3. The closing Nikkei 225 index (N225) data set

The closing Nikkei 225 index (N225) data set covers from 2006/03/03 to 2010/04/01 and totally has 1001 daily data points. The plot of the Nikkei data set is show in the Fig. 12. Based on the literature the first 800 data points (80% of the total sample points, from 2006/03/03 to 2009/06/05) are used as training sample and the remaining 201 data points are used as testing sample (Kao et al. 2013). In similar fashion, the best designed ARIMA and MLP models in the ARIMA-MLP model are found to be $ARIMA(1, 1, 0)$ and a neural network, which is composed of four inputs, nine hidden and one output neurons ($N^{(4,9,1)}$); respectively. The best fitted nonlinear and linear models in the MLP-ARIMA hybrid model are a neural network, which is composed of four inputs, six hidden and one output neurons ($N^{(4,6,1)}$) and $ARIMA(1, 0, 1)$; respectively. The estimated value of the ARIMA-MLP and MLP-ARIMA models for all data and test data are plotted in Fig. 13 to Fig. 16; respectively.

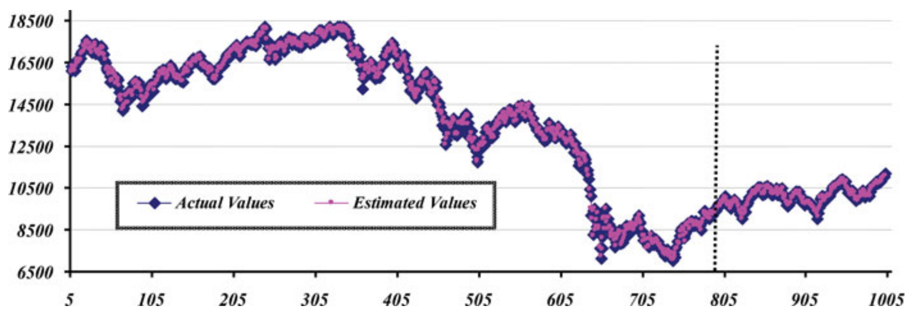

Figure 13. Estimated values of the ARIMA-MLP model for Nikkei (training and test data set).

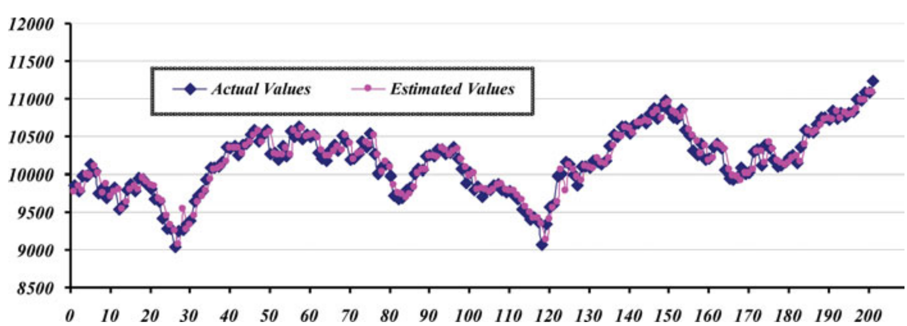

Figure 14. Estimated values of the ARIMA-MLP model for Nikkei (test data set).

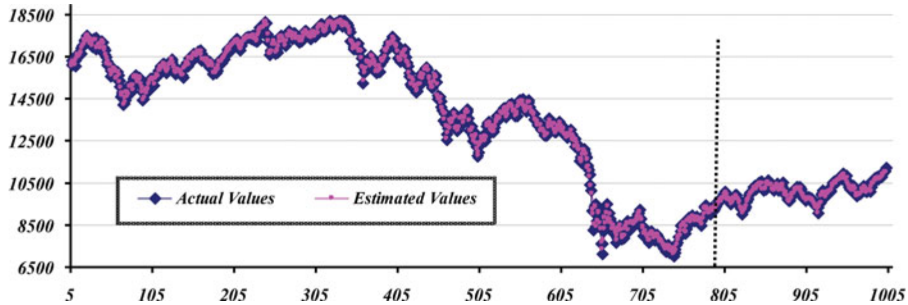

Figure 15. Estimated values of the MLP-ARIMA model for Nikkei (training and test data set).

5. Comparison of forecasting results

In this section, the predictive capabilities of hybrid models are compared together and with either of their components —multilayer perceptrons and autoregressive integrated moving average— in three abovementioned data sets. Four performance indicators including mean absolute error (MAE), mean square error (MSE), mean absolute percentage error (MAPE), and root mean square error (RMSE), which are computed from the following equations, are employed in order to compare forecasting performance of hybrid models and their components.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|e_i| \tag{11}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(e_i)^2 \tag{12}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{|e_i|}{|y_i|} \tag{13}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e_i)^2} \tag{14}$$

5.1. Dow jones forecasting results

Forecasting results of hybrid models and their components for the DJIAI opening index for train and test data sets are summarized in Table 1. Numerical results of DJIAI show that

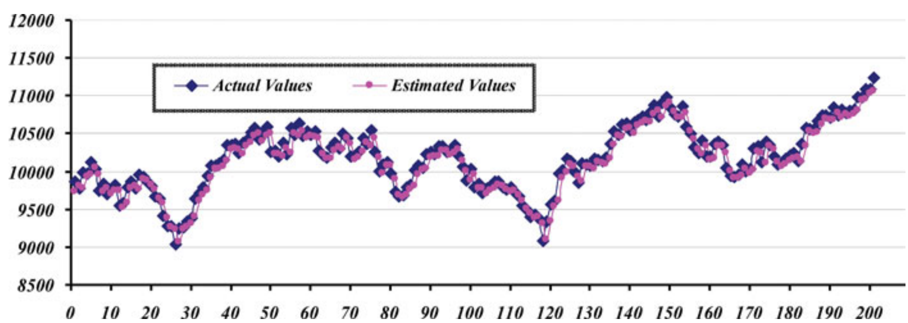

Figure 16. Estimated values of the MLP-ARIMA model for Nikkei (test data set).

Table 1. The performance of models for DJIAI in train and test data sets.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
MAE	245.70	369.85	231.23	366.81	227.75	351.77	229.50	358.42
MSE	119876	239015	107496	221930	103822	204104	105562	183310
MAPE	3.19%	3.56%	3.02%	3.48%	2.93%	3.34%	2.98%	3.32%
RMSE	346.23	488.89	327.87	471.09	322.22	451.78	324.90	428.15

applying both series hybrid models can improve the forecasting accuracy over the ARIMA and MLP models. This may suggest that neither MLP nor ARIMA model captures all of patterns in the data. For example in terms of MSE, the ARIMA-MLP and the MLP-ARIMA model can respectively improve 14.61% and 23.31% over than ARIMA model in the test data. In addition, the ARIMA-MLP and the MLP-ARIMA model can respectively improve 8.03% and 17.40% over than MLP model in the test data. Moreover, by changing the sequence of using ARIMA and MLP models, the MLP-ARIMA model can overall yield slightly better performance than ARIMA-MLP model.

5.2. Shenzhen forecasting results

The forecasting results of the hybrid models, ARIMA and MLP models for SZII closing index are given in Table 2. Similar to the previous case, the forecasting results indicate that both hybrid models significantly outperform ARIMA and MLP models in test data set. In terms of MSE, the ARIMA-MLP and the MLP-ARIMA model can approximately improve 13.78% over than ARIMA model in the test data. In addition, the ARIMA-MLP and the MLP-ARIMA model can roughly improve 2.97% over than ARIMA model in the test data. The MLP-ARIMA model and the ARIMA-MLP models have overall achieved same results.

Table 2. The performance of models for SZII in train and test data sets.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
MAE	224.46	1166.18	215.39	1102.34	212.39	1082.89	210.92	1064.92
MSE	99255	2221776	94577	1974479	91955	1915716	86221	1915422
MAPE	7.30%	10.26%	7.08%	9.81%	7.04%	9.52%	7.02%	9.50%
RMSE	315.05	1490.56	307.53	1405.16	303.24	1384.09	293.63	1383.99

Table 3. The performance of models for N225 in train and test data sets.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
MAE	168.39	105.11	141.98	100.03	135.87	97.397	141.73	96.36
MSE	52056	17734	36699	15199	32207	15379	36593	14808
MAPE	1.34%	1.04%	1.13%	0.98%	1.03%	0.96%	1.13%	0.95%
RMSE	228.16	133.17	191.57	123.29	179.46	124.02	191.29	121.69

Table 4. The average of the improvement percentage of models in comparison with each other in MAE.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
ARIMA	0.00%	0.00%	—	—	—	—	—	—
MLP	8.54%	3.71%	0.00%	0.00%	—	—	—	—
ARIMA/MLP	10.67%	6.46%	2.40%	2.83%	0.00%	0.00%	—	—
MLP/ARIMA	9.49%	6.70%	1.01%	1.12%	− 1.46%	0.28%	0.00%	0.00%

Table 5. The average of the improvement percentage of models in comparison with each other in MSE.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
ARIMA	0.00%	0.00%	—	—	—	—	—	—
MLP	14.85%	10.86%	0.00%	0.00%	—	—	—	—
ARIMA/MLP	19.63%	13.89%	6.14%	3.27%	0.00%	0.00%	—	—
MLP/ARIMA	18.26%	17.86%	3.64%	7.66%	− 3.02%	4.64%	0.00%	0.00%

5.3. Nikkei 225 index forecasting results

The forecasting results of the hybrid models and their components for Nikkei 225 index are given in Table (3). Results of the Nikkei 225 index data set indicate that the ARIMA-MLP model is slightly better than ARIMA and has overall yield same performance with MLP model. On the other hand, the ARIMA-MLP model can only improve the ARIMA model. In terms of MSE, the ARIMA-MLP model can improve 13.28% over than ARIMA model in the test data. While, the MLP-ARIMA model outperforms ARIMA, MLP, and ARIMA-MLP models in test data. In terms of MSE, the MLP-ARIMA can improve 16.50%, 2.57%, and 3.71% over than ARIMA, MLP, and ARIMA-MLP models in the test data. The average of the improvement percentage of models in comparison with each other in MAE, MSE, MAPE, and RMSE are summarized in Table 4 to Table 7; respectively. For example

Table 6. The average of the improvement percentage of models in comparison with each other in MAPE.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
ARIMA	0.00%	0.00%	—	—	—	—	—	—
MLP	8.00%	4.13%	0.00%	0.00%	—	—	—	—
ARIMA/MLP	11.62%	7.03%	4.13%	3.01%	0.00%	0.00%	—	—
MLP/ARIMA	8.70%	7.60%	0.72%	3.61%	− 3.71%	0.62%	0.00%	0.00%

Table 7. The average of the improvement percentage of models in comparison with each other in RMSE.

	ARIMA		MLP		ARIMA-MLP		MLP-ARIMA	
	Train	Test	Train	Test	Train	Test	Train	Test
ARIMA	0.00%	0.00%	—	—	—	—	—	—
MLP	7.91%	5.60%	0.00%	0.00%	—	—	—	—
ARIMA/MLP	10.68%	7.20%	3.15%	1.67%	0.00%	0.00%	—	—
MLP/ARIMA	9.71%	9.40%	1.86%	3.97%	− 1.42%	2.37%	0.00%	0.00%

6. Conclusion

Almost all financial decision makers, i.e. investors, money managers, investment banks, hedge funds, etc. need to predict prices of financial assets such as stocks, bonds, options, interest rates, exchange rates, etc. in order to make accurate financial decisions. It is the main reason that why efforts for improving the efficiency of forecasting models has never been stopped in the finance. However, literature indicates that achieving accurate financial forecasts is an important yet often difficult task facing financial decision makers. Combining different models together is one of the most accepted and widely used methods in the literature for improving the forecasting accuracy. In the literature, several different combination techniques have been developed in order to overcome the deficiencies of single models and yield results that are more accurate. Hybrid techniques that decompose a time series into its linear and nonlinear components are one of the most popular hybrid models, which have been theoretically and practically shown to be successful for single models. These models are jointly used unique advantages of linear and nonlinear models in order to capture different forms of relationship in the time series data.

In this paper, the predictive capabilities of two series hybrid linear/nonlinear of autoregressive integrated moving average (ARIMA) and multilayer perceptrons (MLPs) models, i.e. ARIMA-MLP and MLP-ARIMA, are compared together and with their components. Empirical results with three well-known real data sets of stock prices indicate that using these series hybrid models can be a worthy idea in order to yield more accurate results than both components used separately. In general, it can be theoretically demonstrated that series hybrid models can obtain results that at least is better than one of the component models. On the other hand, obtained results of series hybrid models will not be generally worse than all their components. In addition, it can be generally demonstrated that at least one of these models, i.e. ARIMA-MLP and MLP-ARIMA, can yield more accurate results than both their components. However, in comparison of series hybrid models with themselves, empirical results indicate that the MLP-ARIMA overall outperforms the ARIMA-MLP model. The MLP-ARIMA can averagely improve 16.50%, 2.57%, 2.57%, and 3.71% over than the ARIMA-MLP model in MAE, MSE, MAPE, and RMSE the train data. In addition, the MLP-ARIMA can averagely improve 16.50%, 2.57%, 2.57%, and 3.71% over than the ARIMA-MLP model in MAE, MSE, MAPE, and RMSE in the test data. These results are contrast to more popularity of the ARIMA-MLP models in the literature. However, these results demonstrate that the MLP-ARIMA models can be considered as an appropriate alternative at least for financial time series forecasting.

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