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Which implied volatility provides the best measure of future volatility?

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Abstract The volatility smile/skew phenomenon makes it unclear which implied volatility provides the best measure of the market volatility expectation over the remaining life of the option. Due to the high liquidity of at-the-money option and the low sensitivity of its implied volatility to the price error, the at-the-money implied volatility is often considered a good measure of future volatility. In this paper, we raise the question: is at-the-money implied volatility the best we can do? We provide in this paper an analytical rationale that the implied volatility from option with highest *vega* outperforms the at-the-money implied volatility in terms of forecasting ability, especially for long forecasting horizons. Our empirical findings are consistent with our theoretical argument.

Keywords Option · Volatility Smile · Implied Volatility · Black-Scholes Model · Hypothesis · Informational Content

JEL Classification C02 · C12 · G12 · G14

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1 Introduction

There are two widely used approaches to forecasting the volatility of a financial asset. One of them uses time series techniques (e.g. Hsieh1991; Andersen and Bollerslev 1998; 2008; Anderson et al., 2005) and relies on the past behavior of asset prices to infer the future trend. Time series approach, by nature, is backward looking, using past behavior to project future. An alternative approach is to use reported option prices to infer the volatility expectation from option pricing models (implied volatility approach). There has been a debate over the last two decades about which is the superior volatility estimation method if any in the sense of forecasting future volatility accuracy (Brooks 2002; Poon and Granger 2003; Anderson et al, 2005). In general, the latter approach is preferred because of the forward-looking nature of the implied volatility.

Implied volatility (IV) is widely interpreted as the market expectation of the underlying stock's return volatility over the remaining life of the option. One very common feature in practice that causes difficulty in the line of implied volatility research is that IV obtained from options on the same underlying with different strikes differs; furthermore, deep in-the-money and out-of-money options yield higher implied volatilities than near the money options do, and such phenomenon is often referred to volatility smile/skew phenomenon. By definition, there is only one volatility for the underlying asset. A natural question arises: which implied volatility or combination of IVs provides the best measure of the volatility of the underlying asset over the remaining life of the options?

To answer this question, some researchers, such as Whaley (1982), adopt a least square approach to creating a combined implied volatility estimate which is to solve a least squares problem to determine so called the best implied volatility σ * which minimizes the express $\sum_{i=1}^{N}(\sigma_i-\sigma^*)^2$, where each σ_i represents the IV computed from one of the N available options. We think the least squares approach is appropriate only when the estimation errors are caused by random noise and each IV is viewed equally important. Obviously, the persistence of volatility smile is not just caused by random noise and the informational content in implied volatilities obtained from deep in-the-money and out-of-money options and/or less frequently traded options can not be viewed as equally as that in the implied volatilities from near the money options and /or frequently traded options.

Another common approach is to use different weight schemes. In some cases, a simple average is used; but taking a weighted average is more common as we have seen in the literature. For example, Lataneand and Rendleman (1976) use weighted average by setting weights equal to the option's relative *vega*; Chiras and Manaster (1978) propose a variation in which the weights are set equal to the option's price elasticity with respect to implied volatility. Day and Lewis (1992) and Christensen and Hansen (2002) derive implied volatilities from a trade weighted sum of call option prices. Those adopting weighted average approaches argue that some options are traded with much greater liquidity than others, so their reported prices are expected to contain more information, and higher weight should be put on the implied volatilities obtained from these frequently traded options. Another reason for weighting is to adjust for differing sensitivities of option values to the volatility parameter. Since implied volatilities from deep in-the-money and out-of- money options are more sensitive to the price error, away from money options are down weighted in the average. But exactly what weight should be put on each implied volatility is controversial, and actually, no one explores the reason for their specific weight choices or explains why one scheme forecasts better than another.

Regardless of the complexity of how to select weights, since the plot of implied volatility against the strikes can take many shapes, it is not likely that a single weighing scheme will remove all pricing errors consistently as pointed out by Poon and Granger (2003). For this reason and the liquidity reason, implied volatilities from at-the-money options (denoted as ATMIV) are often used for volatility forecast. Many studies, such as Lamoureus and Lastrapes (1993), Fleming (1998), Christensen and Prabhala (1998) and Gwilymand and Buckle (1999), use single at-the-money implied volatility as an approximate forecast for realized volatility. Their empirical work has found that the at-the-money implied volatility is an efficient, although biased, forecast of subsequent realized volatility. Commercial providers such as Bloomberg and the CBOT also normally use a few at-the-money options to calculate the implied volatility. Though there is no direct economic motivation for regarding the at-the-money implied volatility as the realized volatility forecast, at-the-money options are considered to mitigate the bid/ask and non-continuous price effects and fairly insensitive to small changes in option price.

Beckers (1981) has shown that the implied volatility from at-the-money option outperforms any combination of all the available implied volatilities in his data setting in terms of forecasting ability. In addition, using single at-the-money option can dramatically reduce the computation time. This fact might confirm the old phrase that "simpler is better".

Is at-the-money implied volatility the best we can do? By definition, implied volatility from option with highest *vega* (defined as highest *vega* implied volatility, denoted as HVIV) is least sensitive to the option price error, furthermore, slightly out of the money options have even higher trading volume than at-the-money options. Thus we hypothesize that the highest *vega* implied volatility outperforms the at-the-money implied volatility in terms of forecasting ability, especially for long forecasting horizons.

The outline of the remainder of the paper is as follows. Section 2 discusses the option *vega* which measures the sensitivity of option price to the underlying stock's return volatility, and the mathematical express for the strike price with which option reaches highest *vega* is derived. In Section 3, we present the data and discuss the sample procedure and the test methodology. The empirical results are presented in section 4 and the conclusion follows in section 5.

2 Sensitivity of implied volatility estimation to the price error

The *vega* of an option measures the sensitivity of the option price to the underlying stock's return volatility. Let C denote the theoretical call option price, and σ underlying stock's return volatility, the *vega* of the call option is defined as

$$vega = \frac{\partial C}{\partial \sigma}$$

vega is quoted to show the theoretical price change for every 1 percentage point change in volatility. For example, if the theoretical price is \$2.5 and the *vega* is showing 0.25, then if the volatility moves from 20% to 21% the theoretical price will increase to \$2.75. The reciprocal of the *vega* $\left(\frac{\partial\sigma}{\partial C}\right)$ measures the sensitivity of the implied volatility estimation to the option price change. If the observed option market price is \$2.75 (assume the theoretical price is \$2.5), the implied volatility from that option market price is 21% (assuming the true volatility is 20%, and option has a *vega* of 0.25), as we can see the higher the option *vega*, the less sensitivity to the implied volatility estimation. We can calculate the strike price with which option *vega* reaches its highest value.

The original Black-Scholes (1973) model values the European call option written on stock paying no dividend. Robert Merton (1973) relaxed no dividends assumption and extended the standard Black-Scholes' formula to a dividend paying stock. Let d(<r) be the dividend yield per year, the value of a call option for paid dividend stock can be calculated as:

$$C = Se^{-dT}\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

where

$$d_i = \frac{ln\left[\frac{S}{X}\right] + (r-d)T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = \frac{ln\left[\frac{S}{X}\right] + (r-d)T}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$$

Obviously when d=0, Merton's model is reduced to the Black-Scholes model.

The *vega* of the option is

$$\begin{aligned}\frac{\partial C}{\partial\sigma} &= S.e^{-dT}.\phi(d_1)\left(-\frac{ln\frac{S}{X}+(r-d)T}{\sigma^2\sqrt{T}}+\frac{\sqrt{T}}{2}\right) - X.e^{-(r-d)T}\phi(d_2)\left(-\frac{ln\frac{S}{X}+(r-d)T}{\sigma^2\sqrt{T}}-\frac{\sqrt{T}}{2}\right)\\ &= Se^{-dt}\phi(d_1)\sqrt{T}\left(-\frac{d_2}{\sigma\sqrt{T}}+\frac{Xe^{-(r-d)T}}{S}e^{-\frac{d_1^2-d_2^2}{2}}\frac{d_1}{\sigma\sqrt{T}}\right)\\ &= Se^{-dT}\phi(d_1)\sqrt{T}\left(-\frac{d_2}{\sigma\sqrt{T}}+\frac{d_1}{\sigma\sqrt{\sqrt{T}}}\right) = Se^{-dt}\phi(d_1)\sqrt{T}\end{aligned} \tag{1}$$

where $\phi(.)$ is the standard normal distribution probability density function which is

$$\phi(x) = \frac{1}{\sqrt{2\pi}}.e^{-\frac{x^2}{2}}$$

the maximum of $\frac{\partial C}{\partial\sigma}$ is reached when $\phi(d_1)$ reaches the maximum, which is $\phi(d_1) = \frac{1}{\sqrt{2\pi}}$, in which case, $d_1 = 0$ and strike has to be set as

$$\overline{X} = S.e^{(r-d)T+\frac{\sigma^2 T}{2}}$$

Thus we know that the option with highest *vega* is not at-the-money, but out-of-the-money, and its moneyness depends on the maturity: the longer the maturity, the farther away from the money. Theoretically speaking, the implied volatility from option with the highest *vega* is least sensitive to the option price error. Notice that when the maturity is short, $\overline{X}$ is not much different from S, thus for short maturity option, the *vega* of at-the-money option is nearly the highest, but for long term options, the strike of highest *vega* option can be significantly different from that of at-the-money option. In the next section, we examine the informational content of the implied volatility from option with highest *vega* in comparison to that of at-the-money implied volatility, especially for longer forecasting horizons.

3 Data and testing methodology

To avoid the model misspecification problem, we use S&P 500 index options which are European style to compare the predictive power of at-the-money implied volatilities with highest *vega* implied volatilities. Historical option data and daily dividends are from OptionMetrics. Daily Treasury bill yields (the risk free rates) are from the Federal Reserve Bulletin. Dividend adjusted daily returns are from CRSP. The sample period runs from January 3 rd 2000 to January 7th 2005, spanning a time period of about 5 years.

We use mid bid-ask quotes instead of transaction prices in order to eliminate potential measurement errors in option prices that may result from any potential bias due to the bid-ask bounce (Bakshi et al.1997; 2000). We perform our test for three different forecasting horizons, one month, two months and three months. To avoid the telescoping overlap problem pointed out by (Christensen et al. 2002) that the telescoping overlap structure can induce potentially severe distortions in inferences, we only extract one sample per month and keep options' time to maturity fixed (predetermined fixed maturity intervals). By convention, S&P 500 options expire on the third Saturday of the contract month. Compared to other weekdays, Wednesdays have the fewest holidays. In order to obtain as many samples as possible to increase the statistic power, we specifically choose 31 days as one month forecasting horizon, 59 days for 2 months and 87 days for three months. Thus, most of the volatility measures are extracted on Wednesdays (It is Wednesday when we move back 31 days, or 59 days or 87 days from the third Saturday of the month). If Wednesday is a holiday, which is very rare, the next trading day is used instead, and in such cases, the remaining life of option are 1 day or two less than the predetermined maturity day. Longer maturity options in the sample are not included for two reasons: (1) Samples obtained from longer maturity exhibit a higher degree of overlap which renders the OLS test statistics invalid; (2) Volatility is presumed not to change over time and a constant volatility assumption within a long period of time may not be appropriate.

We first record at-the-money implied volatilities from options with 31 days remaining life. On each third Saturday of the month, we move back 31 days; if it is not a trading day, the next trading day is used instead. We locate a call option on that day that is closest to being at-the-money and record its implied volatility as at-the-money implied volatility, and also record its price using mid bid-ask quotes instead of actual transaction prices. Following Harvey and Whaley (1991) and Jiang and Tian (2005), we use daily cash dividends instead of constant dividend yield, and from dividend adjusted daily return, the realized volatility over the same period, matching the maturities of the corresponding implied volatilities, can be calculated. We assume an average of 252 trading days each year to calculate the annualized volatilities.

We use at-the-money implied volatility, or implied volatility from call option whose strike price is closest to that of at-the-money call option, if at-the-money option is not traded, as an approximation of the true volatility to calculate the highest *vega* strike. The rationale of such an approach is presented in Theorem in Appendix. The implication of Theorem is that though we don't know the true volatility, we can use volatility approximations, such as at-the-money implied volatility as a starting point, to estimate the highest *vega* strike $\overline{\overline{X}} = S.e^{(r-d)T+\frac{\sigma_{At-the-money}^2 T}{2}}$, the implied volatility from option with strike of $\overline{\overline{X}} = S.e^{(r-d)T+\frac{\sigma_{At-the-money}^2 T}{2}}$ is less sensitive to the price error than at-the-money implied volatility if the approximation satisfies $\left|\frac{\sigma_{At-the-money}}{\sigma}\right| < \sqrt{2\left(\frac{r-d}{\sigma^2}+1\right)}$. Prior studies including Christensen and Prabhala (1998), shows that the at-the-money implied volatility is a good approximation of the true volatility, so it is reasonable to assume that $\left|\frac{\sigma_{At-the-money}}{\sigma}\right| < \sqrt{2\left(\frac{r-d}{\sigma^2}+1\right)}$ (Notice that $\sqrt{2\left(\frac{r-d}{\sigma^2}+1\right)} > \sqrt{2} \approx 1.414$). Thus we can theoretically argue that the implied volatility from the option with strike of $\overline{\overline{X}} = S.e^{(r-d)T+\frac{\sigma_{At-the-money}^2 T}{2}}$ is a better forecast than at-the-money implied volatility. For the rest of the paper, we use ATMIV rather than $\sigma_{At\text{-}the\text{-}money}$ to denote the at-the-money implied volatility.

If the option with strike of $\overline{\overline{X}}$(the highest *vega* option approximation) is not traded, we use the one whose strike is closest to $\overline{\overline{X}}$ instead, and record the corresponding implied volatility as the highest *vega* implied volatility. Such a sample procedure may result in some of the at-the-money implied volatility sample being identical to some of the highest *vega* implied volatility sample. The realized volatilities are defined as the standard deviation of the underlying daily stock returns calculated over the period corresponding to the option maturity T.

The same procedure is used to construct at-the-money implied volatilities, highest *vega* implied volatilities and realized volatilities for the maturities of 59 days (two months) and 87 days (almost three months). Since we keep options' time to maturity fixed (31 days, 59 days or 87 days), our sample procedure avoids the telescoping overlap problem which may render the statistics invalid in the regression analysis.

4 Empirical results

We use the log-volatility series instead of the level series in much of our empirical work. Table 1 presents the descriptive statistics of both log and level volatility series. From Table 1, we can see that the distribution of log-volatility series are less skewed and leptokurtic compared to that of the level series, which reveals that the log volatility is more conformable with the normal distribution. Regressions based on the log volatility are thus statistically better specified than those based on the level

Table 1 Descriptive statistics

Statistic	ATMIV	Log ATMIV	HVIV	Log HVIV
(31 days maturity interval)				
Mean	0.2088153	−1.604353	0.2062028	−1.616922
Standard Deviation	0.0588172	0.2777778	0.0581116	0.2775422
Skewness	0.6145503	0.0902665	0.6152065	0.1047059
Excess Kurtosis	−0.2738683	−0.716824	−0.341299	−0.7337042
(59 days maturity interval)				
Mean	0.2059489	−1.60727	0.203792	−1.6188616
Standard Deviation	0.0486857	0.2381238	0.0491025	0.2428735
Skewness	0.3479211	−0.059272	0.3683372	−0.0748369
Excess Kurtosis	−0.6381138	−0.855367	−0.5286316	−0.7695663
(87 days maturity interval)				
Mean	0.2089105	−1.590539	0.2050428	−1.6091568
Standard Deviation	0.0472722	0.2240311	0.0464045	0.223592
Skewness	0.5754891	0.0446712	0.6071596	0.0569869
Excess Kurtosis	0.1629541	−0.468911	0.2685856	−0.4043645

Descriptive statistics for time series of realized volatility, at-the-money IV and highest *vega* IV at three different maturity intervals and their corresponding natural logarithm series. At-the-money implied volatilities are obtained from options being closest to at-the-money options; the highest *vega* strikes are calculated using at-the-money implied volatility as true volatility and the highest *vega* implied volatilities are obtained from options with strikes being closest to the highest *vega* strikes. Realized volatilities are the annualized daily dividend adjusted return volatility matching the corresponding maturity interval of implied volatility measures

series. Starting with means, we note from Table 1 that the means of highest *vega* implied volatilities are closer to the means of realized volatilities over all three horizons. The results also suggest that both at-the-money implied volatilities and highest *vage* implied volatilities are likely to be biased forecasts for subsequently realized volatility, with less bias for the latter.

We run a regression of subsequently realized volatility against volatility forecast (highest *vega* implied volatility or at-the-money implied volatility):

$$LRV_t = \alpha_0 + \alpha_1 LFV_t + e_t$$

where LRV_t denotes the log realized volatility for period t and LFV_t denotes the log volatility forecast at the beginning of period t , and e_t estimation noise term.

For simplicity, we use the same notations ATMIV and HVIV as log at-the-money implied volatility and log highest *vega* implied volatility, respectively, in our regression analysis for the rest of the paper. Following Christensen and Prabhala (1998) and prior research, we test the following three hypotheses:

1. H_0: α_i=0, if volatility measure (ATMIV or HVIV) contains no information about future volatility, the slope coefficient α_i should be zero.
2. H_0 : α_0=0 and α_i=1, if volatility forecast is an unbiased estimate of future realized volatility, then the slope coefficient α_i should be 1 and the intercept α_0, zero.

3. If a volatility measure is efficient, the residuals e_t should be white noise and uncorrelated with any variable in the market's information set.

The OLS regression results for monthly (31 days) non-overlapping sample are reported in Table 2. The results strongly reject the first hypothesis. In both univariate regressions, the slope coefficients are significantly different from zero at all conventional significance levels. This implies that both volatility measures contain some information about future volatility. The F-tests from Panel A (Full samples including all the observations, F_{ATMIV} (2.57)=0.0003 and F_{HVIV} (2,57) = 0.001) also reject the joint hypothesis H_0 : α_0=0 and α_i=1for both regressions at 1% significance level, which indicates that both volatility forecasts are biased forecasts of future volatility. The DW statistics are not significantly different from 2 for both regressions, indicating that the regression residuals are not auto-correlated.

These results are consistent with our expectation that the forecasting ability of ATMIV is not much different from that of HVIV over one month forecasting horizon. This reflects the fact, as we mentioned earlier, that for a short maturity interval (short forecasting horizon, e.g. less than one month), the highest *vega* strike is not much different from the index level. In addition, because the theoretical at-the-money options and highest *vega* options are often not traded, we choose options closest to theoretical ones to be consistent with the sample procedure described earlier, as a result, we find that much of the at-the-money implied volatility is identical to the highest *vega* implied volatility if the maturity interval is not long enough.

Table 2 Realized volatility regressed on volatility forecast (31 days maturity interval)

Intercept	ATMIV	HVIV	Adj. R-square	DW
Panel A (59 observations — Full Sample including observations of ATMIV being equal to HVIV)				
0.0808	1.13468		0.6114	1.802
(0.42)	(9.61)***			
0.09743		1.13614	0.612	1.747
(0.5)		(9.62)***		
The F-statistic for joint hypotheses H_0 : α_0=0 and α_i=1 F_{ATMIV} (2,57)=0.0003***, F_{HVIV} (2,57)=0.0010***				
Panel B (29 observations — Reduced Sample excluding observations of ATMIV being equal to HVIV)				
0.13696	1.17709		0.5492	1.805
(0.43)	(6.03)***			
0.16878		1.17877	0.5487	1.744
(0.52)		(6.02)***		
The F-statistic for joint hypotheses H_0 :α_0=0 and α_i=1, F_{ATMIV} (2,28)=0.0246**, F_{HVIV} (2,28)=0.0629*				

Table 2 reports OLS regression (Dependent variable: Log realized volatility LRV) results for monthly (31 days) non-overlapping sample. Panel A reports the test results for a full sample, including observations of ATMIV being equal to HVIV. The data consist of 59 implied volatilities and realized volatilities observations from the period January 3 rd 2000 to January 7th 2005. Panel B reports the test results for a reduced sample, excluding all the observations of ATMIV being equal to HVIV

The numbers in parentheses are t-statistics. (***), (**) and (*) denote significance level at the 0.01, 0.05, and 0.10 levels, respectively

Table 2 Panel B reports the test results for a reduced sample, excluding from the sample pairs of identical observations of ATMIV being equal to HVIV. The number of observations has subsequently reduced from 59 to 29. Results from Panel B rejects H_0 : α_i =0 for both volatility estimates. The results also show that the performance of ATMIV and HVIV is not significantly different from each other, the latter being slightly better. The F-statistic $F_{ATMIV}(2, 28)$ =0.0246 rejects the null hypothesis H_0:α_0=0 and α_i=1 at the 5% significant level, indicating that at-the-money implied volatility is not an unbiased estimate of future volatility of the underlying stock return. However, with F-statistic at, F_{HVIV} (2, 28)=0.0629, we can not reject the null hypothesis for the highest *vega* option implied volatility at the 5% significant level, indicating that the highest *vega* option implied volatility is likely a more efficient and less biased estimate of the future volatility.

To further compare the predictive power of HVIV with ATMIV, we increase the forecasting horizon to two months. We choose a fixed maturity interval of 59 days in order to extract most of our samples on Wednesdays (as mentioned earlier, if Wednesday is a non-trading day, we choose the next trading day instead). Due to the limited availability of strike prices, some of the ATMIVs may be identical to HVIVs. We run the regressions for both the full and reduced samples, i.e., the samples including and excluding the observations of ATMIV being identical to HVIV.

Table 3 summarizes the OLS results from the univariate regressions for the log volatility measures over the 59 days maturity horizon. Panel A shows the results for the full sample (58 observations). After excluding from the sample pairs of identical observations of ATMIV being equal to HVIV, we reduce the degree of overlap problem, and this is reflected by better Durbin-Watson (DW) statistics. It is also important to note that we avoid the telescoping overlap problem described by Christen et al. (2002) by keeping the time to maturity (59 days) fixed.

Panel B shows the results for the reduced samples (observations of ATMIV being equal to HVIV are excluded). Again, in both univariate regressions, the slope coefficients are significantly different from zero at all conventional significant levels. Contrary to the one month's sample case, both F-statistics (F_{ATMIV} (2,24)=0.4499 and F_{HVIV} (2,24)=0.8035) cannot reject the joint hypothesis H_0:α_0=0 and α_i=1 at any conventional level, with F_{HVIV} (2,24)=0.8035 being higher which indicates that the highest *vega* option implied volatility is likely a more efficient and less biased estimate of the future volatility. Though the results from Panel A may suffer from some serial correlation problems because of the overlapping data, they are consistent with the results in Pane B.

Table 4 reports the OLS regression results for the samples with 87 days fixed maturity interval. As maturity increases, the highest *vega* strike increases, thus, deviates more from index level, however, the overlap problem also increases (the more frequent the sampling within one maturity interval, the greater the degree of overlap in successive observations), and this can be reflected in lower Durbin-Watson (DW) values. In spite of lower Durbin-Watson (DW) values, we are still interested in finding out whether or not the regression results would change if samples with longer horizons were used in analysis. We again concentrate in the regression results from the reduced samples and these results are reported in Table 4, Panel B. The F-statistics (F_{ATMIV}(2,33)=0.0355, F_{HVIV}(2,33)=0.1210) reject the null hypothesis that ATMIV is an unbiased forecast of future volatility at the 5% level,

Table 3 Realized volatility regressed on volatility forecast (59 days maturity interval)

Intercept	ATMIV	HVIV	Adj. R-square	DW
Panel A (58 observations — Full Sample including observations of ATMIV being equal to HVIV)				
0.10381	1.14446		0.5578	1.071
(0.48)	(8.54)***			
0.06972		1.11372	0.5343	1.03
(0.32)		(8.15)***		
The F-statistic for joint hypotheses H_0:α_0 =0 and α_i=1 F_{ATMIV} (2,56)=0.0007***, F_{HVIV} (2,56)=0.0037***				
Panel B (26 observations — Reduced Sample excluding observations of ATMIV being equal to HVIV)				
0.10417	1.10055		0.515	1.51
(0.31)	(5.25)***			
0.09857		1.07707	0.5017	1.49
(0.29)		(5.12)***		
The F-statistics for joint Hypotheses H_0:α_0=0 and α_i=1 F_{ATMIV}(2,24)=0.4499, F_{HVIV}(2,24)=0.8035.				

Table 3 reports OLS regression (Dependent variable: Log realized volatility LRV) results for 59 days maturity interval. Panel A reports the test results for a full sample, including observations of ATMIV being equal to HVIV. The data consist of 58 implied volatilities and realized volatilities observations from the period January 3 rd 2000 to January 7th 2005.

Panel B reports the test results for a reduced sample, excluding all the observations of ATMIV being equal to HVIV. Data used in this table may exhibit some degree of overlap.

The numbers in parentheses are t-statistics. (***), (**) and (*) denote significance level at the 0.01, 0.05, and 0.10 levels, respectively

Table 4 Realized volatility regressed on volatility forecast (87 days maturity interval)

Intercept	ATMIV	HVIV	Adj. R-square	DW
Panel A (58 observations — Full Sample including observations of ATMIV being equal to HVIV)				
0.06788	1.12338		0.4767	0.579
(0.27)	(7.28)***			
0.10928		1.1361	0.486	0.586
(0.44)		(7.41)***		
The F-statistic for joint hypotheses H_0: α_0=0 and α_i =1. F_{ATMIV}(2,56)=0.0015, F_{HVIV}(2,56)=0.0060				
Panel B (35 observations — Reduced Sample excluding observations of ATMIV being equal to HVIV)				
Intercept	ATMIV	HVIV	Adj. R-square	DW
−0.17301	0.9751		0.3045	0.667
(−0.46)	(3.99)***			
−0.15705		0.9661	0.319	0.684
(−0.42)		(4.11)***		
The F-statistics for joint Hypotheses H_0:α_0=0 and α_i=1. F_{ATMIV} (2,33)=0.0355**, F_{HVIV}(2,33)=0.1210				

Table *4* reports the OLS regression (Dependent variable: Log realized volatility LRV) results for the sample with 87 days fixed maturity interval. The numbers in parentheses are t-statistics. . (***), (**) and (*) denote significance level at the 0.01, 0.05, and 0.10 levels, respectively but do not reject that HVIV is an unbiased forecast of future volatility even at the 10% level.

Our empirical results support our rationale that the highest *vega* implied volatility (HVIV) is likely a more efficient and less biased forecast of future volatility, especially for long forecasting horizons. Based on our theoretical argument and our empirical results, in practice, we suggest that implied volatilities from frequently traded out of the money but near the money options (highest *vega* option) be used to estimate future volatility of the underlying asset. The at-the-money option written on low volatility underlying asset with short maturity is actually very close to the highest *vega* option as mentioned earlier; for long forecasting horizons, the difference between the two becomes more visible.

5 Summary and conclusion

In this paper, we show that the degree of the bias of the implied volatility estimation is directly related to the option's moneyness and maturity and such bias can be minimized by choosing option with the highest *vega* to estimate volatility. Based on our theoretical argument, we raise a hypothesis that the implied volatility from options with highest *vega* outperforms at-the-money implied volatility, especially for longer forecasting horizons. We use the S&P 500 index option to test the informational content of the implied volatility from options with highest *vega* in comparison to that of at-the-money implied volatility. To differentiate the two, we increase the forecasting horizon from one month to two months and to three months. Our empirical results confirm our theoretical argument, i.e. the implied volatility from the option with the highest *vega* outperforms at-the-money implied volatility for all three forecasting horizons and the advantage of using option with the highest *vega* over at-the-money implied volatility increases as forecasting horizon increases.

Appendix

Theorem: *Assume the true volatility of underlying stock is σ, $\sigma_A$is a close approximation of the true volatility. If $\left|\frac{\sigma_A}{\sigma}\right| < \sqrt{2\left(\frac{r-d}{\sigma^2}+1\right)}$, then the vega of option with strike of $\overline{\overline{X}} = S.e^{(r-d)T+\frac{\sigma_A^2 T}{2}}$is greater than the vega of at-the-money option. In other words, the sensitivity of implied volatility from option with strike of $\overline{\overline{X}} = S.e^{(r-d)T+\frac{\sigma_A^2 T}{2}}$ to the price error is lower than that of at the money implied volatility.*

Proof of theorem Recall the definition of $d_1 = \frac{\ln\left[\frac{S}{X}\right]+(r-d)T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$

When X=S,

$$d_1 = d_{1S} = \frac{(r-d)T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

When $X = \overline{\overline{X}}$,

$$d_1 = d_{1\overline{\overline{X}}} = -\frac{\frac{\sigma_S^{2T}}{2}}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

It is not difficult to see that when $\left|\frac{\sigma_A}{\sigma}\right| < \sqrt{2\left(\frac{r-d}{\sigma^2}+1\right)}$,

$$-\frac{(r-d)T}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2} < -\frac{\frac{\sigma_A^{2T}}{2}}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} < \frac{(r-d)T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

which is $\left|d_{1\overline{\overline{X}}}\right| < |d_{1s}|$, therefore $\phi(d_{1S}) < \phi\left(d_{1\overline{\overline{X}}}\right)$
According to Eq. (1), we have

$$\left.\frac{\partial C}{\partial \sigma}\right|_{X=S} = Se^{-dT}\phi(d_1)\sqrt{T} < Se^{-dT}\phi\left(d_{1\overline{\overline{X}}}\right)\sqrt{T} = \left.\frac{\partial C}{\partial \sigma}\right|_{X=\overline{\overline{X}}}$$

Thus we have proved **Theorem.**

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