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Tema 2- proiect

Ex 1) Parte de matematică

$x(t) = t;\ t \in [-10;10]$

$T = 20$

Avem o fct imp $\Rightarrow a_0 = a_n = 0$

$$b_n = \frac{1}{10}\int_{-10}^{10} x(t)\cdot \sin\left(\frac{n\pi t}{10}\right) dt = \frac{1}{10}\int_{-10}^{10} t \sin\left(\frac{n\pi t}{10}\right) dt = \frac{1}{10}\left[t\left(-\cos\frac{n\pi t}{10}\right)\frac{10}{n\pi}\right]_{-10}^{10}$$

$$+\int_{-10}^{10}\cos\left(\frac{n\pi t}{10}\right)\cdot\frac{10}{n\pi}dt = \frac{-10}{n\pi}\left(\cos(n\pi)+\cos(n\pi)\right)+\frac{10}{n^2\pi^2}\Big(\underbrace{\sin(n\pi)}_{0} -$$

$$-\underbrace{\sin(n\pi)}_{0}\Big) = -\frac{20}{n\pi}(-1)^n$$

Dezvoltarea funcției în serie Fourier:

$$f(x) = \sum_{n=1}^{\infty} -\frac{20}{n\pi}(-1)^n \sin\left(\frac{n\pi t}{10}\right) = \frac{20}{\pi}\sin\left(\frac{\pi t}{10}\right) - \frac{20}{2\pi}\sin\left(\frac{2\pi t}{10}\right) + \frac{20}{3\pi}\sin\left(\frac{3\pi t}{10}\right) - \dots$$

Ex2) $x(t) = \sin(w_0 t/2) \quad t \in [0;10]$

$T = 10;\ w_0 = \frac{2\pi}{T} \Rightarrow w_0 = \frac{2\pi}{10} \Rightarrow w_0 = \frac{\pi}{5} \Rightarrow x(t) = \sin\left(\frac{\pi t}{10}\right)$ [semnalul este cu dublă alt.]

$\Rightarrow$ funcție pară $\Rightarrow b_n = 0$

Consider 2 perioade: $t \in [-10;10]$

$$a_0 = \frac{1}{10}\int_{-10}^{10} x(t)\,dt = \frac{2}{10}\int_0^{10}\sin\left(\frac{\pi t}{10}\right)dt = \frac{1}{5}\left(-\cos\left(\frac{\pi t}{10}\right)\cdot\frac{10}{\pi}\right)\Bigg|_0^{10} =$$

$$= -\frac{1}{5}\cdot\frac{10}{\pi}\left(\cos\pi - 1\right) = \frac{4}{\pi}$$

$$a_n = \frac{1}{5}\int_0^{10}\sin\left(\frac{\pi t}{10}\right)\cdot\cos\left(\frac{n\pi t}{10}\right)dt = -2\left[\frac{(-1)^n+1}{\pi(n^2-1)}\right]$$

Dezvoltarea în serie Fourier:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{10}\right) = \frac{2}{\pi} + \sum_{n=2}^{\infty}\frac{(-2)\left[(-1)^n+1\right]}{\pi(n^2-1)}\cdot\cos\left(\frac{n\pi t}{10}\right)$$

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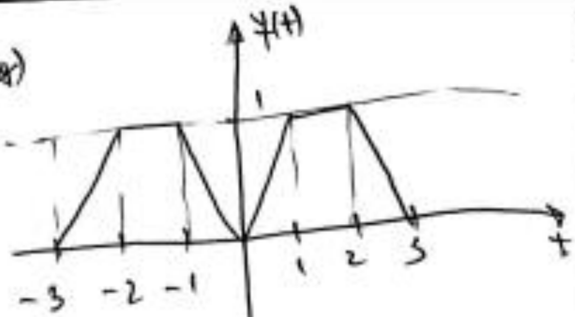
2)(x4) $x(t) = \begin{cases} t, & t\in[0,1] \\ 1, & t\in[1,2) \\ -t+3, & t\in[2,3]\end{cases}$ f. pară (sin cos) $b_n = 0$



$t\in[-3;3]$

$a_0 = \frac{1}{3}\int_{-3}^{3} x(t)\,dt =$

$= \frac13\left(\int_{-3}^{-2} t\,dt + \int_{-2}^{-1} dt + \int_{-1}^{0}(-t+3)\,dt + \int_0^1 t\,dt + \int_1^2 dt + \int_2^3 (t+3)\,dt\right) =$

$= \frac13\left(\frac{t^2}{2}\Big|_{-3}^{-2} + t\Big|_{-2}^{-1} + \left(-\frac{t^2}{2}+3t\right)\Big|_{-1}^{0} + \frac{t^2}{2}\Big|_0^1 + t\Big|_1^2 + \left(-\frac{t^2}{2}+3t\right)\Big|_2^3\right) =$

$= \frac13\left(2-\frac92+2-1+\frac12+3+\frac12+2-1-\frac92+9+2-6\right) = \frac43$

$a_n = \frac23\int_0^3 x(t)\cdot\cos\left(\frac{n\pi t}{3}\right)dt = \frac23\left(\int_0^1 t\cos\left(\frac{n\pi t}{3}\right)dt + \int_1^2\cos\left(\frac{n\pi t}{3}\right)dt + \int_2^3(-t+3)\cos\left(\frac{n\pi t}{3}\right)dt\right) = \frac23\left[t\sin\left(\frac{n\pi t}{3}\right)\cdot\frac{3}{n\pi}\Big|_0^1 - \int_0^1 \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)dt\right.$

$+ \sin\left(\frac{n\pi t}{3}\right)\cdot\frac{3}{n\pi}\Big|_1^2 - t\sin\left(\frac{n\pi t}{3}\right)\cdot\frac{3}{n\pi}\Big|_2^3 + \int_2^3\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)dt +$

$\left.+ 3\cdot\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\Big|_2^3\right] = \frac23\left[\sin\left(\frac{n\pi}{3}\right)\cdot\frac{3}{n\pi} + \frac{9}{(n\pi)^2}\cdot\cos\left(\frac{n\pi t}{3}\right)\Big|_0^1 +\right.$

$+ \frac{3}{n\pi}\cdot\sin\left(\frac{2n\pi}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi}{3}\right) - \underbrace{\frac{9}{n\pi}\cdot\sin(n\pi)}_{0} + \frac{6}{n\pi}\sin\left(\frac{2n\pi}{3}\right) -$

$\left.- \frac{9}{(n\pi)^2}\cos\left(\frac{n\pi t}{3}\right)\Big|_2^3 + \underbrace{\frac{9}{n\pi}\sin(n\pi)} - \frac{9}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right] =$

$= \frac23\cdot\frac{3}{n\pi}\left[\sin\left(\frac{n\pi}{3}\right) + \frac{3}{n\pi}\cdot\cos\left(\frac{n\pi}{3}\right) - \frac{3}{n\pi} + \sin\left(\frac{2n\pi}{3}\right) - \sin\left(\frac{n\pi}{3}\right) +\right.$

$\left.+ 2\sin\left(\frac{2n\pi}{3}\right) - \frac{3}{n\pi}\cos(n\pi) + \frac{3}{n\pi}\cos\left(\frac{2n\pi}{3}\right) - 3\sin\left(\frac{2n\pi}{3}\right)\right] =$

$= \frac{2\cdot3}{n^2\pi^2}\cdot\left(\cos\left(\frac{n\pi}{3}\right) - 1 - (-1)^n + \cos\left(\frac{2n\pi}{3}\right)\right)$.

Seria Fourier:

$f(t) = \frac{2}{3} + \sum_{n=1}\frac{6}{n^2\pi^2}\left[\cos\left(\frac{n\pi}{3}\right) - 1 - (-1)^n + \cos\left(\frac{2n\pi}{3}\right)\right]\cdot\cos\left(\frac{n\pi t}{3}\right)$.

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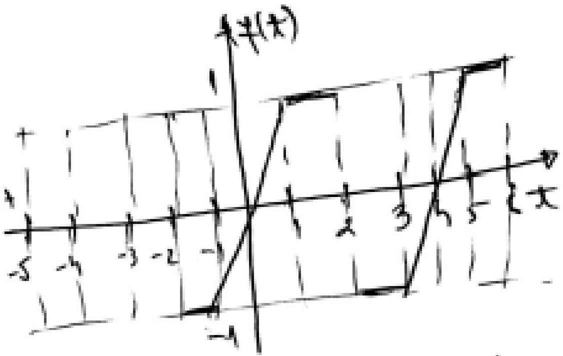
ex5

$$x(t) = \begin{cases} -1; & t \in [-2;-1) \\ t; & t \in [-1;1) \\ 1; & t \in [1;2] \end{cases}$$

f. imp sau f. de orig $\Rightarrow a_0 = a_n = 0$



$$b_n = \frac{1}{2}\int_{-2}^{2} x(t)\sin\left(\frac{n\pi t}{2}\right)dt =$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} -\sin\left(\frac{n\pi t}{2}\right)dt + \int_{-1}^{1} t\sin\left(\frac{n\pi t}{2}\right)dt + \int_{1}^{2}\sin\left(\frac{n\pi t}{2}\right)dt\right] =$$

$$= \frac{1}{2}\left[\cos\left(\frac{n\pi t}{2}\right)\cdot\frac{2}{n\pi}\right]\Big|_{-2}^{-1} - t\cos\left(\frac{n\pi t}{2}\right)\cdot\frac{2}{n\pi}\Big|_{-1}^{1} + \int_{-1}^{1}\frac{2}{n\pi}\cdot\cos\left(\frac{n\pi t}{2}\right)dt -$$

$$- \cos\left(\frac{n\pi t}{2}\right)\cdot\frac{2}{n\pi}\Big|_{1}^{2}\Bigg] = \frac{1}{2}\cdot\frac{2}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos(n\pi) - \cos\left(\frac{n\pi}{2}\right) -\right.$$

$$\left. - \cos\left(\frac{n\pi}{2}\right) + \frac{2}{n\pi}\sin\left(\frac{n\pi t}{2}\right)\Big|_{-1}^{1} - \cos(n\pi) + \cos\left(\frac{n\pi}{2}\right)\right] =$$

$$= \frac{1}{n\pi}\left[-2(-1)^n + \frac{2}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right)\right] = \frac{2}{n\pi}\left[-(-1)^n + \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\right]$$

Seria Fourier:

$$f(t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}\left[(-1)^n + \frac{2}{n\pi}\right]\sin\left(\frac{n\pi t}{2}\right)$$

1. clc;

clear all;

close all;

x=linspace(-10,10,1000);

f=0;

k=0; %contor de iteratie

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```
%Semnalul rampa
X=-10:0.1:10;
Y=-10:0.1:10;

figure(1);

for n=1:1:10

 if mod(n,2)==0
     f = f + (-20/(n*pi))*sin((n*pi*x)/10);%suma pariala Fourier de la pasul curent
 else
     f = f + (20/(n*pi))*sin((n*pi*x)/10);%suma pariala Fourier de la pasul curent
 endif;

 if(n==2 || n==7 || n==4 || n==10)
 k=k+1;
 subplot(2,2,k), line(X,Y,'color','r')
 hold on;
 subplot(2,2,k), plot(x,f,'k');
 title(['n = ', num2str(n)]), grid
 endif;

endfor;
```

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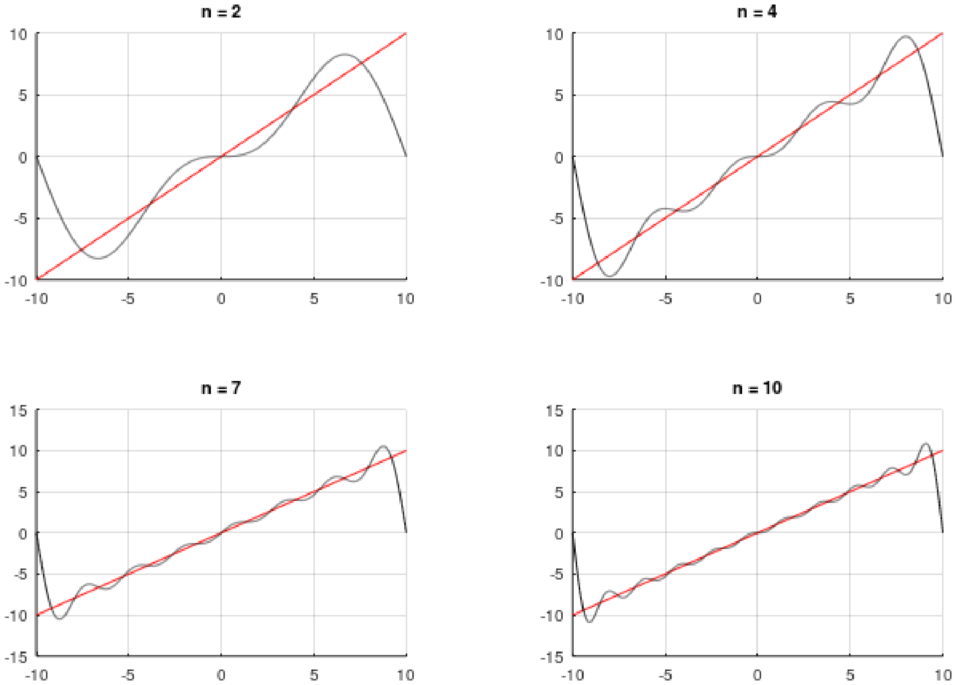
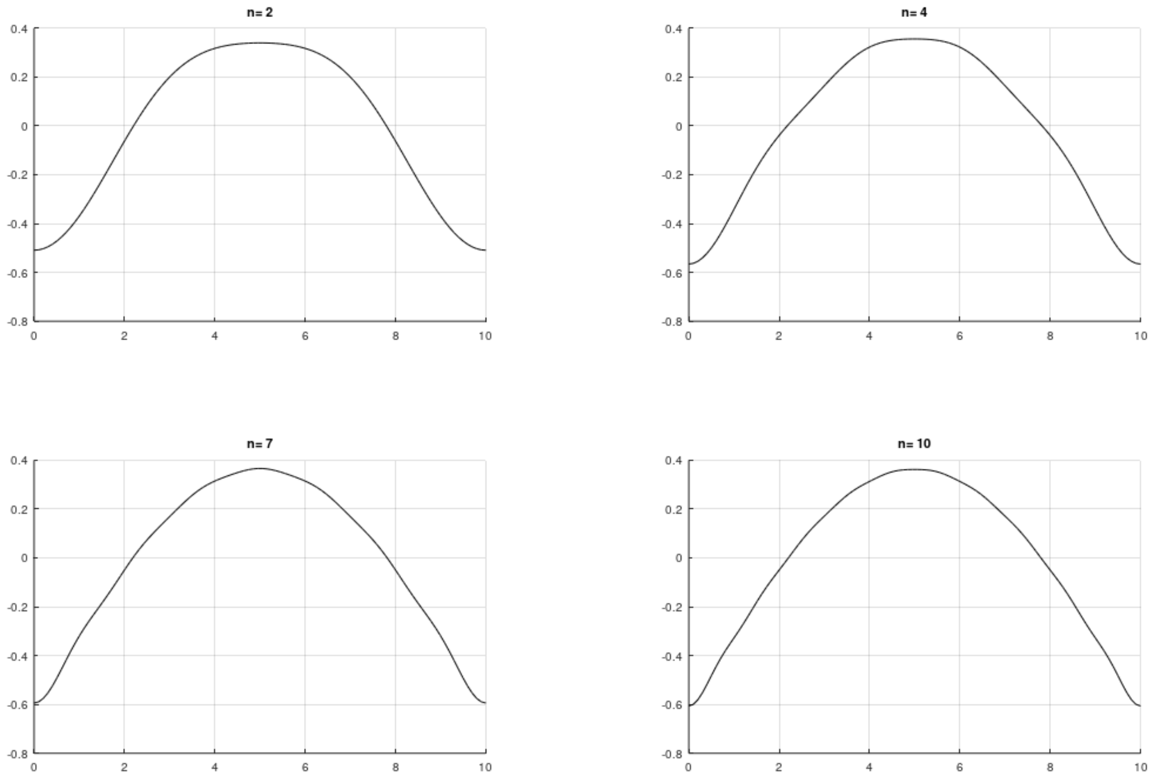


2. clear all;

```
clc;

close all;


x=linspace(0,10,100);

f=0;

k=0;%contor de iteratie


%semnal redresat dubla alternanta


figure(1)


for n=1:1:20


  f=f+4*(-1)^(2*n)/(pi-4*pi*n^2)*cos((n*pi*x)/5); %suma partiala Fourier de la pasul curent

  %f=f+((((-1)^n)*pi-n*pi)/(pi*pi-n*n*pi*pi))*cos(n*pi*x/5);


  if(n==2 || n==4 || n==7 || n==10)
```

```
 k=k+1;

 subplot(2,2,k), %plot(t,y,'color','r')

 hold on;

 subplot(2,2,k), plot(x,f,'k')

 grid

 title(['n= ', num2str(n)])

 endif


endfor
```




```
3. clear all;

clc;

close all;


x=linspace(-10,10,100);
```

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```
f=0;

k=0;%contor de iteratie


%semnal redresat monoalternanta


?

4. clc;

clear all;

close all;




x=linspace(0,3,100);

f=0;

k=0;%contor de iteratie


%Semnalul 4

X=[0,1,2,3];

Y=[0,1,1,0];

figure(1);


for n=1:1:100


 an=1/3*((9/(n*n*pi*pi))*(cos(n*pi/3)+cos(2*n*pi/3)-((-1)^n)-1));%coeficientul an al sumei Fourier de la pasul
curent

 bn=1/3*((9/(n*n*pi*pi))*(sin(2*n*pi/3)+sin(n*pi/3)));%coeficientul bn al sumei Fourier de la pasul curent

 f=f+an*cos(n*pi*x/3)+bn*sin(n*pi*x/3);%suma partiala Fourier de la pasul curent


 if(n==2 || n==7 || n==4 || n==10)

 k=k+1;

 subplot(2,2,k), line(X,Y,'color','r')
```

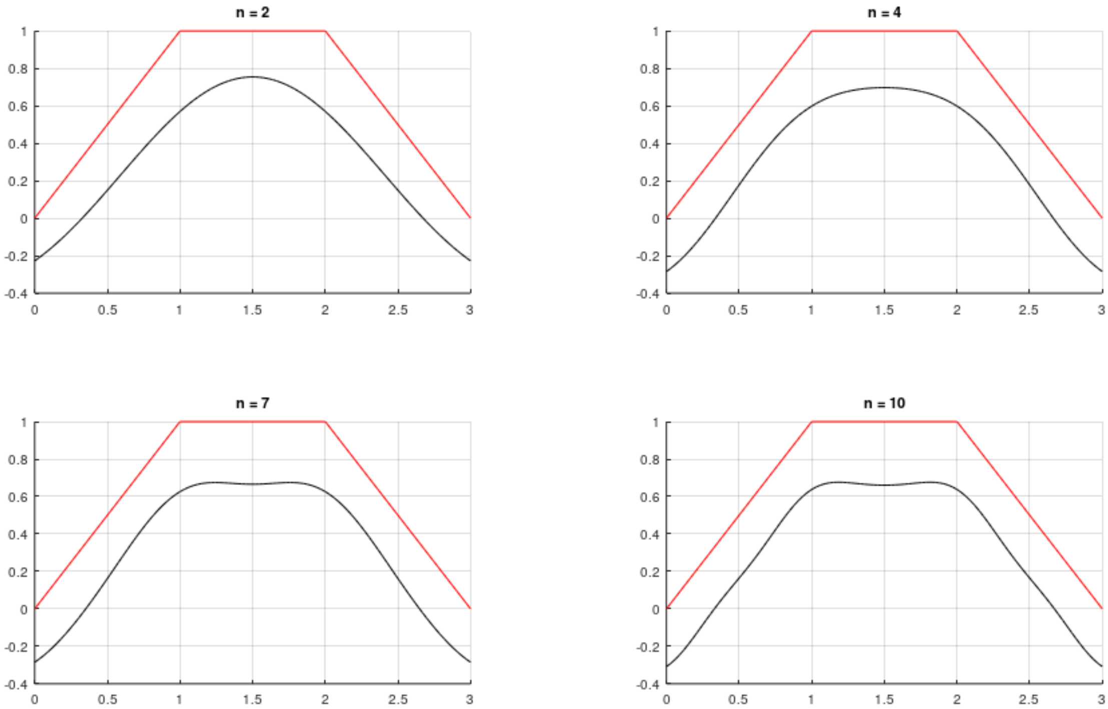
```
 hold on;

 subplot(2,2,k), plot(x,f,'k');

 title(['n = ', num2str(n)]), grid

 endif;


endfor
```




```
5. clc;

clear all;

close all;


x=linspace(-2,2,100);

f=0;

k=0;%contor de iteratie


%Semnalul 5

X=[-2,-1,0,1,2];

Y=[-1,-1,0,1,1];
```

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```
figure(1)


for n=1:1:20


  bn=1/2*((((-4)*((-1)^n))/(n*pi))+(8/(n*n*pi*pi)*sin(n*pi/2)));%coeficientul bn al sumei Fourier de la pasul curent

  f=f+bn*sin(n*pi*x/2);%suma partiala Fourier de la pasul curent


  if(n==2 || n==7 || n==4 || n==10)

  k=k+1;

  subplot(2,2,k), line(X,Y,'color','r')

  hold on;

  subplot(2,2,k), plot(x,f,'k');

  title(['n = ', num2str(n)]), grid

  endif;


endfor
```