① Pp că mașina produce în continuu

Notăm cu A_i → evenimentul că la extragerea i am obținut o piesă bună

$\overline{A}_i$ → — " — — " — o piesă defectă

a) A → evenimentul cerut

$$P(A) = P(A_1 \cap A_2 \dots \cap A_9 \cap \overline{A}_{10}) = P(\underset{i=1}{\overset{9}{\cap}} A_i \cap \overline{A}_{10}) =$$ (IND)

$$= \prod_{i=1}^{9} P(A_i) P(\overline{A}_{10}) = \left(\frac{95}{100}\right)^9 \cdot \frac{5}{100}$$

($\frac{95}{100}$ = " , $\frac{5}{100}$ = ")

b) B → evenimentul cerut

$$P(B) = P(\overline{A}_1 \cup (A_1 \cap \overline{A}_2) \cup (A_1 \cap A_2 \cap \overline{A}_3) \cup$$ (INC)

$$\cup (A_1 \cap A_2 \cap A_3 \cap \overline{A}_4) \cup (A_1 \cap A_2 \cap A_3 \cap A_4 \cap \overline{A}_5)$$ (INC)

$$\cup (A_1 \cap \dots \cap A_5 \cap \overline{A}_6))$$

$$P(B) = P(\overline{A}_1) + P(A_1 \cap \overline{A}_2) + P(A_1 \cap A_2 \cap \overline{A}_3) +$$

$$+ P(A_1 \cap \dots \cap \overline{A}_6)$$ (IND)

(IND)

$$P(B) = 1 - P(A_1) + P(A_1) \cdot (1 - P(A_2)) + \dots + P(A_1) \cdot P(A_2) \cdot (1 - P(A_6))$$

($P(A_1)$ = $\frac{95}{100}$, $P(A_2)$ = $\frac{95}{100}$, $1 - P(A_2)$ = $\frac{5}{100}$)

$$P(B) = \sum_{i=0}^{5} \left(\frac{95}{100}\right)^i \cdot \frac{5}{100}$$

c) $C \rightarrow$ evenimentul dorit

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) \overset{IND}{=} \prod_{i=1}^{4} P(A_i) = \left(\frac{95}{100}\right)^4$$ → Greșit

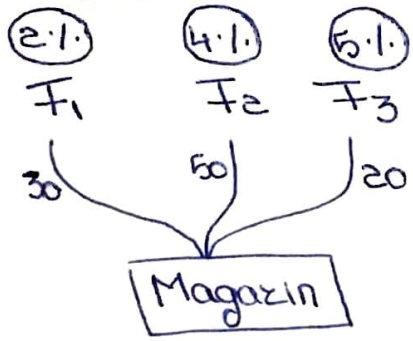
P
$\frac{95}{100}$

$$P(\bar{C}) = P(\bar{A}_1 \cup (A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2 \cap \bar{A}_3) \cup (A_1 \cap A_2 \cap A_3 \cap \bar{A}_4)$$

$$P(\bar{C}) = \sum_{i=0}^{3} \left(\frac{95}{100}\right)^i \cdot \frac{5}{100}$$

$$P(C) = 1 - \sum_{i=0}^{3} \left(\frac{95}{100}\right)^i \cdot \frac{5}{100}$$

(10) Problemă importantă



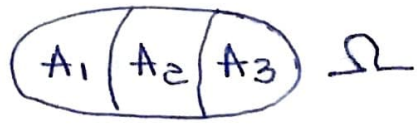
$D \rightarrow$ un cafe. se defectează

$A_i \rightarrow$ evenimentul că un cafe. provine de la F_i $\quad i = \overline{1,3}$

$$P(A_1) = \frac{30}{100} = \frac{3}{10}$$
$$P(A_2) = \frac{5}{10}$$
$$P(A_3) = \frac{2}{10}$$

$$P(D|A_1) = \frac{2}{100}$$
$$P(D|A_2) = \frac{4}{100}$$
$$P(D|A_3) = \frac{5}{100}$$

Folosim formula probabilităților totale



a) $$P(D) = \sum_{i=1}^{3} P(D|A_i) \cdot P(A_i) = \frac{36}{1000} = 0{,}036 = 3{,}6 \cdot 1.$$

b) $$P(A_2|D) = \frac{P(A_2|D)}{P(D)} = \frac{P(D|A_2) \cdot P(A_2)}{P(D)} = \frac{\frac{4}{100} \cdot \frac{5}{10}}{\frac{36}{1000}} =$$

Bayes

$$= \frac{20}{36}$$

$$P(D|A_2) = \frac{P(D \cap A_2)}{P(A_2)}$$

c) $P(D|A_1 \cup A_3) =$

$$= \frac{P(D \cap (A_1 \cup A_3))}{P(A_1 \cup A_3)} =$$

INC

INC

$$= \frac{P((D \cap A_1) \cup (D \cap A_3))}{P(A_1) + P(A_3)} = \frac{P(D \cap A_1) + P(D \cap A_3)}{P(A_1) + P(A_3)} =$$

$$= \frac{P(D|A_1) \cdot P(A_1) + P(D|A_3) \cdot P(A_3)}{P(A_1) + P(A_3)} = \frac{\frac{2}{100} \cdot \frac{3}{10} + \frac{5}{100} \cdot \frac{2}{10}}{\frac{3}{10} + \frac{2}{10}} =$$

$$= 2 \cdot \frac{16}{1000} = \frac{32}{1000} = 0{,}032$$

INC

d) $P(A_1 \cup A_2 | \bar{D}) = \frac{P((A_1 \cup A_2) \cap \bar{D})}{P(\bar{D})} = \frac{P((\bar{D} \cap A_1) \cup (A_2 \cap \bar{D}))}{P(\bar{D})}$

$$= \frac{P(\bar{D} \cap A_1) + P(\bar{D} \cap A_2)}{1 - P(D)} =$$

Obs: $P(\bar{A}|B) = 1 - P(A|B)$

$P(A|\bar{B}) \neq 1 - P(A|B)$

Formule

$$P(\bar{D}|A_1) = \frac{P(\bar{D} \cap A_1)}{P(A_1)}$$

$$P(A_1|\bar{D}) = \frac{P(\bar{D} \cap A_1)}{P(\bar{D})}$$

$$= \frac{P(\bar{D}|A_1) \cdot P(A_1) + P(\bar{D}|A_2) \cdot P(A_2)}{1 - P(D)} =$$

$$= \frac{(1 - P(D|A_1)) P(A_1) + (1 - P(D|A_2)) \cdot P(A_2)}{1 - P(D)} =$$

$$= \frac{\frac{98}{100} \cdot \frac{3}{10} + \frac{96}{100} \cdot \frac{5}{10}}{0{,}964} = \frac{98 \cdot 3 + 96 \cdot 5}{964} = \frac{294 + 480}{964} = 0{,}802$$