Căruțaș Adriana Ștefania
Grupa 243

Tema seminar 4

① a) $X = \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$; $Y = \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$;

$\rightarrow 3X = \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$; $\rightarrow X^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$;

$\rightarrow \cos\left(\frac{\pi}{2}X\right) = \begin{pmatrix} \cos\left(\frac{\pi}{2}\cdot 2\right) & \cos\left(\frac{\pi}{2}\cdot 3\right) \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} \Rightarrow \cos\left(\frac{\pi}{2}X\right) = \begin{pmatrix} -1 & 0 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$

$\rightarrow Y^2 = \begin{pmatrix} 9 & 4 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$; $\rightarrow Y+3 = \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

b) $X = \begin{pmatrix} 5 & 8 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$; $Y = \begin{pmatrix} -1 & 1 \\ \frac{1}{6} & \frac{5}{6} \end{pmatrix}$

$\rightarrow 2X = \begin{pmatrix} 10 & 16 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$; $\rightarrow X^{-3} = \begin{pmatrix} \frac{1}{125} & \frac{1}{5^2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$;

$\rightarrow tg(\pi X) = \begin{pmatrix} tg(5\pi) & tg(2\pi) \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \Rightarrow tg(\pi X) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\rightarrow Y-2 = \begin{pmatrix} -3 & -1 \\ \frac{1}{6} & \frac{5}{6} \end{pmatrix}$; $\rightarrow \|\| = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) $X \cdot \begin{pmatrix} 0 & 9 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$; $Y \cdot \begin{pmatrix} -3 & 1 \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

$X-1 \cdot \begin{pmatrix} -1 & 8 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$; X^{-2} nu se poate calcula

$\sin\left(\frac{\pi}{4} X\right) \cdot \begin{pmatrix} \sin\frac{\pi}{4}\cdot 0 & \sin\frac{9\pi}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \Rightarrow \sin\left(\frac{\pi}{4} X\right) \cdot \begin{pmatrix} 1 & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$;

$Y \cdot 5 \cdot \begin{pmatrix} -15 & 5 \\ \frac{1}{4} & \frac{9}{4} \end{pmatrix}$; $e^Y \cdot \begin{pmatrix} \frac{1}{e^3} & e \\ \frac{1}{4} & \frac{9}{4} \end{pmatrix}$

d) $\rightarrow X \cdot \begin{pmatrix} -3 & 6 \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$; $Y \cdot \begin{pmatrix} e & e^3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

$\rightarrow 2 - X \cdot \begin{pmatrix} 5 & -4 \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$; $\rightarrow X^3 \cdot \begin{pmatrix} -27 & 216 \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\rightarrow \cos\left(\frac{\pi}{6} \cdot X\right) \cdot \begin{pmatrix} \cos\left(-\frac{\pi}{2}\right) & \cos\frac{\pi}{4} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix} \Rightarrow \cos\left(\frac{\pi}{6} \cdot X\right) \cdot \begin{pmatrix} 0 & -1 \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$;

$\rightarrow Y \cdot 1 \begin{pmatrix} \frac{1}{e} & \frac{1}{e^3} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$; $\rightarrow \ln Y \cdot \begin{pmatrix} \ln e & \ln e^3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \Rightarrow \ln Y \cdot \begin{pmatrix} 1 & 3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

2) a) $\rightarrow 2X + 3Y \cdot \begin{pmatrix} 4 & 6 \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} + \begin{pmatrix} -9 & -6 \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} \cdot \begin{pmatrix} -5 & -2 & -3 & 0 \\ \frac{1}{25} & \frac{1}{25} & \frac{16}{25} & \frac{1}{25} \end{pmatrix}$

$: \begin{pmatrix} -5 & -3 & -2 & 0 \\ \frac{1}{25} & \frac{16}{25} & \frac{1}{25} & \frac{1}{25} \end{pmatrix}$

$\rightarrow 3X - Y \cdot \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} - \begin{pmatrix} -3 & -2 \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} \cdot \begin{pmatrix} 9 & 8 & 12 & 11 \\ \frac{1}{25} & \frac{1}{25} & \frac{16}{25} & \frac{1}{25} \end{pmatrix}$

$: \begin{pmatrix} 8 & 9 & 11 & 12 \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{5} & \frac{16}{25} \end{pmatrix}$

$\rightarrow X^2 \cdot Y^3 : \begin{pmatrix} 4 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} \cdot \begin{pmatrix} -27 & -8 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} : \begin{pmatrix} -108 & 32 & 243 & 72 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$

$: \begin{pmatrix} -108 & 32 & 72 & 243 \\ \frac{4}{25} & \frac{1}{25} & \frac{4}{25} & \frac{16}{25} \end{pmatrix}$

b) $\rightarrow X - Y : \begin{pmatrix} 0 & 9 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} - \begin{pmatrix} -3 & 1 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix} : \begin{pmatrix} 3 & -1 & 12 & 8 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix}$

$: \begin{pmatrix} -1 & 3 & 8 & 12 \\ \frac{6}{14} & \frac{1}{14} & \frac{6}{14} & \frac{1}{14} \end{pmatrix}$

$\rightarrow X \cdot Y : \begin{pmatrix} 0 & 9 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} -3 & 1 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix} : \begin{pmatrix} 0 & 0 & -27 & 9 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix} : \begin{pmatrix} -27 & 0 & 9 \\ \frac{1}{14} & \frac{7}{14} & \frac{6}{14} \end{pmatrix}$

$\cos(\pi \cdot XY) : \begin{pmatrix} \cos(-27\pi) & \cos 0\pi & \cos 9\pi \\ \frac{1}{14} & \frac{7}{14} & \frac{6}{14} \end{pmatrix} : \begin{pmatrix} -1 & 1 & -1 \\ \frac{1}{14} & \frac{7}{14} & \frac{6}{14} \end{pmatrix}$

$: \begin{pmatrix} -1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\rightarrow X^2 + 3Y : \begin{pmatrix} 0 & 81 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} -9 & 3 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix} : \begin{pmatrix} -9 & 3 & 72 & 84 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix}$

c) $\rightarrow X + Y : \begin{pmatrix} 5 & 8 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ \frac{1}{6} & \frac{5}{6} \end{pmatrix} : \begin{pmatrix} 4 & 6 & 7 & 9 \\ \frac{1}{2} & \frac{5}{6} & \frac{2}{3} & \frac{10}{6} \end{pmatrix}$

$\rightarrow X \cdot Y : \begin{pmatrix} 5 & 8 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ \frac{1}{6} & \frac{5}{6} \end{pmatrix} : \begin{pmatrix} -5 & 5 & 8 & 10 \\ \frac{1}{9} & \frac{5}{9} & \frac{2}{9} & \frac{10}{9} \end{pmatrix} : \begin{pmatrix} -8 & -5 & 5 & 8 \\ \frac{2}{9} & \frac{1}{9} & \frac{5}{9} & \frac{10}{9} \end{pmatrix}$

$\sin\left(\frac{\pi}{2} \cdot XY\right) : \begin{pmatrix} \sin\left(-\frac{8\pi}{2}\right) & \sin\left(-\frac{5\pi}{2}\right) & \sin\frac{5\pi}{2} & \sin\frac{8\pi}{2} \\ \frac{2}{9} & \frac{1}{9} & \frac{5}{9} & \frac{10}{9} \end{pmatrix}$

$: \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{18} & \frac{12}{18} & \frac{5}{18} \end{pmatrix}$

$$\rightarrow \frac{1}{X} + \frac{1}{Y} : \begin{pmatrix} \frac{1}{5} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ \frac{1}{6} & \frac{5}{6} \end{pmatrix} : \begin{pmatrix} -\frac{4}{5} & \frac{6}{5} & -\frac{7}{18} & \frac{9}{5} \\ \frac{1}{18} & \frac{5}{18} & \frac{1}{18} & \frac{5}{18} \end{pmatrix}$$

$$: \begin{pmatrix} -\frac{7}{18} & -\frac{4}{5} & \frac{6}{5} & \frac{9}{5} \\ \frac{2}{18} & \frac{1}{18} & \frac{5}{18} & \frac{10}{18} \end{pmatrix}$$

d) $\rightarrow X \cdot Y : \begin{pmatrix} -3 & 6 \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix} \begin{pmatrix} e & e^3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} : \begin{pmatrix} -3e & -3e^3 & 6e & 6e^3 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$

$$: \begin{pmatrix} -3e^3 & -3e & 6e & 6e^3 \\ \frac{3}{32} & \frac{1}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$\rightarrow \frac{X}{Y} : \begin{pmatrix} -3 & 6 \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{e} & \frac{1}{e^3} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} : \begin{pmatrix} -\frac{3}{e} & -\frac{3}{e^3} & \frac{6}{e^3} & \frac{6}{e} \\ \frac{1}{32} & \frac{3}{32} & \frac{21}{32} & \frac{7}{32} \end{pmatrix}$

$\rightarrow |X - Y^2| : \left| \begin{pmatrix} -3 & 6 \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix} - \begin{pmatrix} e^2 & e^6 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \right| : \left| \begin{pmatrix} -3-e^2 & -3-e^6 & 6-e^2 & 6-e^6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix} \right|$

$$: \begin{pmatrix} 3+e^2 & 3+e^6 & 6-e^2 & e^6-6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$: \begin{pmatrix} 6-e^2 & e^2+3 & e^6-6 & 3+e^6 \\ \frac{7}{32} & \frac{1}{32} & \frac{21}{32} & \frac{3}{32} \end{pmatrix}$$

③ $p, q \in \mathbb{R}$

$\rightarrow X : \begin{pmatrix} 1 & 2 \\ p & q \end{pmatrix} ; Y : \begin{pmatrix} 3 & 9 \\ 0,1 & \frac{p^2 + 0,02}{0,2} \end{pmatrix}$

$$0,1 + \frac{p^2 + 0,02}{0,2} = 1 \Rightarrow 0,02 + p^2 + 0,09 = 0,2$$

$$\Rightarrow p^2 + 0,04 = 0,20$$

$$\Rightarrow p^2 = 0,16 \Rightarrow p = 0,4$$

$p + q = 1 \Rightarrow q = 1 - p \Rightarrow q = 1 - 0,4 = 0,6$

$p = 0,4$
$q = 0,6$

b) $X: \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{3} & p & q^2 \end{pmatrix}$; $X^2 \begin{pmatrix} 1 & 4 & 9 \\ p & p & p^2 \end{pmatrix}$

$p = \frac{1}{3}$

$\frac{1}{3} + \frac{1}{3} + q^2 = 1 \Rightarrow q^2 = 1 - \frac{2}{3} \Rightarrow q^2 = \frac{1}{3} \Rightarrow q_1 = \frac{1}{\sqrt{3}}$

Dar $q^2 = p^2 \Rightarrow |q| = |p| \Rightarrow q_2 = p = \frac{1}{3}$

$\Rightarrow q_1 \neq q_2$

$\Rightarrow$ nu există v.a. X

c) $X: \begin{pmatrix} -1 & 0 & 1 \\ p & p^2 & q \end{pmatrix}$ $X^4: \begin{pmatrix} 0 & 1 \\ \frac{9}{25} & \frac{16}{25} \end{pmatrix}$

$p^2 = \frac{9}{25} \Rightarrow p = \frac{3}{5}$

$p + q = 1 - \frac{9}{25} \Rightarrow p + q = \frac{16}{25} \Rightarrow q = \frac{16}{25} - \frac{3}{5} = \frac{16}{25} - \frac{15}{25} = \frac{1}{25}$

$p = \frac{15}{25}$

$q = \frac{1}{25}$

d) $X: \begin{pmatrix} -1 & 1 \\ 2p & q \end{pmatrix}$; $Y: \begin{pmatrix} 0 & 1 \\ q & 7q \end{pmatrix}$

$8q = 1 \Rightarrow q = \frac{1}{8}$

$2p + \frac{1}{8} = 1 \Rightarrow 2p = \frac{8-1}{8} = \frac{7}{8} \Rightarrow p = \frac{7}{16}$

$p = \frac{7}{16}$

$q = \frac{1}{8}$

④ a) $\rightarrow P(2X+3Y>1) = 0$

$\rightarrow P(2X+3Y>1 \mid X>0) = \frac{P((2X+3Y>1) \cap (X>0))}{P(X>0)} = \frac{0}{1} = 0$

$\rightarrow P(2X+3Y<3 \mid Y<-2) = \frac{P((2X+3Y<3) \cap (Y<-2))}{P(Y<-2)} = 1$

$\rightarrow P(X^2 \cdot Y^3 > 3) = \frac{1}{25} + \frac{4}{25} + \frac{16}{25} = \frac{21}{25}$

$\rightarrow P(X^2 \cdot Y^3 \leq 3) = \frac{4}{25}$

$\rightarrow P(2X+3Y<3X-Y) = 1$

b) $\rightarrow P(X-Y>0) = \frac{1}{14} + \frac{6}{14} + \frac{1}{14} = \frac{8}{14} = \frac{4}{7}$

$\rightarrow P(X-Y<0 \mid X>0) = \frac{P((X-Y<0) \cap (X>0))}{P(X>0)} = \frac{0}{\frac{1}{2}} = 0$

$\rightarrow P(X-Y>0 \mid Y \leq 0) = \frac{P((X-Y>0) \cap (Y \leq 0))}{P(Y \leq 0)} = \frac{\frac{1}{14} + \frac{1}{14}}{\frac{1}{7}} = 1$

$\rightarrow P(\cos(\pi XY) < \frac{1}{2}) = \frac{1}{2}$

$\rightarrow P(X^2+3Y \geq 3) = \frac{6}{14} + \frac{1}{14} + \frac{6}{14} = \frac{13}{14}$

$\rightarrow P(X-Y < X^2+3Y) = (P(X-Y<-9) \cup P(X^2+3Y=-9)) \cap (P(X-Y<3) \cap P(X^2+3Y=3)) \cap (P(X-Y<12) \cup P(X^2+3Y=12)) \cap (P(X-Y<84) \cap P(X^2+3Y=84))$

$= 0 \cdot \frac{1}{14} + \frac{6}{14} \cdot \frac{6}{14} + \frac{14}{14} \cdot \frac{1}{14} + 1 \cdot \frac{6}{14}$

$= \left(\frac{6}{14}\right)^2 + \frac{7}{14}$

$= \frac{36+98}{14^2} = \frac{134}{196} = \frac{67}{98}$

c) $\to P(X+Y<2) = 0$

$\to P(X+Y>2 \mid X>5) = \frac{P((X+Y>2) \cap (X>5))}{P(X>5)} = 1$

$\to P(X+Y<12 \mid Y<0) = \frac{P((X+Y<12) \cap (Y<0))}{P(Y<0)} = 1$

$\to P(\sin(\frac{\pi}{2} \cdot XY) \le \frac{1}{2}) = \frac{13}{18}$

$\to P(\frac{1}{X} + \frac{1}{Y} < 1 \mid Y<0) = \frac{P((\frac{1}{X}+\frac{1}{Y}<1) \cap (Y<0))}{P(Y<0)} = \frac{\frac{3}{18}}{\frac{1}{6}} = 1$

$\to P(\frac{1}{X} + \frac{1}{Y} < X+Y) = (P(\frac{1}{X}+\frac{1}{Y}<4) \cup P(X+Y=4)) \cap (P(\frac{1}{X}+\frac{1}{Y}<6) \cup P(X+Y=6)) \cap (P(\frac{1}{X}+\frac{1}{Y})<7) \cup P(X+Y=7)) \cap (P(\frac{1}{X}+\frac{1}{Y}<9) \cup P(X+Y=9))$

$= 1 \cdot \frac{1}{18} + 1 \cdot \frac{5}{18} + 1 \cdot \frac{2}{18} + 1 \cdot \frac{10}{18} = 1$

d) $\to P(X \cdot Y \le e^4) = \frac{11}{32}$

$\to P(X \cdot Y > 7 \mid X<0) = \frac{P((X \cdot Y > 7) \cap (X<0))}{P(X<0)} = \frac{0}{\frac{1}{8}} = 0$

$\to P(X \cdot Y < 9 \mid Y>3) = \frac{P((X \cdot Y<9) \cap (Y>3))}{P(Y>3)} = \frac{\frac{3}{32}}{\frac{3}{4}} = \frac{1}{8}$

$\to P(\frac{X}{Y} < 1) = \frac{25}{32}$

$\to P(|X-Y^2| > 3) = \frac{25}{32}$

$\to P(\frac{X}{Y} < |X^2-Y^2|) = (P(\frac{X}{Y} < 6-e^2) \cap P(|X^2-Y^2| = 6-e^2)) \cup (P(\frac{X}{Y} < 3+e^2) \cap P(|X^2-Y^2| = 3+e^2)) \cup (P(\frac{X}{Y} < e^6-6) \cap P(|X^2-Y^2| = e^6-6)) \cup (P(\frac{X}{Y} < 3+e^6) \cap P(|X^2-Y^2| = 3+e^6)) =$

$= 0 \cdot \frac{1}{32} + 1 \cdot \frac{1}{32} + 1 \cdot \frac{21}{32} + 1 \cdot \frac{3}{32}$

$= \frac{25}{32}$