## 2.2

$N_f = ?$ $N_f$ number of traffic flows

All-to-all traffic matrix among servers
All the servers are connected.
So, we need to consider all flows source-dest. between the couple of servers.
In this case, we can use to calculate $N_f$ the binomial coefficient $\binom{a}{b}$, where a means all the servers, which are $N = m^3/4$ and $b = 2$, because are two servers to be connected, i.e. all the possible combination of 2 among N servers.

$$N_f = \binom{N}{2} = \frac{N(N-1)}{2} = \frac{m^3}{4} \cdot \left(\frac{m^3}{4} - 1\right) \cdot \frac{1}{2} = \frac{m^3}{8} \cdot \frac{m^3 - 4}{4} =$$

$$= \boxed{\frac{m^3(m^3-4)}{32}}$$

2.3

Exact value of $\bar{h}$ (FOR FAT-TREE)

$$\bar{h} = \frac{1}{N_f} \sum_{i=1}^{N_f} h_i =$$

CONSIDERIAMO SERVER

$$\underbrace{2 \cdot \left( ! \right)}_{\text{STESSO SWITCH}} + \underbrace{4 \left( ? \right)}_{\text{STESSO POD}} + \underbrace{6 \left( ! \right)}_{\text{POD DIVERSI}}$$

$\left(\frac{n}{2}\right)^2$ core switches

$$\left(\frac{n}{2}\right)$$

$$2 \cdot \left[\frac{n^3}{8}\left(\frac{n}{2} - 1\right)\right] \Rightarrow \frac{n^3}{4} \cdot \frac{1}{2}\left(\frac{n}{2} - 1\right)$$

$$2\left[\left(\frac{m}{2}-1\right)\left(\frac{m^3}{4}\cdot\frac{1}{2}\right)\right]+4\left[\left(\frac{m}{2}-1\right)\left(\frac{m^3}{8}\cdot\frac{m}{2}\right)\right]+6\left[\left(m-1\right)\left(\frac{m^3}{8}\cdot\frac{m^2}{4}\right)\right]$$

## FOR servers connected by same switch

$\boxed{2}$ flows for each path

$\boxed{\frac{m}{2}-1}$ ports of switch for new path (m/2 ports connected to the path interested, but minus 1, bc is the one of the server)

$\boxed{\frac{m^3}{4}\cdot\frac{1}{2}}$ How many different paths from A to B with no duplicates, i.e. only one path from A to B, so not to count the path from B to A, which is the same

$$2\cdot\left(\frac{m}{2}-1\right)\left(\frac{m^3}{4}\cdot\frac{1}{2}\right)$$

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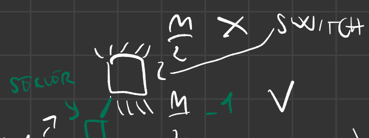
## For servers connected by aggregation layer switch, so same pod




$\boxed{4}$ flows for each path

$\boxed{\frac{m}{2}-1}$ ports of switch for new path (same as above)

$\boxed{\frac{m^3}{4}\cdot\frac{1}{2}\cdot\frac{m}{2}}$ where $\frac{m^3}{4}\cdot\frac{1}{2}$ is the same motivation as above, and $m/2$ means how many aggregation layer switch are in a pod

$$4\cdot\left(\frac{m}{2}-1\right)\left(\frac{m^3}{8}\cdot\frac{m}{2}\right) \qquad 4\cdot1\cdot2\cdot\frac{\cancel{64}^{8}}{\cancel{8}_{1}}=16$$

## For server connected by core layer switch

$\boxed{6}$ flows for each path

$\boxed{m-1}$ ports of switch for new path (for core layer switch, m ports total, minus 1, which is connected to the initial server)

$\boxed{\frac{m^3}{4}\cdot\frac{1}{2}\cdot\frac{m^2}{4}}$ $\frac{m^3}{4}\cdot\frac{1}{2}$ same as previous ones, and $\frac{m^2}{4}$ means the number of core layer switch

$$6\cdot(m-1)\left(\frac{m^3}{8}\cdot\frac{m^2}{4}\right)$$

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## FINAL $\bar{h}$

$$\bar{h} = \frac{1}{N_f}\cdot\sum_{i=1}^{N_f} h_i = \frac{1}{N_f}\left\{2\cdot\left[\left(\frac{m}{2}-1\right)\cdot\frac{m^3}{8}\right] + 4\left[\left(\frac{m}{2}-1\right)\left(\frac{m^3}{8}\cdot\frac{m}{2}\right)\right] + 6\left[(m-1)\left(\frac{m^3}{8}\cdot\frac{m^2}{4}\right)\right]\right\}$$

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$$\left(\frac{m}{2}-1\right)\cdot\frac{m^3}{8} + \left(\frac{m}{2}-1\right)\frac{m^4}{16} + (m-1)\left(\frac{m^5}{32}\right) = \cancel{\frac{m^4}{16}} - \frac{m^3}{8} + \cancel{\frac{m^5}{32}} - \cancel{\frac{m^4}{16}} + \frac{m^6-\cancel{m^5}}{32}$$

$$= \frac{m^6-4m^3}{32} = \frac{m^3(m^3-4)}{32}$$

which is equal to $N_f = \frac{m^3(m^3-4)}{32}$