2.3

Exact value of $\bar{h}$ (FOR FAT-TREE)

$$\bar{h} = \frac{1}{N_s} \sum_{i=1}^{N_s} h_i =$$

CONNESSIONE SERVER

$$\underbrace{2\cdot\left(\ \right)}_{\text{STESSO SWITCH}} + \underbrace{4\left(\ \right)}_{\text{STESSO POD}} + \underbrace{6\left(\ \right)}_{\text{POD DIVERSI}}$$

$\left(\frac{m}{2}\right)^2$ core switches

$$\left(\frac{m}{2}\right)$$

$$2\cdot\left[\frac{m^3}{8}\left(\frac{m}{2}-1\right)\right] \Rightarrow \frac{m^3}{4}\cdot\frac{1}{2}\left(\frac{m}{2}-1\right)$$

$$2\left[\left(\frac{m}{2}-1\right)\left(\frac{m^3}{4}\cdot\frac{1}{2}\right)\right]+4\left[\left(\frac{m}{2}-1\right)\left(\frac{m^3}{8}\cdot\frac{m}{2}\right)\right]+6\left[\left(m-1\right)\left(\frac{m^3}{8}\cdot\frac{m^2}{4}\right)\right]$$

FOR servers connected by same switch

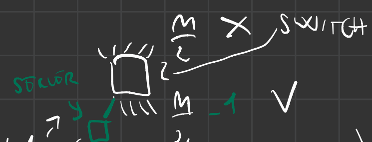
$\boxed{2}$ flows for each path

$\boxed{\frac{m}{2}-1}$ ports of switch for new path (m/2 ports connected to the path interested, but minus 1, bc is the one of the server)

$\boxed{\frac{m^3}{4}\cdot\frac{1}{2}}$ How many different paths from A to B with no duplicates, i.e. only one path from A to B, so not to count the path from B to A, which is the same

$$2\cdot\left(\frac{m}{2}-1\right)\left(\frac{m^3}{4}\cdot\frac{1}{2}\right)$$

For servers connected by aggregation layer switch, so same pod



$\boxed{4}$ flows for each path

$\boxed{\frac{m}{2}-1}$ ports of switch for new path (same as above)

$\boxed{\frac{m^3}{4}\cdot\frac{1}{2}\cdot\frac{m}{2}}$ where $\frac{m^3}{4}\cdot\frac{1}{2}$ is the same motivation as above, and m/2 means how many aggregation layer switch are in a pod

$$4\cdot\left(\frac{m}{2}-1\right)\left(\frac{m^3}{8}\cdot\frac{m}{2}\right) \qquad 4\cdot1\cdot2\cdot\frac{\cancel{64}^{8}}{\cancel{8}_{1}}=16$$

For server connected by core layer switch

$\boxed{6}$ flows for each path

$\boxed{n-1}$ ports of switch for new path (for core layer switch, n ports total, minus 1, which is connected to the initial server)

$\boxed{\frac{n^3}{4} \cdot \frac{1}{2} \cdot \frac{n^2}{4}}$ $\frac{n^3}{4} \cdot \frac{1}{2}$ same as previous ones, and $\frac{n^2}{4}$ means the number of core layer switch

$$6 \cdot (n-1)\left(\frac{n^3}{8} \cdot \frac{n^2}{4}\right)$$

FINAL $\bar{h}$

$$\bar{h} = \frac{1}{N_f} \cdot \sum_{i=1}^{N_f} h_i = \frac{1}{N_f}\left\{2 \cdot \left[\left(\frac{n}{2}-1\right) \cdot \frac{n^3}{8}\right] + 4\left[\left(\frac{n}{2}-1\right)\left(\frac{n^3}{8} \cdot \frac{n}{2}\right)\right]\right.$$
$$\left. + 6\left[(n-1)\left(\frac{n^3}{8} \cdot \frac{n^2}{4}\right)\right]\right\}$$

$$\left(\frac{n}{2}-1\right) \cdot \frac{n^3}{8} + \left(\frac{n}{2}-1\right)\frac{n^4}{16} + (n-1)\left(\frac{n^5}{32}\right) = \cancel{\frac{n^4}{16}} - \frac{n^3}{8} + \cancel{\frac{n^5}{32}} - \cancel{\frac{n^4}{16}} + \frac{n^6 - \cancel{n^5}}{32}$$
$$= \frac{n^6 - 4n^3}{32} = \frac{n^3(n^3-4)}{32}$$

which is equal to $N_f = \frac{n^3(n^3-4)}{32}$