

Chapter 4: 2 (c): For full credit you must write your own version of the secant method. Your function should have the signature

```
function [r,hist] = secant(f,x0,x1,ftol,xtol,Nmax)

end
```

The input values are

- f , the function to find a root of.
- x_0, x_1 the first two iteration values.
- $ftol$, the tolerance for stopping based on the value of f
- $xtol$, the tolerance for stopping based on changes in x
- N_{max} , the maximum number of iterations

Your function should exit with an error if more than N_{max} iterations are used. It should return whenever $|f(x)| < f_{tol}$ or $|x_n - x_{n-1}| < x_{tol}$.

The return values should be r , the estimate of the root's position, and $hist$, a list of all estimates starting with x_0 and x_1 and ending with the final estimate r .

Test that your function works by finding three different ways to call it so that iteration stops for each of the three possible reasons.

To answer the problem in the textbook, you will want to call your function with $x_{tol} = 0$ to ensure that only the f_{tol} condition is used to stop the iteration.

Chapter 4: 3:

Chapter 4: 8:

Chapter 4: 12: For full credit you must use Theorem 4.5.1.

Supplemental 1: Suppose $f(x)$ is a differentiable function on \mathbb{R} and $|f'(x)| \leq 1/2$ for all real numbers. Show that

$$|f(x) - f(y)| \leq \frac{1}{2}|x - y|$$

for all $x, y \in \mathbb{R}$.

Hint: Use Taylor's theorem, the zeroth order version, AKA the Mean Value Theorem. Apply it centered at some point x and then see what the theorem says about $f(y)$.

For context, look at equation (4.22) of the text, which defines a **contraction**. You are showing that if $|f'(x)| \leq 1/2$ for all x then f is a contraction. This is interesting because Theorem 4.5.2 says that every contraction has a unique fixed point, and if you perform fixed point iteration on the contraction then the iterates will converge to the fixed point. You can think of Theorem 4.5.2 as a generalization of Theorem 4.5.1 (you can prove Theorem 4.5.1 directly from the more difficult Theorem 4.5.2).

Chapter 4: 13: You will want to use Theorem 4.5.2!

Chapter 4: 14: