

1. **Exercise 4.28:** Let  $Z$  be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.

- (a)  $P(0 \leq Z \leq 2.17)$
- (b)  $P(0 \leq Z \leq 1)$
- (c)  $P(Z \leq 1.37)$
- (j)  $P(|Z| \leq 2.50)$

**Answer:** Note that for all of the standard normal probabilities we have an appendix that gives us the value of the cdf  $P(Z \leq x) = \phi(x)$  for up to values of  $x$  in the hundredths. Note that,

$$P(a \leq Z \leq b) = \phi(b) - \phi(a).$$

- (a) Calculating using Table A-3,

$$P(0 \leq Z \leq 2.17) = \phi(2.17) - \phi(0) = .9850 - .50 = .4850.$$

- (b) Similarly,

$$P(0 \leq Z \leq 1) = \phi(1) - \phi(0) = .8413 - .50 = .3413.$$

- (c) Similarly,

$$P(Z \leq 1.37) = \phi(1.37) = .9147.$$

- (j) Simplifying the inequality to remove the absolute value,

$$P(|Z| \leq 2.50) = P(-2.50 \leq Z \leq 2.50).$$

Calculating with table A-3,

$$P(-2.50 \leq Z \leq 2.50) = \phi(2.50) - \phi(-2.50) = .9938 - .0062 = .9876$$

2. **Exercise 4.34:** The article “Reliability of DomesticWaste Biofilm Reactors” suggests that substrate concentration ( $mg/cm^3$ ) of influent to a reactor is normally distributed with  $\mu = .30$  and  $\sigma = .06$ .

- (a) What is the probability that the concentration exceeds .50?
- (b) What is the probability that the concentration is at most .20?
- (c) How would you characterize the largest 5% of all concentration values?

**Answer:**

- (a) Note that  $X$  is a non-standard normal distribution, however we can calculate the probability as a standard normal after computing the standardized variable. Computing the standardized variable with,  $X = .50$ ,  $\mu = .30$  and  $\sigma = .06$ ,

$$P(X > .5) = P(Z > \frac{.5 - .30}{.06}) = P(Z > 3.33) = 1 - \phi(3.33) = .0004.$$

- (b) Similarly computing  $P(X < .20)$ ,

$$P(X \leq .20) = P(Z \leq \frac{.2 - .30}{.06}) = P(Z \leq -1.67) = \phi(-1.67) = .475$$

- (c) Recall that we denote the largest 5% of all concentrations values can be denoted as all values greater than the critical value of  $Z_{0.05}$ . Note that our cdf will calculate values up to a certain point, therefore

$$\phi(Z_{0.05}) = 1 - 0.05 = .95.$$

Since we have  $Z_{0.05} = 1.645$  from table A-3 all we have to do is set it equal to the standardizing variable to solve for  $X_{0.05}$ ,

$$\begin{aligned} \frac{X_{0.05} - .30}{.06} &= 1.645 \\ X_{0.05} &= (1.645)(.06) + .3 \\ X_{0.05} &= .3987. \end{aligned}$$

Therefore the top 5% of all concentrations occur when  $X \geq .3987$ .

3. **Exercise 4.50:** In response to concerns about nutritional contents of fast foods, McDonald's has announced that it will use a new cooking oil for its french fries that will decrease substantially trans fatty acid levels and increase the amount of more beneficial polyunsaturated fat. The company claims that 97 out of 100 people cannot detect a difference in taste between the new and old oils. Assuming that this figure is correct (as a long-run proportion), what is the approximate probability that in a random sample of 1000 individuals who have purchased fries at McDonald's,

- (a) At least 40 can taste the difference between the two oils?  
 (b) At most 5% can taste the difference between the two oils?

**Answer:**

- (a)  
 (b)

4. **Exercise 4.38:** There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation .1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

**Answer:**

5. **Exercise 4.42:** The temperature reading from a thermocouple placed in a constant-temperature medium is normally distributed with mean  $\mu$ , the actual temperature of the medium, and standard deviation  $\sigma$ . What would the value of  $\sigma$  have to be to ensure that 95% of all readings are within .18 of  $\mu$ ?

**Answer:**

6. **Exercise 4.60:** Let  $X$  denote the distance (m) that an animal moves from its birth site to the first territorial vacancy it encounters. Suppose that for banner-tailed kangaroo rats,  $X$  has an exponential distribution with parameter  $\lambda = .01386$ ,
- (a) What is the probability that the distance is at most 100 m? At most 200 m? Between 100 and 200 m?
  - (b) What is the probability that distance exceeds the mean distance by more than 2 standard deviations?
  - (c) What is the value of the median distance?

**Answer:**

- (a)
- (b)
- (c)