Exercise 1.2.5: Use the triangle inequality to establish the following inequalities:

(a) $|a - b| \leq |a| + |b|$;

(b) $||a| - |b|| \leq |a - b|$.

Proof. □

Exercise 1.2.6(b), (d): Given a function f and a subset A of its domain, let $f(A)$ represent the range of f over the set A; that is, $f(a) = \{f(x) : x \in A\}$.

(b) Find two sets A and B for which $f(A \cap B) \neq f(A) \cap f(B)$.

(d) Form and prove a conjecture concerning $f(A \cup B)$ and $f(A) \cup f(B)$.

Proof (b). □

Proof (d). □

Exercise 1.2.8: Form the logical negation of each claim. Do not use the easy way out: ”It is not the case that...” is not permitted

(a) For all real numbers satisfying $a < b$, there exists $n \in \mathbb{N}$ such that $a + (1/n) < b$.

(b) Between every two distinct real numbers there is a rational number.

(c) For all natural numbers $n \in \mathbb{N}$, $\sqrt{n}$ is either a natural number or is an irrational number.

(d) Given any real number $x \in \mathbb{R}$ there exists $n \in \mathbb{N}$ satisfying $n > x$.

Solution:

(a)

(b)

(c)

(d)

Exercise 1.2.9: Show that the sequence $(x_1, x_2, x_3, \ldots)$ defined in Example 1.2.7 is bounded above by 2. That is, show that for every $i \in \mathbb{N}$, $x_i \leq 2$.

Proof. □

Exercise 1.3.4: Assume that A and B are nonempty, bounded above, and satisfy $B \subseteq A$. Show that $\sup B \leq \sup A$.

Proof. □

Exercise 1.3.5: Let A be bounded above and let $c \in \mathbb{R}$. Define the sets $c + A = \{a + c : a \in A\}$ and $cA = \{ca : a \in A\}$.

(a) Show that $\sup(c + A) = c + \sup(A)$.

(b) If $c \geq 0$, show that $\sup(cA) = c \sup(A)$.

(c) Postulate a similar statment for $\sup(cA)$ when $c < 0$.

Proof (a). □

Proof (b). □

Statement for part (c):

Exercise 1.3.6: Compute, without proof, the suprema and infima of the following sets.

(a) $\{n \in \mathbb{N} : n^2 < 10\}$.

(b) $\{n/(n + m) : n, m \in \mathbb{N}\}$.

(c) $\{n/(2n + 1) : n \in \mathbb{N}\}$.

(d) $\{n/m : m, n \in \mathbb{N} \text{ with } m + n \leq 10\}$.

Solution:

(a)

(b)

(c)

(d)

Exercise 1.3.7: Prove that if a is an upper bound for A an dif a is also an element of A, then $a = \sup A$.

Proof. □

Exercise 1.3.8: If $\sup A < \sup B$ then show that there exists an element $b \in B$ that is an upper bound for A.

Proof. □