Exercise 4.68: "The special case..." (Note that this problem is just a special case of the Gamma distribution, so you can do parts a, b using the Gamma distribution. For part (b) you can use R, then pgamma(30,shape=alpha, scale=beta).

1. What is the expected value of X? If the time (in minutes) between arrivals of successive customers is exponentially distributed with $\lambda = 5.5$, how much time can be expected to elapse before the tenth customer arrives?

   Solution:
   Since the Erlang distribution is a special case of the gamma, and we know the expected value for the gamma we simply plug in, $X \sim Gamma(10, .5)$,

   $$E(x) = \alpha\beta = n\frac{1}{\lambda} = 10(2) = 20.$$

2. If customer interarrival time is exponentially distributed with $\lambda = .5$, what is the probability that the tenth customer (after the one who has just arrived) will arrive within the next 30 min?

   Solution:
   Simply computing (with R) the $P(X \leq 30)$ where $X \sim Exp(10, .5)$,

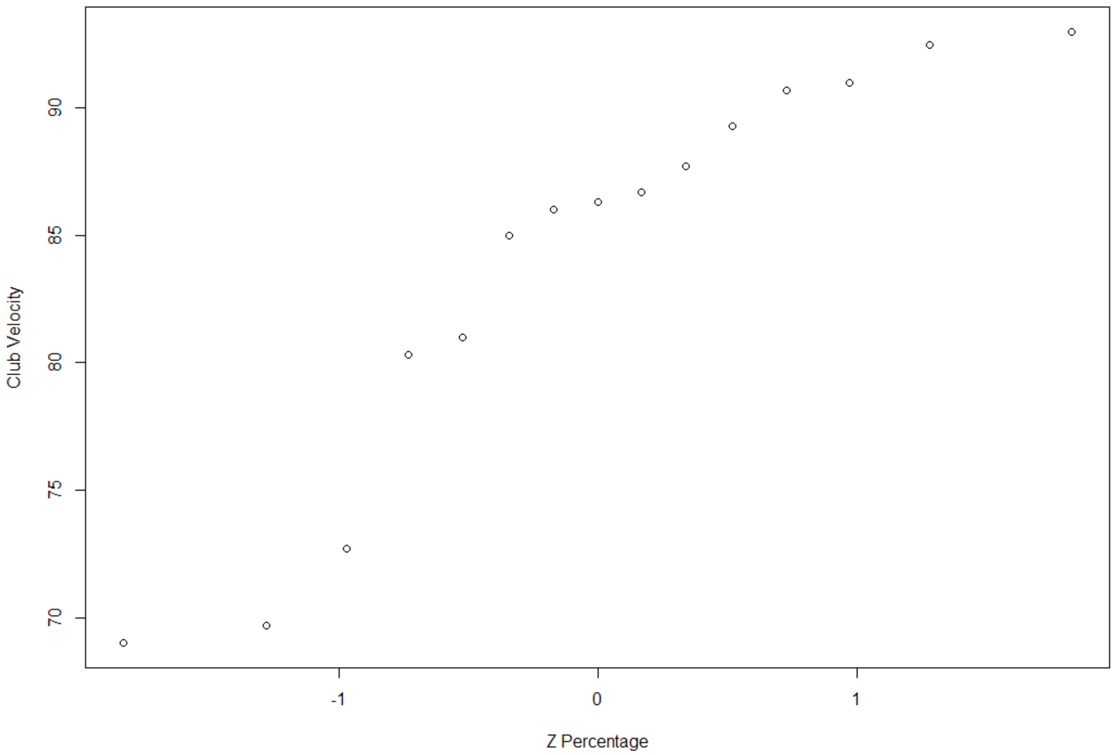
   $$P(X \leq 30) = 0.9301463.$$

Exercise Supplimental 1: We have a data set consisting of grass cover in n = 10 plots: c(0.34, 0.3, 0.9, 0.89, 0.34, 0.72, 0.61, 0.84, 0.17, 0.41). Of the distributions discussed in section 4.6 (Weibull, Lognormal, Beta), which is probably a better choice of model for this data? Why?

Solution:
Since the grass coverage is measured as a percentage, ie (0 - 1) it's most definitely going to be a beta distribution that best models the data. also in most cases id imagine that grass coverage is fairly uniform (depending on the season of course) where the beta function gives the flexibility to model something close to the uniform.

Exercise 4.88: (This technique is a good way to check the normality of a data set)

Solution:
Using *R* to plot the probability plot,



Looking at the plot we can see some slight skew, especially in the lower percentiles, however in large part the majority of the data looks to be normally distributed.

Exercise 5.4: Return to the situation...

1. Determine the marginal pmf of X_1, and then calculate the expected number of customers in line at the express checkout.

   Solution:
   Fist we proceed by calculating the marginal pmf for X_1 by summing over the values

of the table, row wise,

$$\begin{pmatrix} p_{X_1}(0) & .19 \\ p_{X_1}(1) & .3 \\ p_{X_1}(2) & .25 \\ p_{X_1}(3) & .14 \\ p_{X_1}(4) & .12 \\ Otherwise & 0 \end{pmatrix}.$$

Now we compute the expected value like any other discrete random variable (here it is as a dot product),

$$E(X_1) = \begin{pmatrix} .19 \\ .3 \\ .25 \\ .14 \\ .12 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 1.7$$

2. Determine the marginal pmf of X_2

Solution:
Calculating the marginal pmf for X_2 by summing over the values of the table, column wise,

$$\begin{pmatrix} p_{X_2}(0) & p_{X_2}(1) & p_{X_2}(2) & p_{X_2}(3) & Otherwise \\ .19 & .3 & .28 & .23 & 0 \end{pmatrix}$$

3. By inspection of the probabilities $P(X_1 = 4)$, $P(X_2 = 0)$, and $P(X_1 = 4, X_2 = 0)$, are X_1 and X_2 independent random variables? Explain.

Solution:
Note that $P(X_1 = 4) = p_{X_1}(4) = .12$ and that $P(X_2 = 0) = p_{X_2}(0) = .19$. Consider that when variable are independent $P(X_1, X_2) = P(X_1)P(X_2)$ thus since,

$$p(4, 0) = 0 \neq .12(.19) = p_{X_1}(4)P_{X_2}(0).$$

Thus the variables are dependent.

Exercise 5.10: "Annie and Alvie..."

1. What is the joint pdf of X and Y?

   Solution:
   Since the variables X and Y are stated to be independent we know that, $f(x, y) = f_x(x)f_y(y)$. We also know that the marginal pdfs are uniform overthe support $5 \leq x, y \leq 6$ which is an interval size 1. Thus,

$$f(x, y) = \begin{cases} 1 & 5 \leq x, y \leq 6 \\ 0 & Otherwise \end{cases}.$$

2. What is the probability that they both arrive between 5:15 and 5:45?

   Solution:
   First recognize that 15 minutes is a quarter of an hour, and it follows that 45 minutes is 3 quarters of an hour, thus what we mean to solve is $P(5.25 \leq X, Y \leq 5.75)$. Since the variable are independent we can calculate the joint probability by multiplying together the marginal probability,

$$P(5.25 \leq X \leq 5.45) = \int_{5.25}^{5.45} 1dx = .5$$

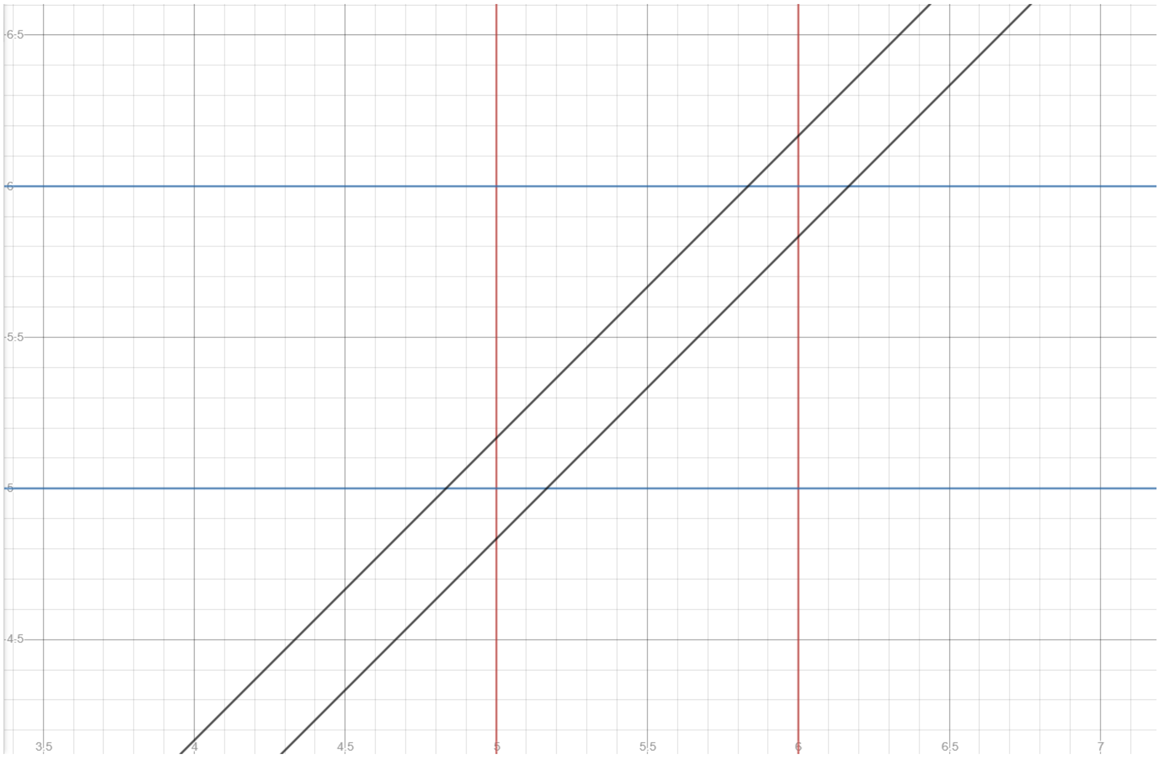
$$P(5.25 \leq Y \leq 5.45) = \int_{5.25}^{5.45} 1dy = .5$$

   Thus we get the following,

$$P(5.25 \leq X, Y \leq 5.75) = .5(.5) = .25$$

3. Integrate the joint pdf over this area $A = \{(x, y) : |x - y| \leq 1/6\}$

   Solution:
   First sketching area that we are going to integrate over, note that in this sketch its the area between the black lines contained by the support of the joint pdf,



Taking advantage of symmetry and compliments we can calculate the probability,

$$P((X, Y) \in A) = P((X, Y)) - P((X, Y) \notin A).$$

Consider the following double integral,

$$\int_5^6 \int_5^6 1dydx - 2\left(\int_{5+\frac{1}{6}}^6 \int_5^{x-\frac{1}{6}} 1dydx\right) = \frac{11}{36}$$

The first part integrates over the whole support and the second integrates over the values $(x, y) \notin A$.