**Exercise Abbott 4.3.9:** Assume $h : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and let $k = \{x : h(x) = 0\}$. Show that $k$ is a closed set.

*Proof.* Suppose $h : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and $k = \{x : h(x) = 0\}$. Let $x$ be a limit point of $k$, by Theorem 3.2.5 there exists a sequence $(a_n) \in k$ such that $lim a_n = x$ where $a_n \neq x$ for all $n \in \mathbb{N}$. By Theorem 4.3.2 (iii) since $h$ is continoues for all $(a_n) \to x$ it follows that $h(a_n) \to h(x)$. Note that since $a_n \in k$ we know that $h(a_n) = 0$ for all $n$ and therefore we know that $h(x) = 0$ an thus by definition we get that $x \in k$. Thus $k$ contains all its limit points and is therefore closed. □

**Exercise Supplemental 1:** a) Show that a continuous function on all of $\mathbb{R}$ that equals zero on the rational numbers must be the zero function

*Proof.* Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuos and that for all $q \in \mathbb{Q}$ we know that $f(q) = 0$. By Theorem 3.2.10 for every $x \in \mathbb{R}$ there exists a sequence $(q_n) \in \mathbb{Q}$ such that $(q_n) \to x$. By the continuity of $f$ we know that $f(q_n) \to f(x)$, since all $q_n \in \mathbb{Q}$ by definitinon of $f$ we know that $f(q_n) = 0$ and thus $f(x) = 0$ for all $x \in \mathbb{R}$.

f(x) = 0 for all x $\in R$ □

b) Suppose $f$ and $g$ are two continuous functions on the real numbers. Is it true that if $f(q) = g(q)$ for all $q \in \mathbb{Q}$, then $f$ and $g$ are the same function?

**Exercise Supplemental 2:** Suppose $K \subseteq \mathbb{R}$ is compact. Show that there exists $x_M \in K$ such that $x_M \geq x$ for all $x \in K$. Then, with very little work, show that there exists $x_m \in K$ such that $x_m \leq x$ for all $x \in K$.

**Exercise Abbott 4.3.7(a):**

**Exercise Abbott 4.4.6:**

**Exercise Abbott 4.4.9:**