1. **Exercise 4.28:** Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.

   (a) $P(0 \leq Z \leq 2.17)$

   (b) $P(0 \leq Z \leq 1)$

   (e) $P(Z \leq 1.37)$

   (j) $P(|Z| \leq 2.50)$

   Answer: Note that for all of the standard normal probabilities we have an appendix that gives us the value of the cdf $P(Z \leq x) = \phi(x)$ for up to values of x in the hundredths. Note that,
$$P(a \leq Z \leq b) = \phi(b) - \phi(a).$$

   (a) Calculating using Table A-3,
$$P(0 \leq Z \leq 2.17) = \phi(2.17) - \phi(0) = .9850 - .50 = .4850.$$

   (b) Similarly,
$$P(0 \leq Z \leq 1) = \phi(1) - \phi(0) = .8413 - .50 = .3413.$$

   (e) Similarly,
$$P(Z \leq 1.37) = \phi(1.37) = .9147.$$

   (j) Simplifying the inequality to remove the absolute value,
$$P(|Z| \leq 2.50) = P(-2.50 \leq Z \leq 2.50).$$
   Calculating with table A-3,
$$P(-2.50 \leq Z \leq 2.50) = \phi(2.50) - \phi(-2.50) = .9938 - .0062 = .9876$$

2. **Exercise 4.34:** The article "Reliability of DomesticWaste Biofilm Reactors" suggests that substrate concentration (mg/cm^3) of influent to a reactor is normally distributed with $\mu = .30$ and $\sigma = .06$.

   (a) What is the probability that the concentration exceeds .50?

   (b) What is the probability that the concentration is at most .20?

   (c) How would you characterize the largest 5% of all concentration values?

   Answer:

(a) Note that X is a non-standard normal distribution, however we can calculate the probability as a standard normal after computing the standardized variable. Computing the standardized variable with, $X = .50$, $\mu = .30$ and $\sigma = .06$,

$$P(X > .5) = P(Z > \frac{.5 - .30}{.06}) = P(Z > 3.33) = 1 - \phi(3.33) = .0004.$$

(b) Similarly computing $P(X < .20)$,

$$P(X \leq .20) = P(Z \leq \frac{.2 - .30}{.06}) = P(Z \leq -1.67) = \phi(-1.67) = .475$$

(c) Recall that we denote the largest 5% of all concentrations values can be denoted as all values greater than the critical value of $Z_{.05}$. Note that our cdf will calculates values up to a certain point, therefore

$$\phi(Z_{0.05}) = 1 - 0.05 = .95.$$

Since we have $Z_{0.05} = 1.645$ from table A-3 all we have to do is set it equal to the standardizing variable to solve for $X_{0.05}$,

$$\begin{aligned} \frac{X_{0.05} - .30}{.06} &= 1.645 \\ X_{0.05} &= (1.645)(.06) + .3 \\ X_{0.05} &= .3987. \end{aligned}$$

Therefore the top 5% of all concentrations occur when $X \geq .3987$.

3. **Exercise 4.50:** In response to concerns about nutritional contents of fast foods, McDonald's has announced that it will use a new cooking oil for its french fries that will decrease substantially trans fatty acid levels and increase the amount of more beneficial polyunsaturated fat. The company claims that 97 out of 100 people cannot detect a difference in taste between the new and old oils. Assuming that this figure is correct (as a long-run proportion), what is the approximate probability that in a random sample of 1000 individuals who have purchased fries at McDonald's,

(a) At least 40 can taste the difference between the two oils?

(b) At most 5% can taste the difference between the two oils?

Answer:

(a) We will Continue by using a standard normal distribution to approximate the Binomial r.v., $X \sim Binom(1000, .97)$ where X is the number of people who can't taste the difference between oils. Checking if X satisfies approximation criteria,

$$\mu = 1000(.97) = 970 \geq 10,$$

$$nq = 1000(.03) = 30 \geq 10.$$

Calculating σ for our approximate normal,

$$\sigma = \sqrt{1000(.97)(.03)} = 5.39.$$

Since at most 40 people tasting the difference is the same as at least 960 people not tasting the difference we know that $P(X \geq 40) = P(X \leq 960)$,

$$P(X \leq 960) \approx P(Z \leq \frac{960 + .5 - 970}{5.39}) = \phi(-1.76) = .039.$$

(b) Similarly we approximate. Note that 5% of 1000 is 50 and saying at most 50 people can detect the difference is the sme as saying at least 950 cannot detect the difference. Hence,

$$P(X \geq 950) = 1-P(X < 950) \approx 1-P(Z < \frac{950 + .5 - 970}{5.39}) = 1-\phi(-3.617) = .9998.$$

4. **Exercise 4.38:** There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation .1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

Answer: Consider the r.v. $X_1 \sim norm(3, .1)$ and $X_2 \sim norm(3.04, .02)$ where X_i is the diameter of the cork produced by the *ith* machine. We will proceed by using the standard normal to calculate $P(2.9 \leq X_i \leq 3.1)$. Consider the first machine,

$$P(2.9 \leq X_1 \leq 3.1) = P(\frac{2.9 - 3}{.1} \leq Z \leq \frac{3.1 - 3}{.1}) = P(-1 \leq Z \leq 1) = \phi(1)-\phi(-1) = .6826.$$

Similarly we calculate the probability for the second machine,

$$P(2.9 \leq X_2 \leq 3.1) = P(\frac{2.9 - 3.04}{.02} \leq Z \leq \frac{3.1 - 3.04}{.02}) = P(-7 \leq Z \leq 3) = \phi(3)-\phi(-7) = .9987.$$

Since machine 2 has a higher probability it is more likely to produce cork within the given range.

5. **Exercise 4.42:** The temperature reading from a thermocouple placed in a constant-temperature medium is normally distributed with mean μ, the actual temperature of the medium, and standard deviation σ. What would the value of σ have

to be to ensure that 95% of all readings are within .18 of μ?

Answer: Recall that by the empirical rule 95% of a normal distribution must lie within 2 standard deviations of the mean. Hence,

$$\mu + 2\sigma = \mu + .18$$
$$\sigma = .09.$$

6. **Exercise 4.60:** Let X denote the distance (m) that an animal moves from its birth site to the first territorial vacancy it encounters. Suppose that for banner-tailed kangaroo rats, X has an exponential distribution with parameter $\lambda = .01386$,

(a) What is the probability that the distance is at most 100 m? At most 200 m? Between 100 and 200 m?

(b) What is the probability that distance exceeds the mean distance by more than 2 standard deviations?

(c) What is the value of the median distance?

Answer: First note that the pdf and cdf for an exponential distribution with $\lambda = .01386$ are,

$$f(x) = \begin{cases} (\lambda)e^{-\lambda(x)} & x \geq 0 \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda(x)} & x \geq 0 \\ 0 & otherwise \end{cases}$$

(a) Calculating the three probabilities using the given cdf,

$$P(X \leq 100) = F(100) = 1 - e^{-\lambda(100)} = .7499.$$

$$P(X \leq 200) = F(200) = 1 - e^{-\lambda(200)} = .9377.$$

$$P(100 \leq X \leq 200) = F(200) - F(100) = .9377 - .7499 = .1878.$$

(b) Recall that for an exponential distribution the mean and standard deviation are the same and are calculated,

$$\mu = \sigma = \frac{1}{\lambda}.$$

Therefore calculating the probability which the distance exceeds the mean by more 2 standard deviations,

$$P(X > \mu{+}2\sigma) = P(X > 216.54) = 1{-}P(X \leq 216.54) = 1{-}F(216.45) = 1{-}(1{-}e^{-\lambda(216.45)}) = .0498.$$

(c) Recall that the median is located at the point where the cdf is equal to .5. Thus,

$$\begin{aligned} F(\widetilde{x}) &= .5 \\ 1 - e^{-\lambda(\widetilde{x})} &= .5 \\ e^{-\lambda(\widetilde{x})} &= .5 \\ \widetilde{x} &= \frac{ln(.5)}{-\lambda} \\ \widetilde{x} &= 50. \end{aligned}$$