

Exercise Supplemental: A random number generator produces two successive values X and Y with the joint distribution, on the support $0 < x, y < 1$

$$f(x, y) = x + y.$$

1. What are the marginal distributions for X and Y .

Solution:

Consider integrating the joint probability distribution over the support on x and y ,

$$f_x(x) = \int_0^1 x + y dy = x + \frac{y^2}{2} \Big|_0^1 = x + \frac{1}{2}$$

$$f_y(y) = \int_0^1 x + y dx = y + \frac{x^2}{2} \Big|_0^1 = y + \frac{1}{2}$$

2. Are X and Y independent why or why not.

Solution:

Note that,

$$f(x, y) \neq f_x(x)f_y(y) = xy + \frac{x}{2} + \frac{y}{2} + \frac{1}{4}$$

Therefore the variable are dependent.