Exercise 3.82: Consider writing into a computer disk and then sending

it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with the parameter $\mu = .2$

1. What is the probability that the disk has exaclty one missing pulse.

   Answer: Using the pmf for a Poissan distribution when $\mu = .2$ and $X = 1$.

   $$P(X = 1) = p(1; .2) = \frac{e^{-.2}.2}{1} = .1637.$$

2. What is the probability that a disk has at least two missing pulses.

   Answer: The probability that the disk has at least two missing pulses is the same as 1 - the probability the disk has at most 1 missing pulse. Therefore,

   $$P(X \geq 2) = 1 - P(X \leq 1) = 1 - F(1; .02) = 1 - .982 = .018.$$

3. If two disks are independently selected what is the probability that neither contains a missing pulse.

   Answer: First let's calculate the probability that the disk has no missing pulses,

   $$P(X = 0) = p(0, .2) = \frac{e^{-.2}}{1} = .819.$$

   Since the drives are independent we simply multiply the probabilities,

   $$P(X_1 \cap X_2) = .819^2 = .671.$$

Exercise 4.2: Suppose the reaction temperature X (inC) in a certain chemical process has a uniform distribution with $A = -5$ and $B = 5$.

1. Compute $P(X < 0)$

   Answer: Since the X has a uniform distribution,

$$P(X < 0) = P(-5 \leq X \leq 0) = \frac{1}{10}(0 + 5) = .50.$$

2. Compute $P(-2.5 < X < 2.5)$

   Answer: Again since X has a uniform distribution over a support length 10,

$$P(-2.5 < X < 2.5) = P(-2.5 \leq X \leq 2.5) = \frac{1}{10}(2.5 - (-2.5)) = .5$$

3. Compute $P(-2 \leq X \leq 3)$

   Answer:

$$P(-2 \leq X \leq 3) = \frac{1}{10}(3 - (-2)) = .5$$

4. For k satisfying $-5 < k < k + 4 < 5$, compute $P(k < X < k + 4)$

   Answer:

$$P(k < X < k + 4) = P(k \leq X \leq k + 4) = \frac{1}{10}(k + 4 - k) = .40.$$

Exercise Extra:

1. Find the mean and variance of the dates. What does this tell you about the distribution of air accidents? (Hint: if they were Poisson,the mean and variance would be similar). Looking at the data, do you have a guess about the pattern?

   Console:

```
> dates = c(14L, 27L, 17L, 15L, 15L, 11L,
9L, 13L, 15L, 14L,17L, 13L, 9L, 13L, 12L,
 6L, 4L, 11L, 8L, 15L, 6L, 7L, 5L, 4L,11L,
 8L, 6L, 8L, 9L, 4L, 9L, 5L, 3L, 9L, 7L)

> fivenum(dates)
[1]  3.0  6.5  9.0 13.5 27.0

> mean(dates)
[1] 10.25714

> var(dates)
[1] 24.72605

> sd(dates)
[1] 4.97253
```

Solution:
So we can see that the mean and median are relatively close, therefore there are not many outliers skewing the data. We can see that the mean, or expected value is nowhere close to the variance so its very Unlikely that this data is Poisson. looking at a histogram the data looks like a flat normal with a slight right skew.

2. Assuming the data was Poisson with mean 10.2, what is the probability of getting exactly 27 accidents in a year? BONUS: What is the probability of getting 27 or more accidents in a year? Does this tell you anything about the patterns of accidents?

Solution:
We simply calculate, $P(X = 27)$, where $X \sim Poisson(10.2)$

$$P(X = 27) = p(27; 10.2) = \frac{e^{-10.2}10.2^{27}}{27!} = 5.826 \times 10^{-6}$$

Exercise Extra: If large earthquakes are a Poisson process with rate 2 per year, what is the probability that you get zero quakes in four years? What IS a Poisson process? Is it a reasonable model for earthquakes (why or why not?)

Solution:
Since we know that large earthquakes are a Poisson process that occur at a rate of 2 per year, in order to calculate the probability that zero quakes occur over the span 4 years we need to calculate μ for our time interval and the evaluate $p(X = 0; \mu)$. Note that $\mu = mean = 4 \cdot 2 = 8$. Thus calculating thr probability,

$$P(X = 0) = \frac{e^{-8}8^0}{0!} = 3.355 \times 10^4$$

The Poisson process is used to is used to model the occurrences of a certain event that appears to have a rate, but occirs completly at random. Event like earthquakes, accident in industrial facilities. Thud this is a reasonable model for earthquakes.

Exercise 3.110: Grasshoppers are distributed at random in a large field according to a Poisson process with parameter $\alpha = 2$ per square yard. How large should the radius R of a circular sampling region be taken so that the probability of find-ing at least one in the region equals .99?

Solution:

First we star by using the pmf for the poisson process to calculate μ given that $P(X \geq 1) = .99$,

$$.99 = P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-\mu}\mu^0}{0!} = 1 - e^{-\mu}. \tag{1}$$

Solving for μ we get,

$$\mu = -ln(.01).$$

Thus we know that the area A of the field must be,

$$A = \frac{\mu}{\alpha} = \frac{\mu}{2} = \frac{-ln(01)}{2}.$$

Solving for the radius, since $A = \pi R^2$,

$$R = \sqrt{\frac{-ln(01)}{\pi 2}} = .8561.$$

Exercise 4.4: Let X denote the vibratory stress on a wind turbine blade ar a particular wind speed in a wind tunnel, with pdf,

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Verify that $f(x;\theta)$ is a legitimate pdf

   Solution:

   Integrating $f(x;\theta)$ over the support,

$$\int_0^\infty \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}} = -e^{\frac{-x^2}{2\theta^2}}|_0^\infty = 0 - (-1) = 1$$

This the given pdf is valid.

2. Suppose $\theta = 100$ (a value suggested by a graph in the article). What is the probability that X is at most 200? Less than 200? At least 200?
   Solution:
   Since we had to solve for the cdf to find out if we have a valid pdf, we can compute all these probabilities by simply plugging in the values,
$$P(X \leq 200) = -e^{\frac{-x^2}{2(100)^2}}|_0^{200} = .8647$$
   Since our variable is a continuos random variable,
$$P(X < 200) = P(X \leq 200) = .8647.$$
$$P(X \geq 200) = 1 - P(X < 200) = 1 - .8647 = .1353.$$

Exercise 4.10: A family of pdf's that has been used to approximate the distribution of income, city population size, and size of firms is the Pareto family. The family has two parameters, k and u, both ¿ 0, and the pdf is,

$$f(x; k, \theta) = \begin{cases} \dfrac{k \cdot \theta^k}{x^{k+1}} & x \geq \theta \\ 0 & x < \theta \end{cases}$$

1. Sketch a graph of the pdf.
   Solution:
   I offer a sketch with my words, The graph is $f(x) = 0$ until $x = \theta$ then it looks like $f(x) = 1/x$ where $f(\theta) = \frac{k}{\theta}$.

2. Verify that the total area under the graph equals 1.
   Solution:
   Integrating over the non-zero support,
$$\begin{aligned} \int_\theta^\infty \frac{k\theta^k}{x^{k+1}} &= k\theta^k \int_\theta^\infty \frac{1}{x^{k+1}}, \\ &= x^{-k}|_\theta^\infty(-\theta^k), \\ &= (0 - \theta^{-k})(-\theta^k), \\ &= 1. \end{aligned}$$

3. Find the closed form for the cdf, for $b > \theta$.

   Solution:
   Using the pdf to solve for $P(X \leq b)$,

$$P(X \leq b) \int_{\theta}^{b} \frac{k\theta^k}{x^{k+1}} = k\theta^k \int_{\theta}^{b} \frac{1}{x^{k+1}},$$
$$= x^{-k}|_{\theta}^{b}(-\theta^k),$$
$$= (b^{-}k - \theta^{-k})(-\theta^k).$$

4. Find the closed form for the cdf, for $b > a > \theta$.

   Solution:
   Using the pdf to solve for $P(a \leq X \leq b)$,

$$P(a \leq X \leq b) \int_{a}^{b} \frac{k\theta^k}{x^{k+1}} = k\theta^k \int_{a}^{b} \frac{1}{x^{k+1}},$$
$$= x^{-k}|_{a}^{b}(-\theta^k),$$
$$= (b^{-}k - a^{-k})(-\theta^k).$$

Exercise 4.12: The cdf for X (5 measurement error) of Exercise 3 is,

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right) & -2 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

1. Compute $P(X < 0)$,

   Solution:
   Plugging into the given cdf,

$$P(X < 0) = P(X \leq 0) = F(0) = .5.$$

2. Compute $P(-1 < X < 1)$,

   Solution:
   Plugging into the given cdf,

$$P(-1 < X < 1) = P(X \leq 1) - P(X \leq -1) = F(1) - F(-1) = \frac{11}{16} = .6875.$$

3. Compute $P(.5 < X)$,

   Solution:
   Plugging into the given cdf,

$$P(.5 < X) = 1 - P(X \leq .5) = 1 - F(.5) = .3164.$$

Exercise 4.22: