**Exercise 2.1:** Consider the feasible region defined by the constrains,

$$1 - x_1^2 - x_2^2 \geq 0,\ \sqrt{2} - x_1 - x_2 \geq 0,\ \text{and } x_2 \geq 0.$$

For each of the following points, determine whether the point is feasible or infeasible and (if it is feasible) whether it is interior to or on the boundary of each of the constraints:

1. $x_a = (\frac{1}{2}, \frac{1}{2})^T$.

   **Solution:**
   Checking constraints,

   $$1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 1 - \frac{2}{4} = \frac{1}{2} > 0$$

   $$\sqrt{2} - (\frac{1}{2}) - (\frac{1}{2}) = \sqrt{2} - 1 > 0$$

   and clearly $\frac{1}{2} > 0$. Thus $x_a$ is a feasible point, and since we have strict inequalities on all the constraints we know that $x_a$ is interior to all constraints (Defined in Chapter 2 p.44).

2. $x_b = (1, 0)^T$

   **Solution:**
   Checking all constraints,

   $$1 - 1^2 - 0^2 = 1 - 1 = 0$$

   $$\sqrt{2} - (1) - (0) = \sqrt{2} - 1 > 0$$

   and since $x_2 = 0$ we know that $x_b$ is a feasible point. Note that $x_b$ is on the boundary of $1 - x_1^2 - x_2^2 \geq 0$ and $x_2 \geq 0$, and interior to $\sqrt{2} - x_1 - x_2 \geq 0$.

3. $x_c = (-1, 0)^T$

   **Solution:**
   Checking all constraints,

   $$1 - (-1)^2 - 0^2 = 1 - 1 = 0$$

   $$\sqrt{2} - (-1) - (0) = \sqrt{2} + 1 > 0$$

   and since $x_2 = 0$ we know that $x_b$ is a feasible point. Note that $x_c$ is on the boundary of $1 - x_1^2 - x_2^2 \geq 0$ and $x_2 \geq 0$, and interior to $\sqrt{2} - x_1 - x_2 \geq 0$.

4. $x_d = (-\frac{1}{2}, 0)^T$