1. Determine the number of edges in a complete graph on $n$ vertices.

   *Proof.* Suppose $G$ is a complete graph on $n$ vertices. There exists $\binom{n}{2}$ ways of pairing vertices. Since $G$ is a complete graph, each pair of vertices correspond to an edge. Hence there are $\binom{n}{2}$, or $\frac{n(n-1)}{2}$ edges.

   □

2. Let $d \in \mathbb{N}$ and $V = \{0,1\}^d$. That is, $V$ is the set of all binary sequences of length $d$. Define a graph on $V$ in which two sequences form an edge if and only if they differ in exactly one position. (This graph is called the **d-dimensional cube**.)

   (a) Draw and label the vertices of the 1-, 2-, and 3-dimensional cube.

   (b) Determine the average degree, number of edges, diameter, girth and circumference of the $d$-dimensional cube.

3. Let $G$ be a graph containing a cycle $C$, and assume that $G$ contains a path of length at least $k$ between two vertices of $C$. Show that $G$ contains a cycle of length at least $\sqrt{k}$.

4. Proposition 1.3.2 states that . Is this bound best possible? Prove your answer is correct.

5. Show that for every graph $G$, $\text{rad}(G) \leq \text{diam}(G) \leq 2\text{rad}(G)$.