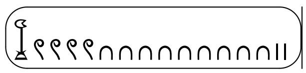
Section 1.2

Exercise 1: Express each of the given numbers in Egyptian hieroglyphics.

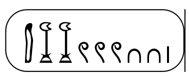
1. 1,492

   Solution:

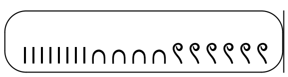
   

2. 12,321

   Solution:

Exercise 2: Write the following egyptian number in our system.

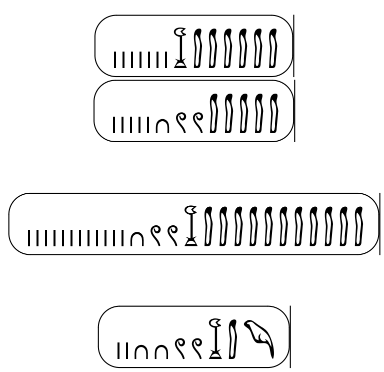


Solution:

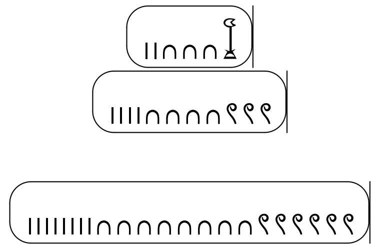
648

Exercise 3: Perform the indicated operations and express the answers in hieroglyphics.

1. Addition



2. Subtract



Exercise 5: Write the Ionian Greek numerals corresponding to 396.
Solution:
tau koppa digamma

Exercise 11: Write the corresponding Roman Numerals

1. 1,492

   Solution:
   $MCDXCII$

2. 3,040,279

   Solution:
   $\overline{MMMXL}CCLXXIX$

Exercise 12: Convert the following Roman Numeral, $MDCCXLVIII$
Solution:

$$1000 + 700 + 40 + 8 = 1748$$

Section 1.3

Exercise 1: Express the following in Babylonian cuneiform notation, 12,345.
Solution:
We must convert 12345 to base 60, then use the cuneiform notation to represent our base 60 representation. Converting we get,

$$123456 \div 3600 \approx 3.$$

Dividing the remainder by 60 we get,

$$(12345 - (3600 * 3)) \div 60 \approx 25.$$

Calculating the final remainder,

$$(1545 - (60 * 25)) = 45$$

Thus in base 60 we get that $12345 \rightarrow (3, 25, 45)_{60}$. Applying cuneiform notation,

Exercise 2: Translate the following number into our system,

Solution:
Rewriting the cuneiform notation, we get that $(14, 40, 6)_{60}$. Multiplying by the base 60 digits, we get that,

$$(3600 * 14) + (60 * 40) + (6) = 52806$$

Exercise 3: Express the fractions $\frac{1}{6}$, $\frac{1}{40}$, and $\frac{5}{12}$ in sexagesimal notation.

Solution:
For the fraction $\frac{1}{6}$ we can already see that,

$$\frac{1}{6} = \frac{10}{60}.$$

Therefore we get that,

$$\frac{1}{6} =;$$ 

For the fraction $\frac{1}{40}$ we can see that $\frac{1}{60}$ fits into $\frac{1}{40}$ with a remainder of $\frac{1}{120}$. Since,

$$\frac{1}{120} = \frac{30}{3600}.$$

Therefore we get the following,

$$\frac{1}{40} =;$$ , 

For the function $\frac{5}{12}$ we can simply see that for the function,

$$\frac{1}{12} = \frac{5}{60}.$$

Applying addition to get $\frac{5}{12}$,

$$\frac{5}{12} =;$$ 

Exercise 4: Convert $12; 3, 45$ from sexagesimal notation into our system.
Solution:
Applying our algorithm to convert we get that,

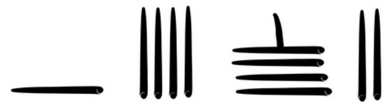
$$(12 * 1) + (3 * \frac{1}{60}) + (45 * \frac{1}{3600}) = 12 + \frac{1}{20} + \frac{1}{80}.$$

Simplifying we get that,

$$12; 3, 45 = 12 + \frac{1}{16} = \frac{193}{16}$$

Exercise 6: Write 1,492 in Chinese counting-rod numerals

Solution:



Exercise 7: Convert the following Chinese counting-rod numerals into our system,

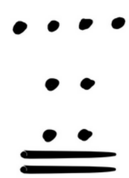
Solution:

Using the conversion table we get the following, 7,725

Exercise 13: Write the Mayan Priest numerals corresponding to 1492,

Solution:

First we divide 1492 by 360 and get, 4 with a remainder of 52. Then we divide 52 by 20 to get 2 with a remainder of 12. Therefore the following is the Mayan Priest representation of 1492,



Exercise 14: Convert the following Mayan Priest numeral into our system,

Solution:

Multiplying by base for each digit we get that,

$$(13 * 7200) + (0 * 360) + (5 * 20) + (7) = 93707.$$

Section 2.3

Exercise 1: Use the egyptian method of doubling to find $18 * 25$

Solution:
Taking the smaller term in the product and doubling it until we find a left hand exponent of 2 that is larger than 25,

$$\begin{array}{r}1|18\\2|36\\4|27\\8|144\\16|288\\32|576\end{array}$$

Note that $16 + 8 + 1 = 25$ thus we get that the product $18 * 25 = 288 + 144 + 18 = 450$

Exercise 2: Find, in Egyptian fashion the following quotients.

1. $184 \div 8$

   Solution:
   Again taking the smaller term from the quotient and doubling it to create a table,

   $$\begin{array}{r}1|8\\2|16\\4|32\\8|64\\16|128\end{array}$$

   Now we need to see which terms on the right hand side of the table sum to 184. Note that,

   $$128 + 32 + 16 + 8 = 184$$

   Summing the corresponding terms on the left hand side we arrive at the answer,

   $$16 + 4 + 2 + 1 = 23$$

2. $19 \div 8$

   Solution:
   Taking the smaller term from the quotient and constructing the table again.

This time we will need to divide by 2 as well since 8 is not a divisor of 19.

$$\overline{8}|1$$
$$\overline{4}|2$$
$$\overline{2}|4$$
$$1|8$$
$$2|16$$

Summing again we can see that, $16 + 2 + 1 = 19$. Thus summing on the left hand side we get that,

$$2 + \overline{4} + \overline{8}$$

Exercise 3: Use the Egyptian method of multiplication to calculate $(11 + \overline{2} + \overline{8})37$.
Solution:
In this case we will construct the table again with $(11 + \overline{2} + \overline{8})$

$$1|11 + \overline{2} + \overline{8}$$
$$2|23 + \overline{4}$$
$$4|46 + \overline{2}$$
$$8|93$$
$$16|186$$
$$32|372$$

Note that $32 + 4 + 1 = 37$, therefore summing the corresponding right hand values we get,

$$372 + 46 + \overline{2} + 11 + \overline{2} + \overline{8} = 430 + \overline{8}$$

Exercise 5: Problem 309 of the Rhind Papyrus asks the reader to find a quantity such that $\overline{\overline{3}} + \overline{10}$ of it will make 10. Do this as the Egyptians would have sone, first by confirming that,

$$13(\overline{\overline{3}} + \overline{10}) = 9 + \frac{29}{30}$$

Then determine by what amount $\overline{\overline{3}} + \overline{10}$ should be multiplied to give $\overline{30}$.
Solution:
The question suggests that when using the method of false position we should let $x = 13$. Therefore finding the product of $13(\overline{\overline{3}} + \overline{10})$ by a table,

$$1|\overline{\overline{3}} + \overline{10}$$
$$2|1 + \overline{3} + \overline{5}$$
$$4|2 + \overline{\overline{3}} + \overline{5} + \overline{6} + \overline{30}$$
$$8|5 + \overline{\overline{3}} + \overline{5} + \overline{6} + \overline{15} + \overline{30}$$

Summing to 13 on the left hand side we see that, $13 = 8 + 4 + 1$. Summing the corresponding left hand values, we find that,

$$\overline{\overline{3}} + \overline{10} + 2 + \overline{\overline{3}} + \overline{5} + \overline{6} + \overline{30} + 5 + \overline{\overline{3}} + \overline{5} + \overline{6} + \overline{15} + \overline{30}$$

We get the following in Egyptian form,

$$9 + \overline{3} + \overline{5} + \overline{6} + \overline{10} + \overline{15} + \overline{16} + \overline{30} + \overline{240}$$

Expanding and adding in our system we do find that,

$$\overline{3} + \overline{5} + \overline{6} + \overline{10} + \overline{15} + \overline{16} + \overline{30} + \overline{240} = \frac{29}{30}.$$

Solving for the factor that we need to multiply $(\overline{\overline{3}} + \overline{10})$ using our system,

$$\begin{aligned} x(\frac{2}{3} + \frac{1}{10}) &= \frac{1}{30} \\ \frac{23x}{30} &= \frac{1}{30} \\ x &= \frac{1}{23} \end{aligned}$$

Thus the quantity is $13 + \overline{23}$.

Exercise 19: Divide 9 loaves equally among 10 men, solve by false position.
Solution:
First suppose that the problem would be written in this form

$$10x = 9.$$

Lets suppose that the quantity $x = \overline{10}$, this gives us that

$$10 * \overline{10} = 1.$$

To make 1 into 9 we simply multiply the quantity by 9. Therefore we must solve $\overline{10} * 9$. With a table we get that,

$$1|\overline{10}$$

$$2|\overline{5}$$

$$4|\overline{5} + \overline{6} + \overline{30}$$

$$8|\overline{3} + \overline{5} + \overline{6} + \overline{15} + \overline{30}$$

Note that $8 + 1 = 9$ and thus summing the corresponding right hand values gives us our answer,

$$x = \overline{3} + \overline{5} + \overline{6} + \overline{10} + \overline{15} + \overline{30} = (\frac{9}{10})$$

Exercise 20: What quantity and it's $\frac{1}{5}$ added together become 21.
Solution:
Consider this problem written in our notation,

$$x + \frac{x}{5} = 21.$$

Using the method of false position, lets suppose that $x = 5$, therefore we get the following,

$$5 + 1 = 6.$$

Note that 6 must be multiplied by $\frac{21}{6} = 3 + \overline{2}$ to obtain the right answer and therefore we know that the quantity is the product $5 * (3 + \overline{2})$. We could use doubling to find this product but I think an egyptian student could see that,

$$5 * (3 + \overline{2}) = 17 + \overline{2}$$

Substituting back into the original problem with $x = 17 + \overline{2}$ we can see that,

$$(17 + \overline{2}) + \frac{5 * (3 + \overline{2})}{5} = 20 + \overline{2} + \overline{2} = 21.$$