Section 5.3

Exercise 21: Bhaskara, 1150. What number divided by 6 leaves a remainder of 5, divided by 5 leaves a remainder of 4, divided by 4 leaves a remainder of 3, and divided by 3 leaves a remainder of 2?

$$N \cong 5 mod 6,$$
$$N \cong 4 mod 5,$$
$$N \cong 3 mod 4,$$
$$N \cong 2 mod 3.$$

Solution:
To solve this problem we must first compute the product M, of the moduli m_i,

$$M = 6 * 5 * 4 * 3 = 360.$$

The next step involves computing all M_i such that

$$M_i = \frac{M}{m_i}.$$

Therefore we get the following,

$$M_1 = \frac{360}{6} = 60,$$
$$M_2 = \frac{360}{5} = 72,$$
$$M_3 = \frac{360}{4} = 90,$$
$$M_4 = \frac{360}{3} = 120.$$

Now we reduce each M_i mod m_i, so find a P_i such that,

$$M_i = P_i \mod m_i.$$

Doing that we get,

$$60 \equiv 0 \mod 6.$$
$$72 \equiv 2 \mod 5.$$
$$90 \equiv 2 \mod 4.$$
$$120 \equiv 0 \mod 3.$$

Now we need to find one for each P_i so solving for some x_i that gives,

$$P_i x_i \equiv 1 \mod m_i.$$

Doing this we get,

$$
\begin{aligned}
(0)(1) &\equiv 1 \mod 6\\
(2)(4) &\equiv 1 \mod 5\\
(2)(5) &\equiv 1 \mod 4\\
(0)(1) &\equiv 1 \mod 3
\end{aligned}
$$

Section 5.5

Exercise 1: Solve the following quadratic equqitons with the araic method of complete the square.

1. $x^2 + 12x = 64$

   Solution:
   First note that this is a type 4 problem with the form $ax^2 + bx = 2$. We complete the square by noting that,
   $$(x + 6)^2 = x^2 + 12x + 36.$$
   So adding 36 to both sides we get that,
   $$
   \begin{aligned}
   x^2 + 12x + 36 &= 64 + 36,\\
   (x + 6)^2 &= 100,\\
   (x + 6)^2 &= 10^2.
   \end{aligned}
   $$
   Therefore $x = 4, -12$.

2. $3x^2 + 10x = 32$

   Solution:
   From the hint lets multiply both sides of the equation by 3 and and simplify the form of our equantio with a substitution of $y = 3x$,
   $$
   \begin{aligned}
   3x^2 + 10x &= 32,\\
   3(3x^2 + 10x) &= 3(32),\\
   9x^2 + 30x &= 96,\\
   (3x)^2 + 10(3x) &= 96,\\
   (y)^2 + 10(y) &= 96.
   \end{aligned}
   $$

Now our problem is a type 4 problem with the form $ax^2 + bx = 2$. We complete the square by noting that,

$$(y+5)^2 = y^2 + 10y + 25.$$

So adding 25 to both sides,

$$\begin{aligned} y^2 + 10y + 25 &= 96 + 25, \\ (y+5)^2 &= 121, \\ (y+5)^2 &= (11)^2. \end{aligned}$$

Thus we get that $y = 6, -16$ and since $y = 3x$ we get that $x = 2, \frac{-16}{3}$

Exercise 7: 1. Show that the cubic equation $x^3 + b^2c = b^2x$ can be solved by finding the intersection of the parabola $x^2 = by$ and the hyperbola $y^2 + cx = x^2$.

Solution:
We can show that the intersection of $x^2 = by$ and $y^2 + cx = x^2$ gives $x^3 + b^2c = b^2x$ through algebra. First solve the first equation for y,

$$y = \frac{x^2}{b}.$$

Now substituting into the second equation and doing some algebra to get the third equation.

$$\begin{aligned} (\frac{x^2}{b})^2 + cx &= x^2, \\ \frac{x^4}{b^2} + cx &= x^2, \\ \frac{x^4}{b^2} + cx &= x^2, \\ \frac{x^3}{b^2} + c &= x, \\ x^3 + b^2c &= b^2x. \end{aligned}$$

Therefore where the two conic sections intersect we get the solution to the cubic.

2. Show that the cubic equation $x^3 + c = ax^2$ can be solved by finding the intersection of the parabola $y^2 + cx = ac$ and the rectangular hyperbola $xy = c$.

Solution:
Again we can show this through algebra. Solving the rectangular hyperbola for y,

$$y = \frac{c}{x}.$$

Substotutinog into the parabola and doing some algebra to get the cubic,

$$\begin{aligned} y^2 + cx &= ac, \\ (\frac{c}{x})^2 + cx &= ac, \\ \frac{c^2}{x^2} + cx &= ac, \\ c^2 + cx^3 &= acx^2, \\ c + x^3 &= ax^2, \\ x^3 + c &= ax^2. \end{aligned}$$

Thus where the two conic sections intersect we get the solution to the cubic.

Additional Problems

Exercise 1:

Exercise 2:

Reflection

1.

2.