Section 4.3

Exercise 6: Prove that 7 is the only prime of the form [Hint factor $n^3 - 1$ into $(n-1)(n^2 + n + 1)$.],

$$n^3 - 1$$

Solution:
Considering the hint let's factor $n^3 - 1$ into $(n-1)(n^2 + n + 1)$. Now let z be a prime number and suppose that,

$$z = (n - 1)(n^2 + n + 1).$$

Since z is prime by the fundamental theorem of arithmetic we know that either $(n - 1) = 1$ or $(n^2 + n + 1) = 1$. Suppose that $(n^2 + n + 1) = 1$, through some algebra we can see that,

$$\begin{aligned}(n^2 + n + 1) &= 1,\\ n^2 + n &= 0,\\ n(n + 1) &= 0.\end{aligned}$$

Therefore we have that $n = 0, -1$ however substituting back into our original equation we get,

$$(0)^3 - 1 = -1$$

$$(-1)^3 - 1 = -2$$

and neither solution is prime therefore it must be the case that $(n - 1) = 1$ which gives us $n = 2$. Substituting $n = 2$ we get,

$$(2)^3 - 1 = 7.$$

Thus 7 is the only prime of the form $n^3 - 1$.

Exercise 7: Find a set of four consecutive odd integers of which three are primes, and a set of five consecutive odd integers of which four are prime.

Solution:
For the set of four consecutive odd integers consider the set,

$$3, 5, 7, 9.$$

Note that $3, 5, 7$ are prime, and 9 is not since $9 = 3^2$. For the set of five consecutive odd numbers consider,

$$3, 5, 7, 9, 11.$$

Note that 11 is prime, so there are now four prime numbers.

Exercise 15: Find the $gcd(272, 1479)$.

Solution:
We will proceed by using the Euclidean algorithm,

$$\begin{aligned} 1479 &= (5) * 272 + 119, \\ 272 &= (2) * 119 + 34, \\ 119 &= (3) * 34 + 17, \\ 34 &= (2) * 17 + 0. \end{aligned}$$

Therefore we get that $gcd(272, 1479) = 17$.

Exercise 16b: Use the Euclidean algorithm to find x and y such that,

$$gcd(24, 138) = 24x + 138y.$$

Solution:
First we must use the Euclidean algorithm to find $gcd(24, 138)$,

$$\begin{aligned} 138 &= (5)24 + 18, \\ 24 &= (1)18 + 6, \\ 18 &= (3)6 + 0. \end{aligned}$$

Therefore we have that $gcd(24, 138) = 6$. To solve for x and y we use the Euclidean algorithm backwards,

$$\begin{aligned} 6 &= 24 - 18, \\ 6 &= 24 - (138 - (5)24), \\ 6 &= 24 - 138 + (5)24, \\ 6 &= (6)24 + (-1)138. \end{aligned}$$

Therefore we have that $x = 6$ and $y = -1$.

Section 4.4

Exercise 4b:

From Ptolemy's value $chord(1) = 1;2,50$ and using an inscribed 360-gon to approximate the circumference of a circle, obtain his approximation of π.

Solution:
We know that the π is the ratio between a circle's circumference and diameter. Using Ptolemy's value for a the length of a 1 degree chord, we can find an approximation for the circumference using the perimeter of an inscribed 360-gon. Note that eac side of an inscribed 360-gon will have the length $chord(1) = 1;2,50$ so we know that,

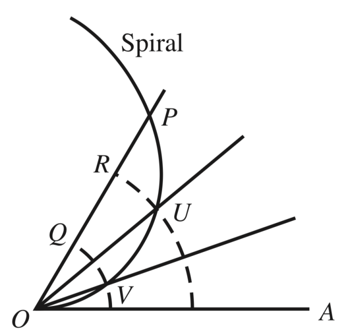
$$C \approx 360 * 1;2,50 = 6,0; *1;2,50$$

Dividing by a diameter of $2,0;$ we get that,

$$\begin{aligned}\pi &\approx \frac{6,0;}{2,0;} * 1;2,50 \\ &= 3; *1;2,50 \\ &= 3;6;150 \\ &\approx 3.14166\end{aligned}$$

Section 4.5

Exercise 10: Given a spiral, place the angle to be trisected so that the vertex and the initial side of the angle coincide with the initial point of the spiral and the initial position OA of the rotating ray. Let the terminal side of the angle intersect the spiral of P. Trisect the segment OP at the points Q and R, and draw circles with center at O and with OQ and OR as radii. Prove that if these circles meet the spiral in points U and V , then the lines OU and OV will trisect $\angle AOP$.



Solution:
Recall that the Archimedean spiral is defined by a parametric equation of the form,

$$r = a\theta$$

for some $a > 0$. Note that by construction we have,

$$OP = a(\angle AOP).$$

Solving for a gives us,

$$a = \frac{OP}{\angle AOP}.$$

Solving for $\angle AOU$ in terms of $\angle AOP$ using the fact that by construction $OU = \frac{2}{3}OP$,

$$\begin{aligned} OU &= \frac{OP}{\angle AOP}(\angle AOU), \\ \frac{2}{3}OP &= \frac{OP}{\angle AOP}(\angle AOU), \\ \frac{2}{3} &= \frac{1}{\angle AOP}(\angle AOU), \\ \frac{2}{3}\angle AOP &= \angle AOU. \end{aligned}$$

Similarly solving for for $\angle AOV$ in terms of $\angle AOP$ using the fact that by construction $OV = \frac{1}{3}OP$,

$$\begin{aligned} OV &= \frac{OP}{\angle AOP}(\angle AOV), \\ \frac{1}{3}OP &= \frac{OP}{\angle AOP}(\angle AOV), \\ \frac{1}{3} &= \frac{1}{\angle AOP}(\angle AOV), \\ \frac{1}{3}\angle AOP &= \angle AOV. \end{aligned}$$

Thus $\angle AOP$ is trisected by points V and U.

Reflection:

1. I had a really hard time doing sexagesimal division in problem 4b. At first I was solving for the circumference multiplying the 360 by the chord length and then dividing by 120. It was much easier to simplify the 360/120 to get 3 and then multiply by the chord length.

2. I don't know if we were supposed to do something special for problem 7 but I was able to find both sequences by just considering the first few primes.