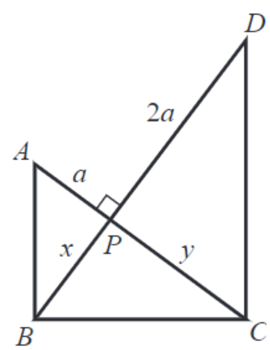
Section 3.4

2. The following solution to the continued mean proportionals problem is often attributed to Plato, although it could hardly be his in view of his objection to mechanical constructions. Consider two right triangles ABC and BCD, lying on the same side of he common leg BC. Suppose that the hypotenuses AC and BD intersect perpendicularly at point P, and are constructed in such a way that $AP = a$ and $DP = 2a$. Prove that $x = BP$ and $y = CP$ are the required mean proportionals between a and $2a$, that is,
$$\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}.$$



Answer: Note that since ABC and BCD are right triangles with the following property,
$$\angle ABC = \angle DCB = 90°$$
Note that these angles sum to $180°$ and are the interior angles to segments AB and DC. Therefore by Euclid's Parallel postulate AB——DC. With segments AC and BD as transversals we get the following equalities through alternate interior angles.
$$\angle BAC = \angle DCA,$$
$$\angle ABD = \angle CDB.$$
Therefore we get $\triangle APB \sim \triangle CPD$ by AA similarity. Note that by the construction of point P and vertical angles we know that,
$$\angle DPA = \angle BPC = \angle APB = \angle CPD = 90°.$$
Since the angles of $\triangle BCD$ and $\triangle CPD$ sum $180°$ we get the following through algebra,
$$\angle CDB + \angle DCP + 90° = \angle CDB + \angle CBD + 90°,$$
$$\angle DCP = \angle CBD.$$

Note that $\angle CBP = \angle CBD = \angle DCP$ and $\angle BPC = \angle CPD$ therefore by AA similarity $\triangle BPC \sim \triangle CPD$. Thus by the transitivity of similar triangles we know that $\triangle APB \sim \triangle CPD \sim \triangle BPC$ and the desired proportional relationship,

$$\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}$$

is derived from the ratio between the legs of each right triangle.

4. **Answer:**

Intro to GeogGebra worksheet
Section 4.2

1. **Answer:**

7. **Answer:**

11. **Answer:**

12. **Answer:**

Reflection:

1.

2.