Exercise 1: What is a row vector, what is a column vector?

Solution:
row and column vectors are simply vectors, but in the context of matrices. A matrix has two spaces/dimensions, the row space and column space. We can think of row and column vectors as matrices with a single dimension, i.e a 1xn matrix would be a column vector and a nx1 matrix would be a row vector.

Exercise 2: The following is an rxc matrix. What are r and c, as well as the matrices transpose.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

Solution:
A is a 2x4 matrix, where $r = 2$ and $c = 4$, The following is the transpose,

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Exercise 3: Give an example of an identity matrix and an example of a diagonal matrix.

Solution:
The identity matrix is a diagonal matrix with all ones on the diagonal. The following is the 3x3 identity matrix,

$$I_{3x3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity matrices are a subset of diagonal matrices. The following is an example of diagonal matrix,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Exercise 4: Is the following a symmetric matrix? why?

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 6 \\ 5 & 6 & 100 \end{bmatrix}$$

Solution:
A is a symmetric matrix, since it has the property that $A = A^T$.

Exercise 5: Consider the following matrices,

A =
$$\begin{matrix} 1 & 2 & 3 \\ 4 & 1 & 1 \end{matrix}$$

B=
$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

C=
$$\begin{matrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{matrix}$$

D =
$$\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$$

E=
$$\begin{matrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{matrix}$$

a. Is $A + B$ undefined?

Solution:
Matrix addition is done entrywise. Note A is a 2x3 matrix and B is 2x2 so this operations would be undefined.

b. Is AB undefined?

Solution:
Matrix multiplication requires the inner dimensions to be equal. Note A is a 2x3 matrix and B is 2x2, since $3 \neq 2$ this operation would be undefined.

c. Is BA undefined?

Solution:
The inner dimensions do match, so BA is well defined.

d. Is $E'D$ undefined?

Solution:
Note that E' is a 4x2 matrix and D is a 2x2 matrix. This operation is well defined.

e. Is $3E$ undefined?

Solution:
$3E$ is well defined, we just multiply every term in E by 3.

Exercise 6: Find the inverse of the following matrix, then compute $G^{-1}G$

$$G = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Solution:
There is a formula for computing the inverse of a 2x2 matrix, which gives the following,

$$G^{-1} = -\frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

Finally we know that the product of a matrix and its inverse is simply the identity. For this case we get $G^{-1}G = I_{2x2}$.

Exercise 7: Consider the following matrix,

$$Q = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

Is $e = [111]'$ and eigenvector? If so show that it's eigenvalue is 6.
Solution:
We can check this by computing Qe. Doing so we get,

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Exercise 8: a. What is a p-value? If you knew a test had a p-value of .01 and it was performed at the $\alpha = .05$ significance level, would you reject the null hypothesis?

Solution:
The p-value is the probability of obtaining the test statistic under the assumption of the null hypothesis. In the situation described above we would reject the null hypothesis since p-value is smaller than the significance level. The probability of attaining the test statistics is under the $\alpha = .05$ significance level is essentially zero.

b. What does the significance level of a test tell you?

Solution:
The significance level of a test is the probability of rejecting the null hypothesis when it is true.

c. What is the difference between a parameter and a statistic?

Solution:
A parameter describes an entire population, a statistic descirbes a sample of the population.

d. If you create a 95% confidence interval for a parameter what does that tell you?

Solution:
95% percent of all samples taken to estimate the parameter will give a value inside the interval.