Stat 605 – Spatial Statistics – Spring 2022
Homework 5. Due: Friday, February 11, midnight.

1. Use R to plot two Matern semi-variograms on the same graph, one with smoothness parameter $\nu = \kappa = 1/2$, another with $\nu = \kappa = 5$; in each case, use range parameter $a = \phi = 1$. Use a nugget $\tau^2 = 1.0$ and a partial sill of $\sigma^2 = 1.5$. Which of the two semivariograms corresponds to smoother realizations, and how can you tell?

   The geoR package has a function called `matern` that calculates a matern covariogram; the values that this function returns lie between 0 and 1. (Note that I use a to denote the range parameter for the matern semivariogram; geoR uses ϕ. What I call ν ("nu"), geoR calls κ ("kappa").

   Here is sample R code for two matern covariograms:

```
curve( matern(x, phi=1.2, kappa=0.75), from = 0, to = 5.0 )
curve( matern(x, phi=1.2, kappa=1.25), add=TRUE, lty=2 )
```

2. Continuing analysis using R for the scallop data set using centered lats/longs.

   (a) Add borders to the data set that has centered latitudes and longitudes. (See page 81 of lecture notes.)

   (b) Use the `pred_grid` function to create a suitable grid of prediction locations (using range of values for the centered latitudes and longitudes). (Page 84 of lecture notes)

   (c) Use the `krige.control` function followed by `krige.conv` to carry out universal kriging for log(catch) at the prediction locations in (b). The R code on pages 96 and 97 may be helpful, as they show you how to handle non-constant trend. You'll probably just need `trend.d="2nd", trend.l="cte"` rather than the additional codes in the notes.

   (d) State the estimated regression equation, $\mathbb{E}(Y(\boldsymbol{s})) = \ ?$

   (e) Plot the resulting smoothed map using the centered coordinates. (Modify the R code on pages 86 and 87 of the lecture notes.)

   Typo alert! Every place you see "`my_plot_results`" in the lecture notes, it should say "`one_plot`", which is the function on page 86. I have posted the `one_plot` function on Canvas. Remember that you have to type `library(sp)` before you call the `one_plot` function the first time in an R session.

   (f) Create a linear interpolation plot of log(catch) using the `interp` function in the akima package.

   Here's the code I used on page 82 of the lecture notes:

```
lons <- Ksat$coords[,1]; lats <- Ksat$coords[,2]
bar <- interp( lons, lats, log(Ksat$data), linear=TRUE)
my_grays <- gray( (25:59)/64 )
image(bar, xlim=range(lons),ylim=range(lats),
      col=my_grays, main="Akima on log(Ksat) data")
contour(bar, labcex=1.5, add=TRUE)
```

   (g) Comment briefly on your results in (e) and (f). (Are the plots essentially the same? Are there any noticable differences?)

3. Kriging weights. Use the spherical semi-variogram model with sill = 4.0, range = 2.0, and nugget = 0 for the following locations:

longitude	0	0	1	1	1.1	0.9
latitude	0	1	1	0	-0.1	-0.1

Construct a plot of these locations and calculate the kriging weights at each of the following five locations, $\boldsymbol{s}_0$:

$$(0.5, 0.5),\ \ (0.1, 0.9),\ \ (0.25, 0.75),\ \ (1.1, -0.1),\ \ (1.05, -0.05)$$

In each case, comment on the weights, using your plot to assist in your description. For example, for each $\boldsymbol{s}_0$, which observation locations have the largest weights, and does this make sense?

Here is sample R code, which you will need to modify:

```
my_locs <- rbind(  c(1,1),  c(1,2),  c(3,3)  )
my_kr_control <- krige.control(cov.model="spherical", cov.pars=c(4,2))
krweights(coords=my_locs, loc=c(0,0), krige=my_kr_control)
```