Multivariable Spatial Prediction and Model Validation

Stefano Fochesatto

What is Multivariable Spatial Prediction

- Methods we have discussed in class offer prediction of one variable at a time.
- Recall,

$$\hat{y}(s_0) = \sum_{i=1}^{n} \lambda_i y(s_i)$$

  - Ordinary Kriging; λ_i's are constrained by relative distances.
  - Universal Kriging; λ_i's are also constrained by trends and covariates.
- For Universal Kriging we found λ_i's by minimizing MSPE,

$$MSPE = \mathbb{E}\left[\left(Y(s_0) - \sum_{i=1}^{n} \lambda_i y(s_i)\right)^2\right],$$

  by the following constraint (lagrange multipliers),

$$\bar{\lambda}^T X = x_0^T.$$

- To find the $\bar{\lambda}$ using LeastSquares it was necessary to estimate a variogram for Y.

What is Multivariable Spatial Prediction

- What if we don't have x_0 where we want to predict s_0, but we still have secondary data that has information?!
- Cokriging methods follow analogously in derivation; and allow us to directly use existing spatial correlations from secondary-data,

$$\hat{y}(s_0) = \sum_{i=1}^{n} \lambda_i y_1(s_i) + \sum_{j=1}^{k} \lambda_j y_2(s_j) + \dots$$

- In solving for λ_i and λ_i using LeastSquares it becomes necessary to estimate variograms and crossvariograms for all Y_i.

What is Multivariable Spatial Prediction

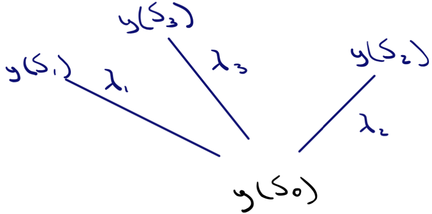
- The goal is the spatial prediction of multiple variables simultaneously.
- Multivariable Spatial Prediction is an extension of Cokriging.
- 'It is shown that the cokriging predictor for one variable at a time is identical to the predictor of that same variable in the multivariable predictor.' Ver Hoef & Cressie (1993)
- The constraints for solving for the weights change when cokriging a variable at a time.

Kriging Overview

- 'The truth of the matter is, when someone says 'kriging' I kind of blend it all together in my mind.' - M. Short.
- same tbh

Kriging Overview

Kriging



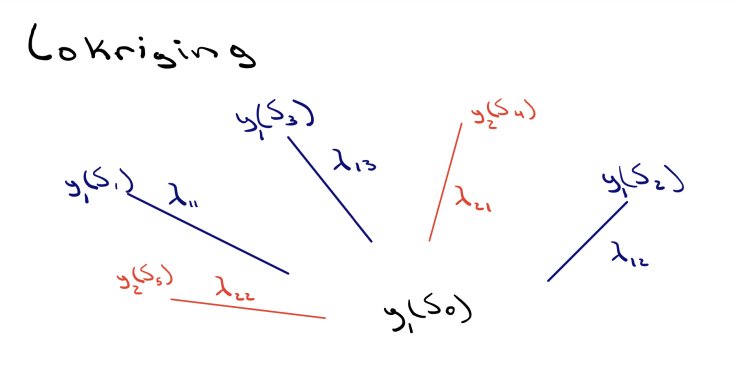
Ordinary:

λ_i Constrained by relative distance

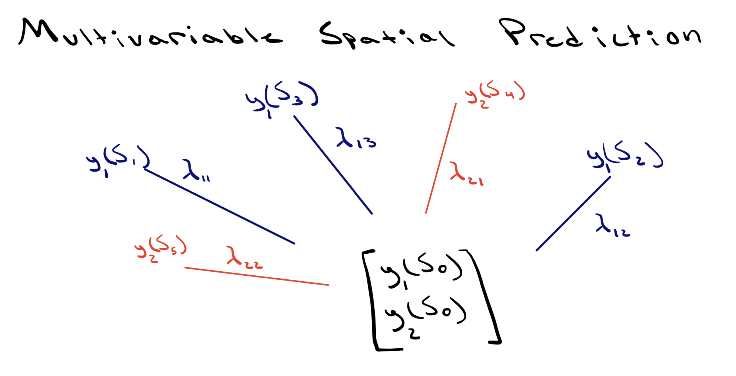
Universal:

λ_i Constrained by Possible Covariates

Kriging Overview



Kriging Overview

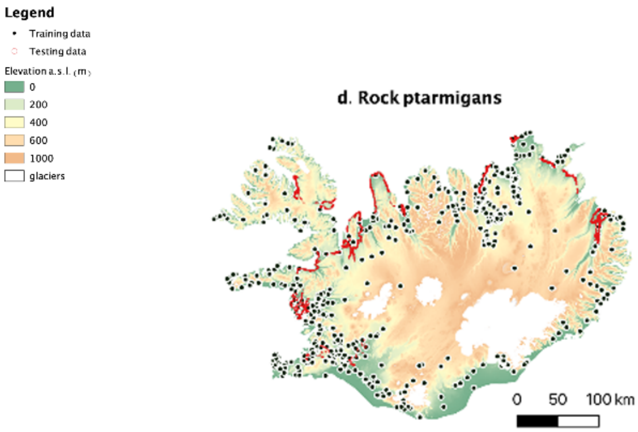


Application For Model Validation in Ecology

Our Data

- Study put together through several Icelandic environmental agencies, in conjunction with the University of Iceland and the UAF Institute of Arctic Biology.
- Machine learning models (Random Forest and TreeNet) were used to model RIO (relative index of occurrence).
- Nationwide and long term (1860-2021) Rock ptarmigan occurrence data (GBIF).
- Separate occurrence data from the Icelandic Institute of Natural History (2005-2010) for model validation.
- 11 Environmental Layers (May, June, July of 2021).

Our Data


Legend
Training data
Testing data
Elevation a.s.l. (m)
0
200
400
600
1000
glaciers
d. Rock ptarmigans
0
50
100 km

Our Data

- Why use Multivariable Spatial Prediction?
    - Separate occurrence data does not have the associated predictors (environmental layers).
    - Current Models are validated with OOB/Cross-Validation.
    - Fun exercise!