Multivariable Spatial Prediction and Model Validation

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What is Multivariable Spatial Prediction

- Methods we have discussed in class offer prediction of one variable at a time.
- Recall,

$$\hat{y}(s_0) = \sum_{i=1}^{n} \lambda_i y(s_i)$$

  - Ordinary Kriging; λ_i's are constrained by relative distances.
  - Universal Kriging; λ_i's are also constrained by trends and covariates.
- For Universal Kriging we found λ_i's by minimizing MSPE,

$$MSPE = \mathbb{E}\left[\left(Y(s_0) - \sum_{i=1}^{n} \lambda_i y(s_i)\right)^2\right],$$

  by the following constraint (lagrange multipliers),

$$\bar{\lambda}^T X = x_0^T.$$

- To find the $\bar{\lambda}$ using LeastSquares it was necessary to estimate a variogram for Y.

What is Multivariable Spatial Prediction

- What if we don't have x_0 where we want to predict s_0, but we still have secondary data that has information?!

- Cokriging methods follow analogously in derivation; and allow us to directly use existing spatial correlations from secondary-data,

$$\hat{y}(s_0) = \sum_{i=1}^{n} \lambda_i y_1(s_i) + \sum_{j=1}^{k} \lambda_j y_2(s_j) + \dots$$

- In solving for λ_i and λ_i using LeastSquares it becomes necessary to estimate variograms and crossvariograms for all Y_i.

What is Multivariable Spatial Prediction

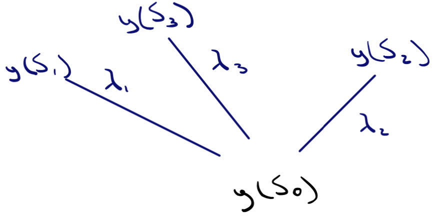
- The goal is the spatial prediction of multiple variables simultaneously.
- Multivariable Spatial Prediction is an extension of Cokriging.
- 'It is shown that the cokriging predictor for one variable at a time is identical to the predictor of that same variable in the multivariable predictor.' Ver Hoef & Cressie (1993)
- The constraints for solving for the weights change when cokriging a variable at a time.

Kriging Overview

- 'The truth of the matter is, when someone says 'kriging' I kind of blend it all together in my mind.' - M. Short.
- same tbh

Kriging Overview

Kriging



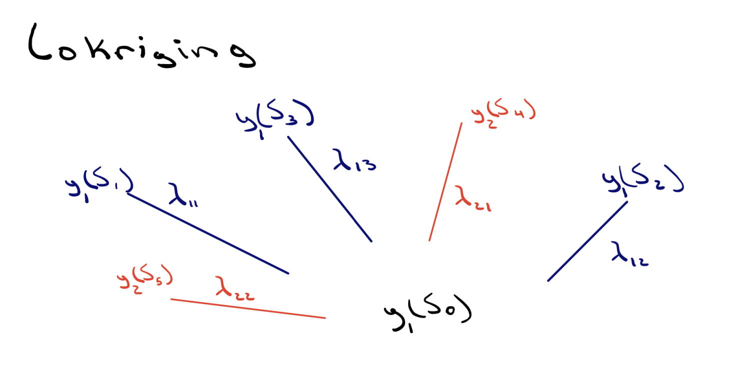
Ordinary:

λ_i Constrained by relative distance

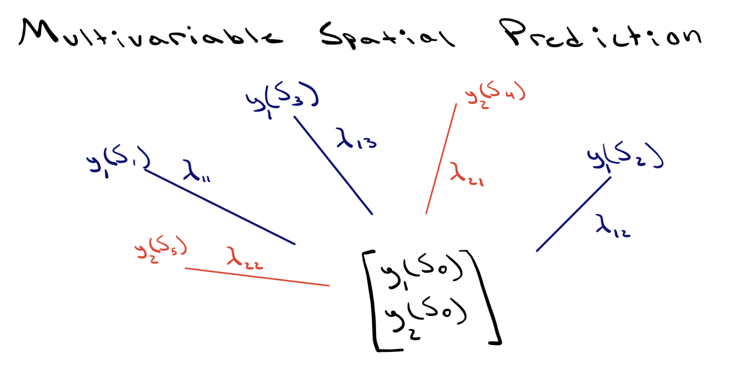
Universal:

λ_i Constrained by Possible Covariates

Kriging Overview



Kriging Overview

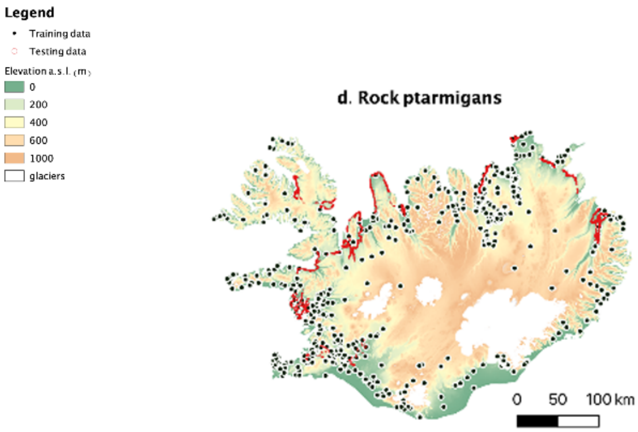


Application to Model Validation in Ecology

Application to Model Validation in Ecology

- Study put together through several Icelandic environmental agencies, in conjunction with the University of Iceland and the UAF Institute of Arctic Biology.
- Machine learning models (Random Forest and TreeNet) were used to model RIO (relative index of occurrence).
- Nationwide and long term (1860-2021) Rock ptarmigan occurrence data (GBIF).
- Separate occurrence data from the Icelandic Institute of Natural History (2005-2010) for model validation.
- 11 Environmental Layers (May, June, July of 2021).

Application to Model Validation in Ecology



- Why use Multivariable Spatial Prediction?
    - Separate occurrence data does not have the associated predictors (environmental layers).
    - Current Models are validated with OOB/Cross-Validation.
    - Temper Expectations.

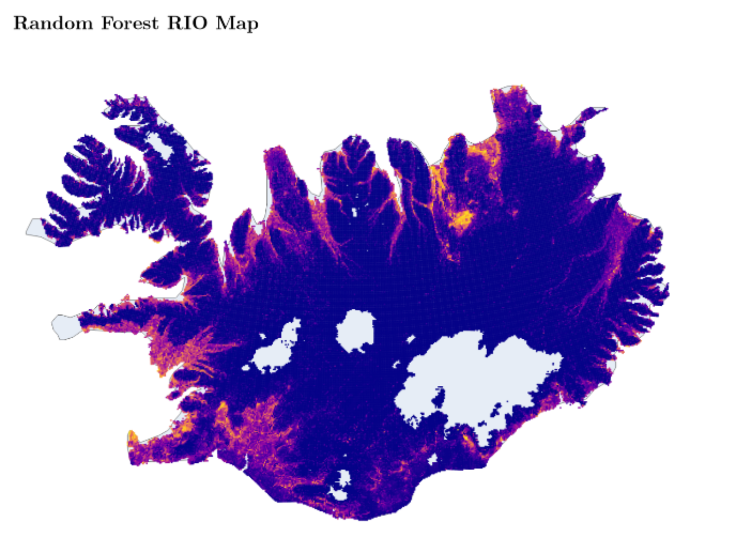
Application to Model Validation in Ecology

- A tiny bit of code ...

```r
library(gstat)
g <- gstat(NULL, id = "NDVImax250", form = NDVImax250 ~ x + y + I(x^2) + I(y^2) + x*y, data=Data.Subsample)
g <- gstat(g, id = "JJA_tavg25", form = JJA_tavg25 ~  x + y + I(x^2) + I(y^2), data=Data.Subsample)
g <- gstat(g, id = "JJA_ppt_av", form = JJA_ppt_av ~  x + y + I(x^2) + I(y^2) + I(x*y), data=Data.Subsample)
g <- gstat(g, id = "JJA_mean_w", form = JJA_mean_w ~ 1, data=Data.Subsample)
g <- gstat(g, id = "dem250"    , form = dem250 ~ 1, data=Data.Subsample)
g <- gstat(g, id = "soil250"   , form = soil250 ~ 1, data=Data.Subsample)
g <- gstat(g, id = "veg250"    , form = veg250 ~ 1, data=Data.Subsample)
g <- gstat(g, id = "slope250"  , form = slope250 ~ 1 , data=Data.Subsample)
```

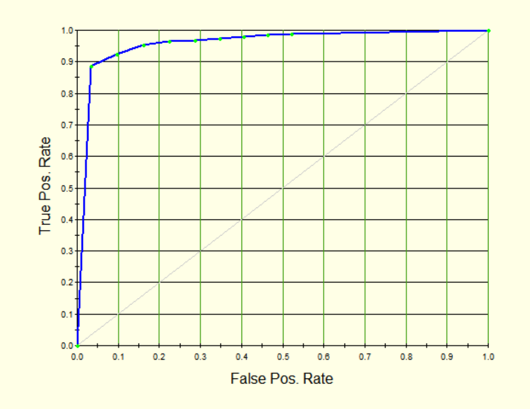
```r
g <- fit.lmc(v.cross, g, fit.method = 7, correct.diagonal=1.01)
```

Results



Results

- Accuracy of 91.98% on out of bag samples.



Results

- Accuracy of 72.91% on outside data with kriged predictors.