Please see the rules below.

1. A map f between spaces is **proper** if the preimage of any compact set is compact. Let X and Y be locally compact Hausdorff spaces. Show that a continuous map $f : X \to Y$ extends to a continuous map $f^* : X^* \to Y^*$ between the 1-point compactifications if and only if it is proper.

2. The Möbius band is the quotient of $[0,1] \times \mathbb{R}$ where $(0, y) \sim (1, -y)$.

a) Show that the Möbius band is a 2-manifold.

b) Show that the Möbius band is homotopy equivalent to a circle.

c) No rigor please, just a picture or two: what familiar space is the 1-point compacti-ficaiton of the Möbius band?

3.

a) Lee 3-22 b

b) Show $\mathbb{RP}^n$ is Hausdorff. (Hurrah Closed Map Theorem!)

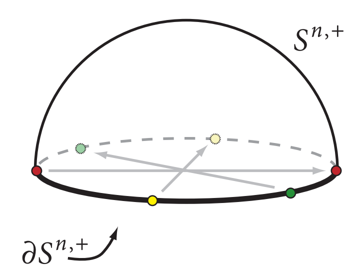
4. Consider the metric on $\mathbb{R}$ given by $\bar{d}(x, y) = \min(|x - y|, 1)$. For $z, w \in \mathbb{R}^\omega$, define

$$d(z, w) = \sum_{k=1}^{\infty} 2^{-k} \bar{d}(z_k, w_k).$$

It can be easily shown (you need not show it) that d is a metric. Prove that the topology that this metric induces on $\mathbb{R}^\omega$ is the product topology.

5. a) Show that the upper half sphere $S^{n,+}$ with antipodal points on $\partial S^{n,+}$ identified is homeomorphic to $\mathbb{RP}^n$.

b) Consider $X = \{(xy, yz, zx, x^2, y^2, z^2) \in \mathbb{R}^6 : x^2 + y^2 + z^2 = 1\}$. Prove that this set is homeomorphic to $\mathbb{RP}^2$.



Gluing a half sphere.

6. Lee 8-11 (a,b)

7. Lee 10-11

8. Lee 9-5

9. Let $\pi : X \to Y$ be a quotient map. Suppose Y is connected and each fiber $\pi^{-1}(y)$ is connecteed. Show that X is connected.

10. Recall that a set $A \subset X$ is a retract of X if there is a continuous $f : X \to A$ such that $f(a) = a$ for all $a \in A$.

a) Show that if X is Hausdorff and A is a retract of X then A is closed.

b) Let A be a two point subset of $\mathbb{R}^2$. Show that it is not a retract of $\mathbb{R}^2$.

c) Show that the closed ball $\overline{\mathbb{B}^2}$ is a retract of $\mathbb{R}^2$.

d) Show that S^1 is not a retract of $\mathbb{R}^2$.

Rules and format:

- You are welcome to discuss this exam with me (David Maxwell) to ask for hints and so forth.
- If you find a suspected typo, please contact me as soon as possible and I will communicate it to the class if needed.
- You may not discuss the exam with anyone else until after the due date/time.
- You may not use the internet.
- You are permitted to reference either of the two texts used for the class (Lee or Crossley). No other resources are permitted.
- Each problem is weighted equally (10 points each).
- The due date/time is absolutely firm.
- We will schedule a hints sessions for this exam.