

Week 1 homework problems

1. Below are three samples of a random variable. Which sample displays the greatest variance? Why?

Sample 1 $\{21.45, 22.93, 31.86, 19.37, 20.87\}$

Sample 2 $\{182, 186, 179, 184, 187\}$

Sample 3 $\{14.3, 14.7, 10.0, 14.8, 14.6\}$

2. A random sample of dogs studied by a veterinarian was found to have the following lifespans, in years: $\{14, 18, 6, 9, 13\}$. The veterinarian inquired about each dog's mother and found the mothers' lifespans to be: $\{10, 19, 13, 8, 24\}$. What is the sample covariance between offsprings' lifespans and mothers' lifespans?
3. Consider X , a random variable with a standard normal distribution. What is the probability that X is greater than -0.4 ?
4. Consider V , a random variable with a $N(13, 400)$ distribution. What is the probability that V is less than or equal to -7 ?
5. Let the expected value of random variable X be a , the expected value of Y be b , and the expected value of Z be c . Find $E(4 - 2X + 3Y - 10Z)$.
6. Let the variance of random variable X be 3, the variance of Y be 12, and the variance of Z be 9, and let X , Y , and Z be uncorrelated. Find $V(4 - 2X + 3Y - 10Z)$.
7. If random variable A is distributed $N(3, 8)$ and an independent random variable B is distributed $N(-3, 10)$, what is the distribution of $mA + nB$ for some constants m and n ?
8. Angie records the lengths of 15 poems from ancient Greece and finds the average wordcount of the poems is 456 with a standard deviation of 192. What is the standard error of Angie's estimator of the mean wordcount for all Greek poems?

9. Consider the mean of a random sample of size 75, \bar{X} . If S^2 is the sample variance and the population is normally distributed with mean μ , what is the distribution of

$$\frac{\bar{X} - \mu}{S/\sqrt{75}}?$$

10. The mean weight of peanuts in a sample of size 16 from a barrel is 0.09 ounces. The standard deviation of the sample is 0.015 ounces. What is a 90% confidence interval for the mean weight of all peanuts in the barrel? Assume peanut weights in the barrel are normally distributed.

11. The weights, in pounds, of a team of sixteen male athletes are as follow:

188.5, 183.0, 194.5, 185.0, 214.0, 203.5, 186.0, 178.5 186.0, 184.5, 204.0, 184.5, 195.5, 202.5, 174.0, 183.0

Assume that this team is representative of all athletes in the league. Test the hypothesis that the mean weight of the league is greater than 190. Assume that athlete weights in the league are normally distributed and use an α level of 0.01.

12. A doctor is performing a clinical trial of a research medication for easing symptoms of emphysema. She recruits 50 patients with the disease and randomly assigns 25 to the treatment group and 25 to the placebo group. After 6 weeks, she measures each patient's blood oxygen level and compares the sample means of the two groups. She calculates a test statistic of 4.38 and a p -value of 0.0001. She concludes that, since $0.0001 < 0.05$, the medication significantly raises blood oxygen levels compared to the placebo. She calculates that the power of her test is 0.7.

What would constitute a type I error in this scenario and what is its probability? What would constitute a type II error in this scenario and what is its probability?