Assignment #7

Due Monday 1 November, 2021 at the start of class

Please read Lectures 12, 13, 14, 15, 16, and 17 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. Then do the following exercises.

P18. *Compare this problem to Exercise 12.1, done in class, but the numbers are not as clean.*

Suppose A is a 100×100 matrix with $\|A\|_2 = 10$ and $\|A\|_F = 11$. Give the sharpest possible lower bound on the 2-norm condition number of A. (*Hint. Write everything in terms of singular values, and then think about best cases for* $\kappa_2(A)$.)

P19. *This problem is comparable to Exercise 13.3, which was done in class.*

Consider the polynomial $p(x) = (x-3)^{10} = x^{10} - 30x^9 + 405x^8 - 3240x^7 + 17010x^6 - 61236x^5 + 153090x^4 - 262440x^3 + 295245x^2 - 196830x + 59049$.

(a) Plot $p(x)$ for $x =$ `2.85:0.01:3.15`, evaluating $p(x)$ via its coefficients $1, -30, 405, \ldots$

(b) Plot $p(x)$ again, now using its expression $(x-3)^{10}$. (*Put it on the same graph.*)

(c) In two or three sentences, compare and contrast the bad behavior here with the ill-conditioning phenomenon in Example 12.5 on page 92, i.e. Wilkinson's example.

P20. *This is a reading assignment. You can answer the questions by scanning the document for answers, but that would be un-sporting. Do the right thing! Actually read it! It's good.*

Please read the following 12 page encyclopedia entry:

> L. N. Trefethen, *Numerical Analysis*, in W. T. Gowers, editor, Princeton Companion to Mathematics, Princeton U. Press, 2008.
> people.maths.ox.ac.uk/trefethen/NAessay.pdf

Answer the following questions with a sentence or two at most:

(i) Give a one-sentence version of Trefethen's definition of "numerical analysis."
(ii) Is analysis of rounding errors the main business of numerical analysis? If not, what is?
(iii) Gaussian elimination with pivoting is a matrix factorization. State it.
(iv) Trefethen refers to Householder triangularization, Algorithm 10.1 in our textbook, as "QR factorization". But then what does the "QR algorithm" do?
(v) Which of the major "algorithmic developments in the history of numerical analysis" have we already covered in MATH 614? Which do you think we will (or should) cover?
(vi) What is the "central dogma" of numerical linear algebra?

Exercise 12.2.

Exercise 13.2. Do parts (a) and (b) only.

Exercise 14.1. Do parts (a), (b), (c), (e), and (f) only.

Exercise 14.2.

Exercise 15.1. Do parts (a), (b), (c), and (d) only.