

Exercise 1: Suppose we know the values from the populations,

Plot	Number of Weeds
1	4
2	8
3	5
4	7
5	5
6	12

Taking a sample consisting of plots 1, 3, 5 and 6,

1. Compute the mean and standard deviation of the population. Is this a parameter or a statistic?

Solution:

Computing the mean of the population,

$$\mu = \frac{4 + 8 + 5 + 7 + 5 + 12}{6} = \frac{41}{6} \approx 6.833$$

Computing the standard deviation of the population,

$$\sigma = \left(\frac{(4 - \mu)^2 + (8 - \mu)^2 + (5 - \mu)^2 + (7 - \mu)^2 + (5 - \mu)^2 + (12 - \mu)^2}{6} \right)^{\frac{1}{2}} = \left(\frac{257}{6} \right)^{\frac{1}{2}} \approx 2.62$$

These are parameters since they are computed with the entire population.

2. Compute the mean and standard deviation of the sample. Is this a parameter or a statistic?

Solution:

Computing the mean of the sample,

$$\bar{x} = \frac{4 + 5 + 5 + 12}{4} = \frac{13}{2} = 6.5$$

Computing the standard deviation,

$$S = \left(\frac{(4 - \bar{x})^2 + (5 - \bar{x})^2 + (5 - \bar{x})^2 + (12 - \bar{x})^2}{4} \right)^{\frac{1}{2}} = \left(\frac{41}{4} \right)^{\frac{1}{2}} \approx 3.69$$

These are statistics since they are computed using only a sample of the population.

Exercise 2: What IS a sampling distribution?

The sampling distribution of a statistic describes the probability of the values that that statistic can take on.

Exercise 3: We are interested in the average number of trees on three small islands. Unknown to us, the numbers are 20 trees, 30 trees, and 25 trees. We can only afford to visit two islands. We use a simple random sample without replacement, so that all samples are equally likely.

1. Write out all possible samples,

Index	Sample
1	20, 30
2	20, 25
3	25, 30

2. Find the sampling distribution of the average sample.

Index	Sample	\hat{x}	$P(\hat{X} = \hat{x})$
1	20, 30	25	1/3
2	20, 25	22.5	1/3
3	25, 30	27.5	1/3

3. Find the expected value of the sampling distribution, is this an unbiased estimator?

$$E(\hat{X}) = \frac{1}{3}(25 + 22.5 + 27.5) = 25$$

$$\mu = \frac{20 + 25 + 30}{3} = 25$$

Since $E(\hat{X}) = \mu$ we know that $E(\hat{X})$ is an unbiased estimator.

$$V(\hat{X}) = \frac{1}{3}(25^2 + 22.5^2 + 27.5^2) - 25^2 \approx 4.166$$

4. What are two desirable properties in an estimator?

A good estimator is unbiased, has the minimal variance, and high resistance to outliers.

Exercise 5: Suppose we have estimated moose populations on two islands. Each island was sampled independently. The estimated count from the first island is $X = 120$ with a standard deviation (standard error) equal to 10. The estimated count from the second island is $Y = 200$ with a standard error of 40. Find an estimate of the total moose on both islands, along with a standard error for this total. Then create a 95 percent confidence interval for the total number of moose on both islands.

Solution:

Consider the following estimator τ for the total number of moose on both islands,

$$E(\tau) = E(X + Y) = E(X) + E(Y) = 120 + 200 = 320$$

Note that the total number of sampling units $N = 2$ so we get the following, Therefore we also know that,

$$V(\tau) = V(X + Y) = V(X) + V(Y) = 10^2 + 40^2 = 1700.$$

Computing the standard error we get that,

$$\tau_{S.E} = \frac{1700^{1/2}}{2} \approx 29.15$$

Finally we get a 95 percent confidence interval of

$$95_{CI} = (320 + 2(29.15), 320 - 2(29.15)) = (378.3, 261.7)$$