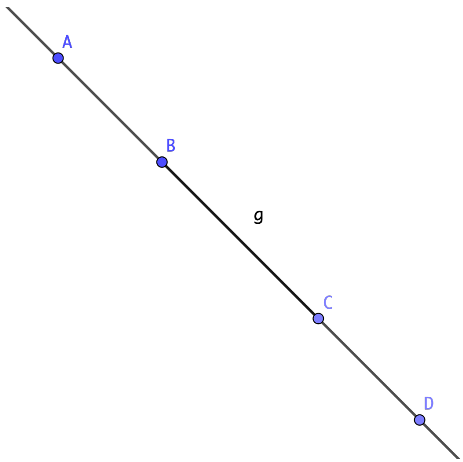
Exercise ($\square$ 4.1.1). Prove that if A, B, C and D are collinear points in that order and $\overline{AB} \cong \overline{CD}$ then $\overline{AC} \cong \overline{BD}$

Proof. Suppose A, B, C and D are collinear points in that order and $\overline{AB} \cong \overline{CD}$,

FIGURE 1. Scene described in 4.1.1



Consider the following, by the Sum of Lengths axiom,

$$||\overline{AB}|| + ||\overline{BC}|| = ||\overline{AC}||$$

$$||\overline{BC}|| + ||\overline{CD}|| = ||\overline{BD}||$$

Solving for $||\overline{AB}||$ and $||\overline{CD}||$

$$||\overline{AC}|| - ||\overline{BC}|| = ||\overline{AB}||$$

$$||\overline{BD}|| - ||\overline{BC}|| = ||\overline{CD}||$$

By congruence we know,

$$||\overline{AB}|| = ||\overline{CD}||$$

By Substitution,

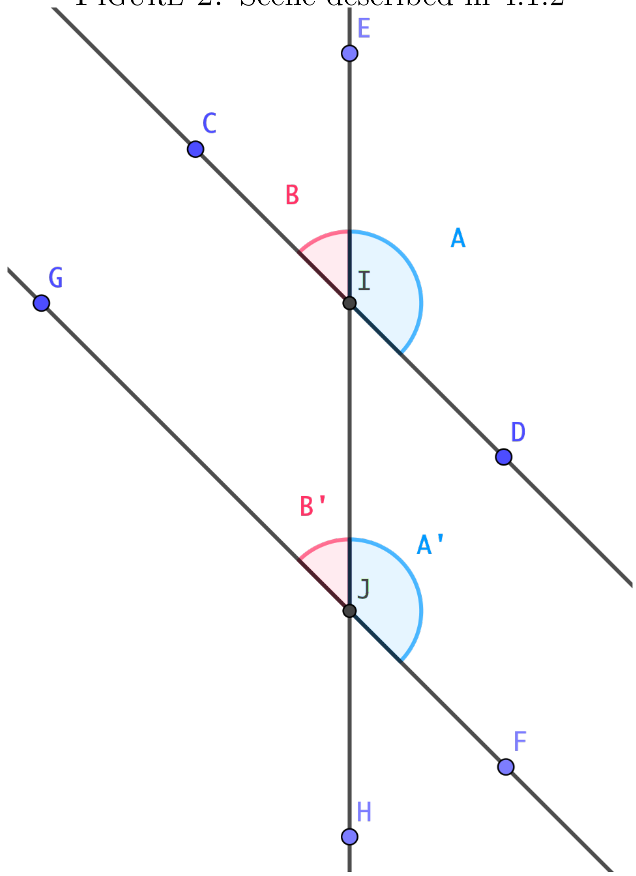
$$||\overline{AC}|| - ||\overline{BC}| = ||\overline{BD}|| - ||\overline{BC}||$$

$$||\overline{AC}|| = ||\overline{BD}||$$

Thus by measurement $\overline{AC} \cong \overline{BD}$. $\square$

Exercise (☐ 4.1.2). Prove that if a pair of parallel lines is cut by a transversal, the corresponding angles are congruent

FIGURE 2. Scene described in 4.1.2



Proof. [Answer:] (Contradiction) Suppose that a pair of parallel lines is cut by a transversal, and the corresponding angles are not congruent. Since the corresponding angles are not congruent we know that $\angle EID \neq \angle EJF$.

Case 1: Suppose $||\angle EID|| > ||\angle EJF||$. We know that,

$$||\angle EID|| + ||\angle HID|| = 180^o.$$

By substitution we know that,

$$||\angle EJF|| + ||\angle HID|| < 180^o.$$

By Euclid's parallel postulate

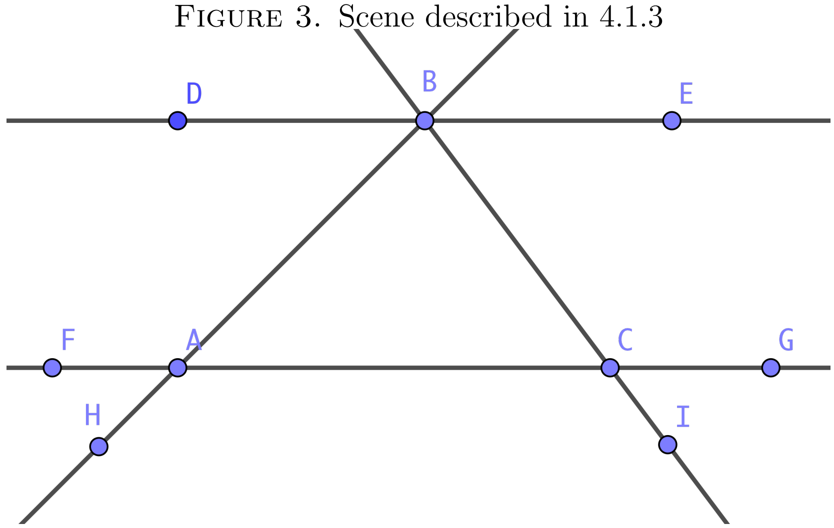
If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on the side on which the angles sum to less than two right angles. (p.48)

Since $\angle EJF$ and $\angle HID$ are two interior angles on the same side of the transversal that sum to less than 180^o we know that the two lines $\overline{GF}$ and $\overline{CD}$ must intersect on the same side of the transversal. Thus $\overline{GF}$ and $\overline{CD}$ are both parallel and not parallel.

$\square$

Exercise ($\square$ 4.1.3). Prove that the measurements of three interior angles of a triangle sum to 180^o

Figure 3. Scene described in 4.1.3



Proof. Consider $\triangle ABC$. Now construct a line that is parallel to one side of the triangle and also incident to one point of the triangle. Using our diagram we can say that $\overline{DE}||\overline{AC}$. Note that the lines $\overline{AB}$ and $\overline{CB}$ are by definition, transversal lines. We have just proved that the corresponding angles of a pair of lines cut by a transversal are congruent, therefore we can make the following claims,

$$||\angle ABE|| = ||\angle HAC||$$

$$||\angle CBD|| = ||\angle ICA||$$

We can also make the following claims because we know that the sum of all the angles on a single side of a line must add up to 180^o,

$$||\angle ABD|| + ||\angle ABC|| + ||\angle CBE|| = 180^o$$

$$||\angle HAC|| + ||\angle CAB|| = 180^o$$

$$||\angle ICA|| + ||\angle BCA|| = 180^o$$

With all these claims, WTS that $||\angle CAB|| = ||\angle ABD||$.
Note,

$$180^o - ||\angle ABD|| = ||\angle ABE||$$

Also note,

$$180^o - ||\angle CAB|| = ||\angle HAC||$$

Since $||\angle ABE|| = ||\angle HAC||$ we know that by cancellation,

$$||\angle CAB|| = ||\angle ABD||$$

By a similar argument WTS that $||\angle BCA|| = ||\angle CBE||$.
Note,

$$180^o - ||\angle CBE|| = ||\angle CBD||$$

Also note,

$$180^o - ||\angle BCA|| = ||\angle ICA||$$

Since $||\angle CBD|| = ||\angle ICA||$ we know that by cancellation,

$$||\angle BCA|| = ||\angle CBE||$$

Thus by substitution into $||\angle ABD|| + ||\angle ABC|| + ||\angle CBE|| = 180^o$ it must be true that,

$$||\angle CAB|| + ||\angle ABC|| + ||\angle BCA|| = 180^o$$

$\square$