Exercise (□.2.1). In this problem, we consider distinct pencils L and K in $\mathbb{R}^3$. Give a true statement about $L \cap K$, the set of lines contained in both pencils, and then prove your statement. Your statement and proof might include two more more cases.

Statement: If L and K are distinct pencils in $\mathbb{R}^3$ defined by different properties then $|L \cap K|$ can be at most 1.

Proof. Suppose two distinct pencils L and K in $\mathbb{R}^3$ and L and K are defined as pencils by different properties, without loss of generality letters say that K is a set of parallel lines and L is a set of lines which intersect at a single point. By our definition of K we know that each line in K can be described by the same direction vector, and by our definition of L we know that each line in L is described by a unique direction vector. Note since all lines in K are described by one direction vector, and all lines in L have unique direction vectors, there can only ever be at most one line in $L \cap K$.

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