(1) (Problem □6.1)

If a full mesh $\mathcal{M}$ is not empty, then $\mathcal{M}$ contains at least six elements.

Answer: Suppose M is a full mesh. Note that M must have at least 3 points because it is a mesh, this was proven in page 78 problems 3 and 4. Since M is defined as a full mesh, for every pair of distinct points P,Q the line $\overline{PQ}$ must also be in the mesh. It then follows that for each distinct pair of points there must also exist a distinct line. Counting the pairs for the smallest case full mesh,

$$\binom{3}{2} = 3. \tag{1}$$

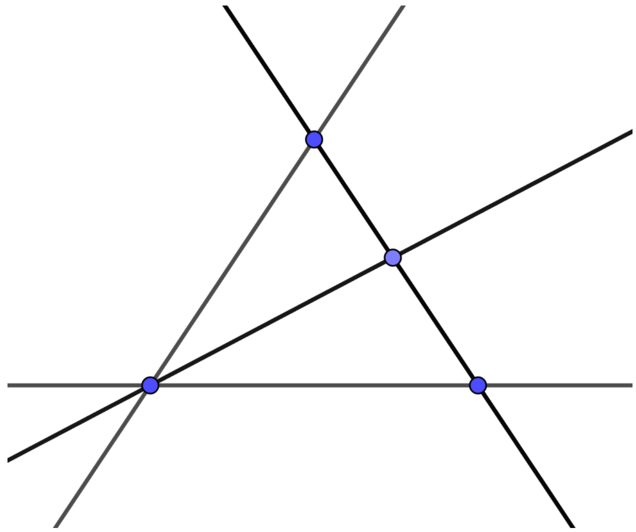
Therefore M must also have at least 3 lines. Thus M must have at least six elements.

(2) (Problem □6.2)

If a full mesh $\mathcal{M}$ contains at least four points, then $\mathcal{M}$ contains infinitely many elements.

Answer: Consider the following full mesh with 4 points, and 4 lines.

FIGURE 1.

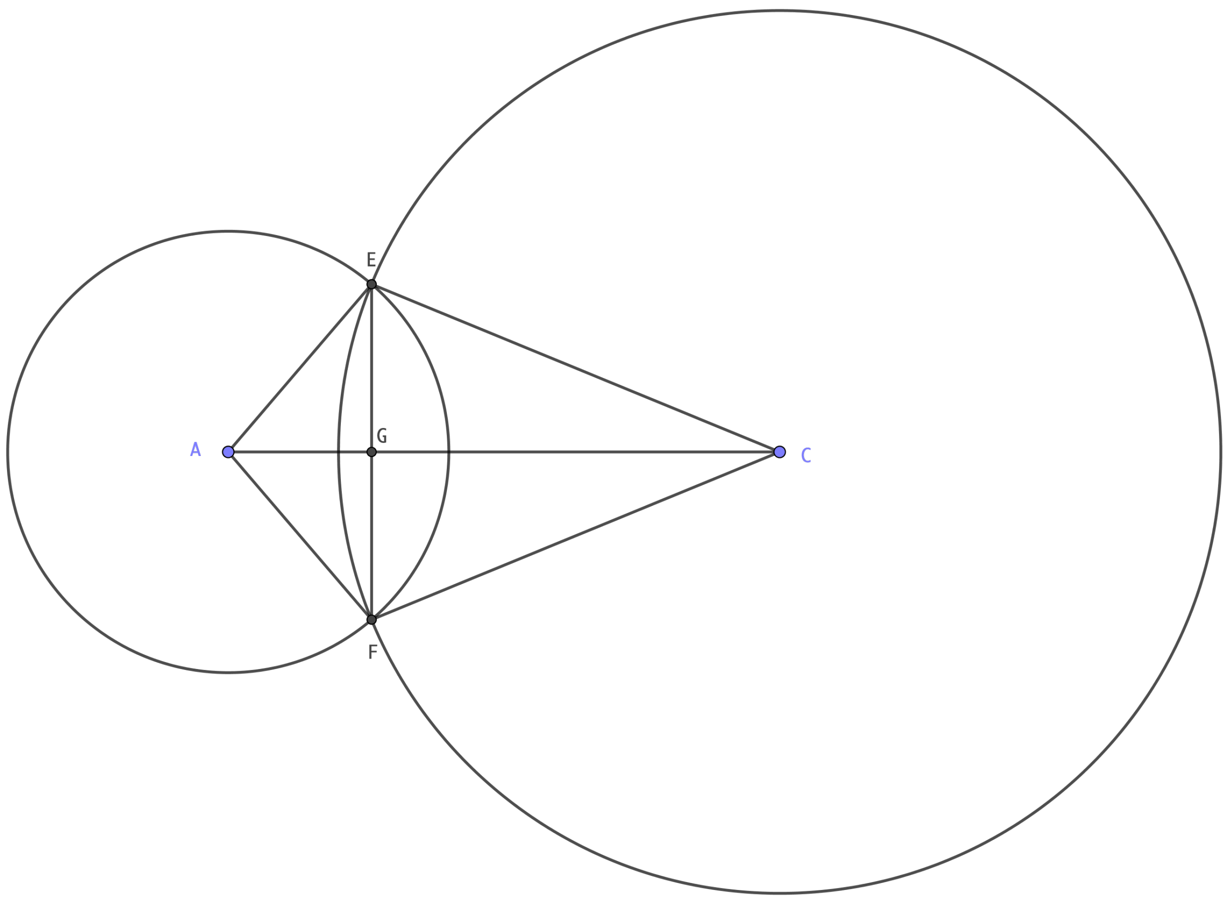


(3) (Page 23 #8)

If two circles intersect, the straight line which joins their centers is the right bisector of their common chord. (Note, a circle is the set of points equidistant from a specified point called the centre.)

Answer: Let 2 circles with center points A and C intersect with common chord $\overline{EF}$. Let point G be the intersection of $\overline{AC}$ and $\overline{EF}$. Consider the following figure.

FIGURE 2.



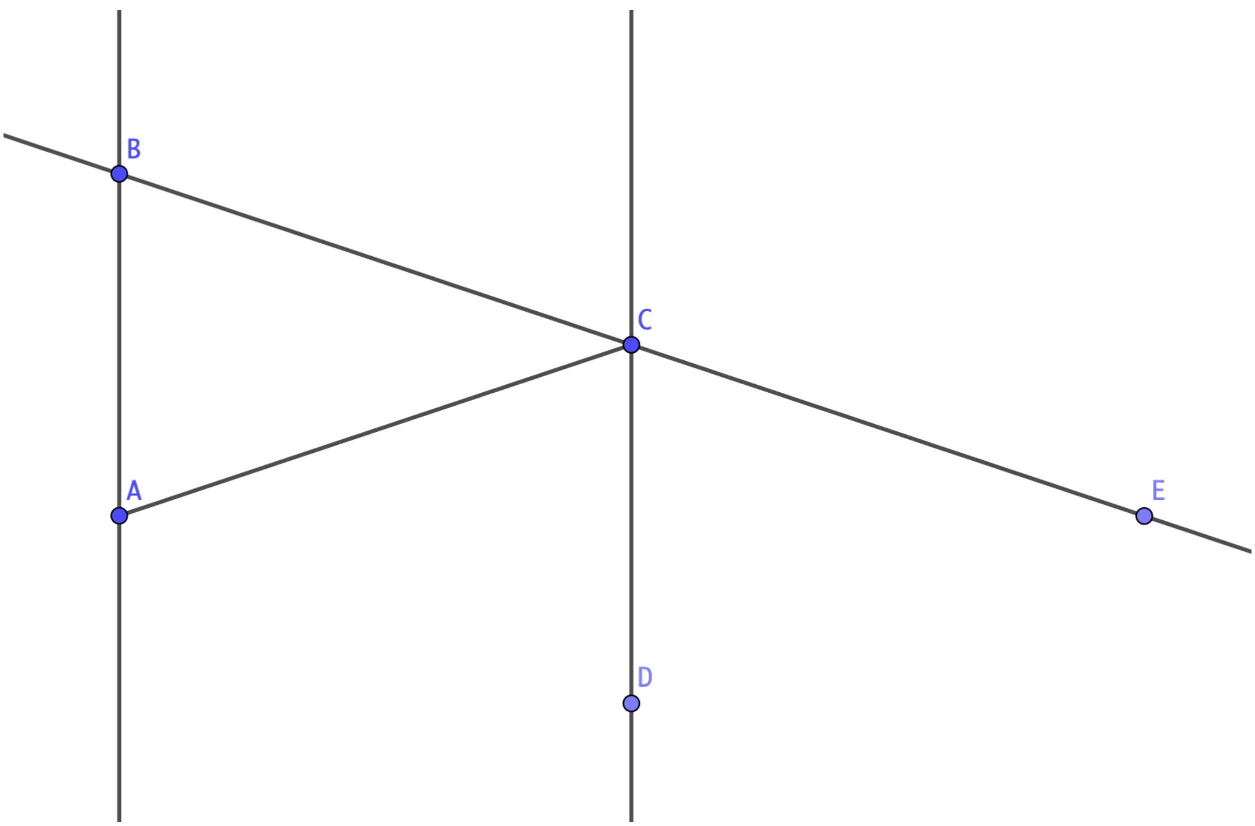
Since lines $\overline{AE}$ and $\overline{AF}$ are radii of circle we know that $\overline{AE} \cong \overline{AF}$ and similarly $\overline{CE} \cong \overline{CF}$. Thus it follows that $\triangle AEC \cong \triangle AFC$ by SSS which gives us that $\angle GAF \cong \angle GAE$. It then follows that $\triangle GAE \cong \triangle GAF$ by SAS. Therefore $\angle EGA = \angle FGA$. We know that $\angle EGA = \angle FGA = \angle EGC = \angle FGC$ by Vertical Angle Theorem. The only way to have 4 equivalent angles divide 360° is if they all equal 90°.

(4) (Page 38 #1)

If the bisector of an exterior angle of a triangle is parallel to the opposite side, the triangle must have two of its angles equal.

Answer: Consider triangle $\triangle ABC$ with exterior angle $\angle ACE$ such that $\overline{CD}$ bisects $\angle ACE$. Also suppose that $\overline{CD}$ is parallel to $\overline{BA}$.

Figure 3.



Since $\overline{CD}$ is a bisector of angle $\angle ACE$ we know that angles $\angle ACD = \angle DCE$. Since $\overline{CD}$ is parallel to $\overline{BA}$ with transversal $\overline{AC}$ we know by alternate interior angles, that $\angle BAC = \angle ACD$. Considering the same set of parallel lines but with transversal

$\overline{BE}$ we know by corresponding angles that $\angle ABC = \angle DCE$. Through substitution,

$$\angle ACD = \angle DCE, \tag{2}$$

$$\angle BAC = \angle ABC. \tag{3}$$

Thus $\triangle ABC$ has two equal angles.