

## Section 4.5:

### Exercise 4.5.19:

- (1) The number of pivot columns of a matrix equals the dimension of its column space.

**Answer:** True. By Theorem 6 which says that the pivot columns of a matrix  $A$  form a basis for the  $\text{Col}(A)$ . Therefore it must be true that the number of pivot columns is equal to the number of pivots.

- (2) A plane in  $\mathbb{R}^3$  is a two-dimensional subspace of  $\mathbb{R}^3$ .

**Answer:** False. Consider a plane that does not go through the origin, It cannot be a subspace.

- (3) The dimension of vector space  $\mathcal{P}_4$  is 4.

**Answer:** False.  $\mathcal{P}_4$  is all fourth degree polynomials, which have 5 coefficients. Therefore the dimension of  $\mathcal{P}_4$  is 5.

- (4) If the  $\dim V = n$  and  $S$  is linearly independent set in  $V$ , then  $S$  is a basis for  $V$ .

**Answer:** False. It is possible that the set  $S$  does not span  $V$  therefore it is not always a basis. Consider any  $V = \mathbb{R}^2$  and  $S = [1, 1]$ ,  $S$  is a linearly independent set that contains one vector, but It doesn't span  $\mathbb{R}^2$  therefore it is not a basis.

- (5) If a set  $\{v_1, \dots, v_p\}$  spans a finite-dimensional vector space  $V$  and if  $T$  is a set of more than  $p$  vectors in  $V$ , then  $T$  is linearly dependent.

**Answer:** True. By spanning set theorem.

**Exercise 4.5.22:** The first four Laguerre polynomials are  $1$ ,  $1 - t$ ,  $2 - 4t + t^2$ , and  $6 - 18t + 9t^2 - t^3$   
**Solution:**

**Exercise 4.5.24:** **Solution:**

## Section 4.6:

**Exercise 4.6.17:** **Solution:**

**Exercise 4.6.20:** **Solution:**

**Exercise 4.6.28:** **Solution:**

## Section 4.7:

**Exercise 4.7.7:** **Solution:**

**Exercise 4.7.9:** **Solution:**

**Exercise 4.7.14: Solution:**

**Exercise 4.7.16: Solution:**