Text 8.10: Let f be the function satisfying $f(0) = 1$, $f(1) = 2$, and $f(2) = 0$. A quadratic spline interpolant $r(x)$ is defines as a piecewise quadratic that interpolates f at the nodes $(x_0 = 0, x_1 = 1, x_2 = 2)$ and whose first derivative is continuous throughout the interval. Find the quadratic spline interpolant of f that also satisfies $r'(0) = 0$.

Solution:
Note that since $r(x)$ is a piecewise quadratic interpolant on nodes $(x_0 = 0, x_1 = 1, x_2 = 2)$, it must be of the form,

$$r(x) = \begin{cases} a_0x^2 + b_0x + c_0 & 0 \leq x \leq 1 \\ a_1x^2 + b_1x + c_1 & 1 < x \leq 2 \end{cases}$$

Note that our interpolant has the property that $r(x) = f(x)$ and $r'(x) = f'(x)$ on our sample points there fore we get the following system of equations,

$$a_0(0)^2 + b_0(0) + c_0(1) = 1,$$

$$a_0(1)^2 + b_0(1) + c_0(1) = 1,$$

$$a_0 2(0) + b_0(1) + c_0(0) = 0.$$

Solving we get that $a_0 = 1, b_0 = 0, c_0 = 1$, which gives us the function,

$$r(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 1 \end{cases}$$

Differentiating we get $r'(1) = 2$. Setting up a new system to solve a_1, b_1, c_1.

$$a_1(1)^2 + b_1(1) + c_1(1) = 2,$$

$$a_1(2)^2 + b_1(2) + c_1(1) = 0,$$

$$a_1 2(1) + b_1(1) + c_1(0) = 2.$$

Solving we get that $a_1 = -4, b_1 = 10, c_1 = -4$. Therefore we get,

$$r(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 1 \\ -4x^2 + 10x - 4 & 1 < x \leq 2 \end{cases}$$

Text 8.12: Show that the following function is a natural cubic spline through the points $(0, 1)$, $(1, 1)$, $(2, 0)$, and $(3, 10)$:

$$s(x) = \begin{cases} 1 + x - x^3 & 0 \leq x < 1 \\ 1 - 2(x-1) - 3(x-1)^2 + 4(x-1)^3 & 1 \leq x < 2 \\ 4(x-2) + 9(x-2)^2 - 3(x-2)^3 & 2 \leq x \leq 3 \end{cases}$$

Text 8.13:

Text 8.14:

Text 10.1:

Text 10.2: