

**Supplemental 1:** Finish your code from question 19 of the worksheet on implementing LU decomposition with partial pivoting. Then use this code together with the course page `lsolve` and your own `usolve` to solve  $A\mathbf{x} = \mathbf{b}$  where

$$A = \begin{pmatrix} 9 & 3 & 2 & 0 & 7 \\ 7 & 6 & 9 & 6 & 4 \\ 2 & 7 & 7 & 8 & 2 \\ 0 & 9 & 7 & 2 & 2 \\ 7 & 3 & 6 & 4 & 3 \end{pmatrix}$$

and  $\mathbf{b} = [35, 58, 53, 37, 39]^T$ . For the record, the true solution is  $\mathbf{x} = [0, 1, 2, 3, 4]^T$ .

**Solution:**

The following is a modified LUPivot Function that takes an augmented matrix in the form of the matrix  $A$  and the vector  $\mathbf{b}$  and returns the pivoted  $LU$  factorization as well as the pivoted vector  $\mathbf{b}'$  where  $P\mathbf{b} = \mathbf{b}'$ . The `Usolve` and `LSolve` function remain unchanged from last week.

**Code:**

```
function [L,A, b] = LUPivot(A, b)
%This function takes an NxN matrix A and vector b and returns an LU
%factorization and permuted b.

n = size(A,2);
L = zeros(n);

for k = 1:n %Initializes the diagonal of L
    L(k,k) = 1;
end

P = zeros(1,n);
for i = 1:n-1 % Iterates through columns of A

    %U pivot
    [M, I] = max(A(i,:));
    tmp = A(i,:);
    A(i,:) = A(I,:);
    A(I,:) = tmp;

    % Vector pivot
    tmp = b(i,:);
```

```

    b(i,:) = b(I,:);
    b(I,:) = tmp;

    %L Pivot
    tmp = L(i, 1:i-1);
    L(i, 1:i-1) = L(I, 1:i-1);
    L(I, 1:i-1) = tmp;

    for j = i+1:n % Iterates through Rows of A

        x = A(j,i)/A(i,i); %Calculates factor for Gauss Elim

        L(j,i) = x; %Stores factor in L

        for k = 1:n % Iterates through current row and performs
                    %Gaussian Elim step
            A(j,k) = A(j,k) - (A(i,k)*x);
        end

    end

end
end

```

Finally we can write a function that solves a matrix equation using *LU* factorization with partial pivoting,

### Code:

```

function x = PivotSolve(A, b)
%This function solves a linear system where A
%is NxN

[L,U,z] = LUPivot(A, b);

y = lsolve(L,z);

x = usolve(U,y);

```

### Console

```
>> A = [9 3 2 0 7; 7 6 9 6 4; 2 7 7 8 2; 0 9 7 2 2; 7 3 6 4 3]
```

```
A =
```

```

    9    3    2    0    7
    7    6    9    6    4
    2    7    7    8    2
    0    9    7    2    2
    7    3    6    4    3
```

```
>> b = [35, 58, 53, 37, 39]
```

```
b =
```

```

    35    58    53    37    39
```

```
>> PivotSolve(A,b')
```

```
ans =
```

```

    0.0000    1.0000    2.0000    3.0000    4.0000
```

**Exercise 7.8:** Compute the 2-norm, the 1-norm and the  $\infty$ -norm of,

$$v = \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix}$$

**Solution:**

By definition we can compute all vector norms easily,

$$\|v\|_2 = \sqrt{4^2 + 5^2 + (-6)^2} = \sqrt{77}$$

$$\|v\|_1 = |4| + |5| + |-6| = 15$$

$$\|v\|_\infty = \max\{|v|\} = 6$$