Computational Physics Laboratory report

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Contents

1 Error analysis

1.0.1 Approximating exponentials

Results

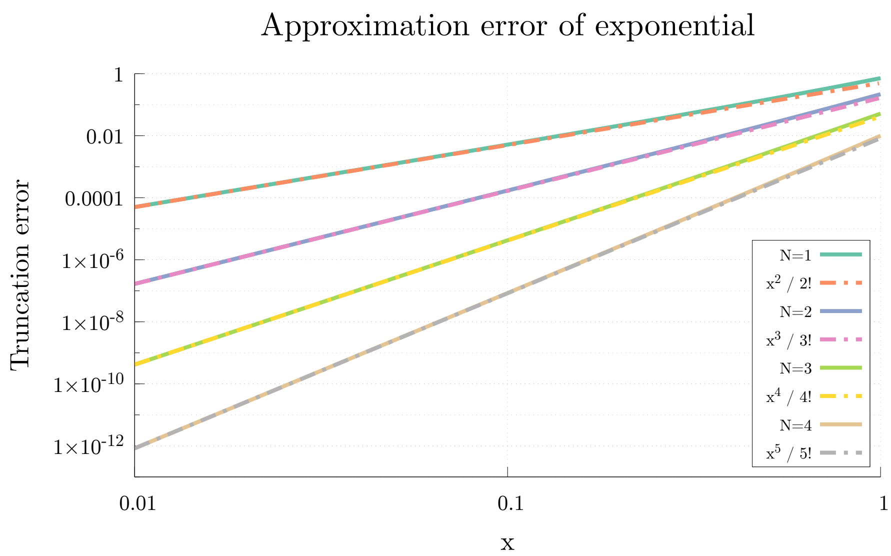

Figure 1: Truncation errors when approximating e^x through a truncated power series. N is the degree of the polinomyal used for the approximation. The plot of $x^{N+1}/(N+1)$ has been added for reference.

Remarks

- Horner's method was used in polynomial evaluation, because it is faster and reduces roundoff errors. A naive implementation of a polynomial computation requires more operations ($O(N^2)$) and, for small values of x, adds together numbers of order 1 and x^N, while Horner's method is $O(N)$ and keeps the additions only between terms of order 1 and x.

- By Taylor's theorem, if $\hat{f}(x)$ is the power series of e^x truncated after the N-th term, the error is of order $O(x^{N+1})$ as x approaches 0. Computing an additional term of the series shows that:

$$|e^x - \hat{f}(x)| = \frac{x^{N+1}}{(N+1)!} + o(x^{N+1})$$

  It is then not surprising that the approximation error in Figure 1 is indistinguishable from $x^{N+1}/(N+1)$ for small values of x. For larger values, the contribution of higher order terms in the series becomes ever more important, and hence the divergence from $x^{N+1}/(N+1)$.

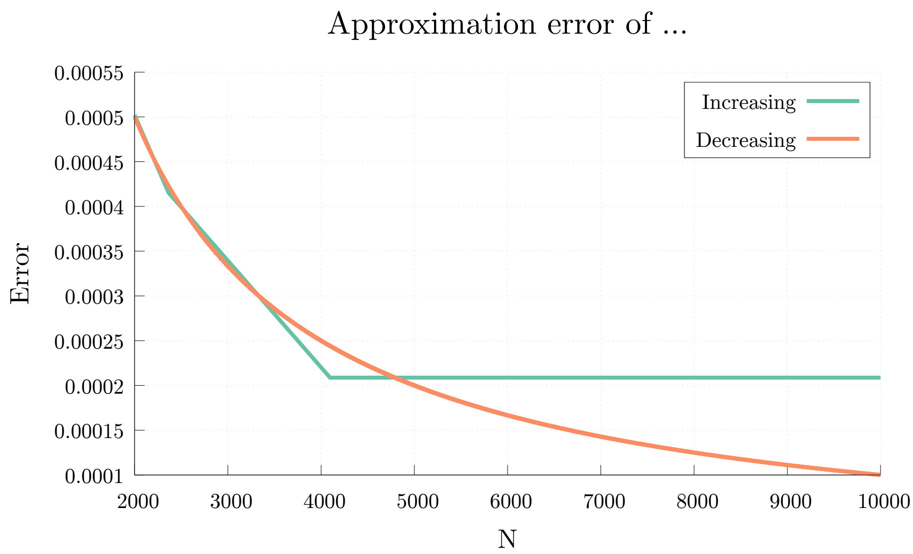

Figure 2:

1.2 Floating-point arithmetic and roundoff errors

1.2.1 Computing the Basel problem

1.3 Error propagation and condition number

1.3.1 Computing statistical momenta

1.3.2 Condition number: study of a simple algorithm

2 Linear systems

test text

2.1 Forward- and back-substitution

2.2 LUP Decomposition

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2.3

3 Interpolation

4 Roots of nonlinear equations

5 Numerical integration

5.1 Newton-Cotes formula

5.1.1 Trapezoidal rule

test

5.1.2 Simpson's rule

test

5.2 Free-nodes integration

5.2.1 Nodes and weights of Gauss-Legendre rule

something

Remarks

5.2.2 Integrals with Gauss-Legendre rule

5.3 Advanced topics in integration

6 Ordinary differential equations