TASK: PROOF THAT GIVEN (x, λ) and (z, μ) IF $\lambda \neq \mu \Rightarrow \langle x, z \rangle = 0$

$$\lambda \langle x, z \rangle \underset{\text{bilinearity}}{=} \langle \lambda x, z \rangle \underset{\substack{\text{eigenvalue}\\\text{eigenvector}}}{=} \langle Mx, z \rangle \underset{\substack{\text{dot}\\\text{product}}}{=} (Mx)^T z =$$

$$\underset{\text{transpose}}{=} x^T M^T z \underset{\substack{M \text{ is}\\\text{symmetric}}}{=} x^T M z \underset{\substack{\text{eigenvalue}\\\text{eigenvector}}}{=} x^T \mu z = \mu x^T z \underset{\substack{\text{dot}\\\text{product}}}{=} \mu \langle x, z \rangle$$

- $\lambda \langle x, z \rangle = \mu \langle x, z \rangle$ SINCE $\lambda \neq \mu$ THEN IT MUST BE:

$\langle x, z \rangle = 0$ ∎